AARHUS UNIVERSITET

BROWNIAN MOTION

AND

CLASSICAL POTENTIAL THEORY

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Lecture Notes Series
NO. 47.

MATEMATISK INSTITUT

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P R E F A C E

In these notes we present some basic classical potential theory. Although language of Brownian motion is used the discussion is more analytic than probabilistic.

Chapters 4 to 7 describe the principal results whereas the first three chapters are of a more or less general character. Ample hints are provided for almost all the exercises.

Senior under graduate level mathematics is necessary prerequisites for reading this set of notes.

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CONTENTS

CHAPTER 1

Martingales

§1 Optional sampling, inequalities, and convergence

We will not be needing too much martingale theory. What little is done is to make the notes more complete. In the following we assume given a fixed probability space $(\Omega, \mathbb{B}, P)$. All σ-fields considered are <u>sub-σ-fields of $\mathbb{B}$</u>. In some of the examples some knowledge of the relevant concepts is needed.

Let $F_0, F_1, \ldots$ be a (finite or infinite) sequence of σ-fields and $X_0, X_1, \ldots$ a sequence of random variables. $\{X_i\}$ is said to be <u>adapted</u> to F_i if X_i is F_i-measurable. An integer valued (possibly ∞) random variable T is called a <u>stopping time</u> relative to F_i if $(T = i) \in F_i$ for all i. For a stopping time T, the σ-field F_T consists of the events A for which $A \cap (T = i) \in F_i$ for all i. If $F_0 \subset F_1 \subset \ldots$ is increasing and $T \geq S$ are stopping times $F_T \supset F_S$; if F_i are decreasing and $T \geq S$ are stopping times $F_T \subset F_S$. These are easily verified. <u>Until further notice F_i will be an increasing sequence of σ-fields</u>.

A sequence $\{X_i\}$ of random variables having expectations and adapted to $\{F_i\}$ is called a <u>super martingale</u> if

$$E[X_{i+1} \mid F_i] \leq X_i, \quad i = 0, 1, 2, \ldots, \tag{1}$$