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Problems

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In an attempt to add to the gaiety of nations, I am offering cash prizes for most of the questions below. The prizes, indicated by the formulae '(£n)' following questions, are not for solving the problems but for communicating the solutions (whether yours or others') to me. They will be paid in cheques drawn on a UK bank. I may pay out part of a prize for a partial solution or a consistency result if it seems to me to be a substantial step forward.

David Fremlin

Notation Most terms may be found in Fremlin 74 or Fremlin 84; see also Fremlin 02 and Fremlin 03. $\mathfrak{m}$ is the least cardinal for which MA($\mathfrak{m}$) is false; thus '$\mathfrak{m} = \mathfrak{c}$' is Martin's axiom. $\mathfrak{p} = \mathfrak{m}_{\sigma\text{-centered}}$ is the least cardinal for which $\mathrm{MA}_{\sigma\text{-centered}}(\mathfrak{p})$ is false. $\mathfrak{b}$ is the least cardinal of any subset A of $\mathbb{N}^{\mathbb{N}}$ such that there is no $g \in \mathbb{N}^{\mathbb{N}}$ for which $\{n : f(n) > g(n)\}$ is finite for every $f \in A$.

For problems marked † you can find notes in `https://www1.essex.ac.uk/maths/staff/fremlin/problems.htm`.

D. Let (X, Σ, μ) be a measure space. A **lifting** of μ is a Boolean homomorphism $\theta : \Sigma \to \Sigma$ such that $\theta E = \emptyset$ whenever $\mu E = 0$ and $\mu(E \triangle \theta E) = 0$ for every $E \in \Sigma$. It is known that (a) every complete probability space has a lifting (see Fremlin 02, §341) (b) subject to the continuum hypothesis, every probability space of Maharam type at most $\mathfrak{c}^+$ has a lifting (see Fremlin 08, §535) (c) it is possible (that is, relatively consistent with ZFC) that the restriction of Lebesgue measure to the Borel σ-algebra of $[0, 1]$ has no lifting (see Shelah 83 or Just 92 or Burke 93).

Problem Must there be (that is, is it a theorem of ZFC) that there is a probability space without a lifting? (£33.)

Remark By Maharam's theorem, it is enough to consider $\nu_\kappa \restriction \Sigma$ where κ is a cardinal, Σ is the Baire σ-algebra of $\{0, 1\}^\kappa$ and ν_κ is the usual measure on $\{0, 1\}^\kappa$. (The **Baire σ-algebra** of a topological space is the smallest σ-algebra with respect to which every continuous function is measurable; for a compact zero-dimensional space, as in this case, it is the σ-algebra generated by the open-and-closed sets.) What about $\kappa = \omega_3$? (£25.)

See also **DO, EJ** below.

F. Let K be a compact convex set in a (non-locally-convex) Hausdorff linear topological space E, and U a neighbourhood of 0 in E. Is there necessarily a continuous function $T : E \to K$ such that $Tx - x \in U$ for every $x \in K$? What if $E = L^0([0, 1])$? (£2.)

H. Let E be a Banach lattice which is a symmetric sequence space (i.e., a solid linear subspace of $\mathbb{R}^{\mathbb{N}}$ which is invariant under permutations of $\mathbb{N}$). Must E have the approximation property?

R. (W.Moran) Can it be that

$$\lim_{n \to \mathcal{F}} E_n = \{\alpha : \{n : \alpha \in E_n\} \in \mathcal{F}\}$$

is negligible whenever $\langle E_n \rangle_{n \in \mathbb{N}}$ is a stochastically independent sequence of measurable subsets of $[0, 1]$ such that $\lim_{n \to \infty} \mu E_n = 0$, and $\mathcal{F}$ is a non-principal ultrafilter on $\mathbb{N}$? (£40.) (If $\mathfrak{m} = \mathfrak{c}$, no; see Fremlin & Talagrand 79 and Talagrand 84, 9-1-3. Also Shelah & Fremlin 93.)

W†. (L.Schwartz) A **Borel measure** on a topological space X is a measure with domain the Borel σ-algebra of X. A Hausdorff topological space X is **Radon** if every totally finite Borel measure on X is inner regular with respect to the compact sets. Can it be that the product of two Radon spaces is always Radon? (£30.) (If *either* there is an atomlessly-measurable cardinal *or* $\mathfrak{m} = \mathfrak{c}$, no; see Wage 80 and Fremlin n82a.)

See also **DZ**, **FM** below.

X. Let E be a Dedekind complete Banach lattice for which the norm is Fatou and the norm topology is Levi (i.e., if A is a non-empty upwards-directed subset of the unit ball of E, then $\sup A$ exists in E and $\|\sup A\| \le 1$). Does it follow that there is a positive contractive projection from the bidual E^{**} of E onto E? (£8.) (See SCHAEFER 74, §IV.4.)

Z†. Assume that $\mathfrak{m} > \omega_1$. Give $\mathbb{R}^{[0,1]}$ the topology $\mathfrak{T}_p$ of pointwise convergence, and write $\mathcal{L}^0(\mathcal{B})$ for the subset of $\mathbb{R}^{[0,1]}$ consisting of Borel measurable functions. For $K \subseteq \mathbb{R}^{[0,1]}$ write $\overline{\Gamma(K)}$ for its $\mathfrak{T}_p$-closed convex hull in $\mathbb{R}^{[0,1]}$. Is $\overline{\Gamma(K)}$ a subset of $\mathcal{L}^0(\mathcal{B})$ whenever $K \subseteq \mathcal{L}^0(\mathcal{B})$ is $\mathfrak{T}_p$-compact? (£8.) (If $\mathfrak{c} = \omega_1$, no; see TALAGRAND 84, 10-1-1, or FREMLIN 03, 463Ye. If there is a real-valued-measurable cardinal, yes; see FREMLIN N95.)

AB†. If E is a Riesz space, a norm $\| \, \|$ on E is **weakly Fatou** if it is a Riesz norm (that is, $\|x\| \le \|y\|$ whenever $|x| \le |y|$) and there is a constant $\alpha \ge 0$ such that $\|x\| \le \alpha \sup_{y \in A} \|y\|$ whenever A is a non-empty upwards-directed subset of E with supremum x. Many of the results proved in FREMLIN 74 for Fatou norms are true also for weakly Fatou norms. There are Banach lattices with weakly Fatou norms which have no equivalent Fatou norms (ELLIOTT P18). But is there a weakly Fatou norm on $C([0,1])$ which is not equivalent to any Fatou norm? (£9.)

AC†. (A.Bellow) Can there be a probability space X and a set K of real-valued measurable functions on X such that (i) K is compact for the topology $\mathfrak{T}_p$ of pointwise convergence inherited from $\mathbb{R}^X$ (ii) K is separated by the measure, that is, if two functions in K agree almost everywhere they are identical (iii) K is not metrizable for $\mathfrak{T}_p$? (£40.) (If $\mathfrak{m} = \mathfrak{c}$, no; see FREMLIN 08, §536.) What if X is a completely regular Hausdorff space, K consists only of continuous functions, and μ is a Borel measure which is **τ-additive**, that is, $\mu(\bigcup \mathcal{G}) = \sup_{G \in \mathcal{G}} \mu G$ whenever $\mathcal{G}$ is an upwards-directed family of open sets? (£14.)

AD. (J.Bourgain & F.Delbaen.) Give $\mathbb{R}^{[0,1]}$ the topology $\mathfrak{T}_p$ of pointwise convergence, and write $\mathcal{L}^0(\Sigma)$ for the subset of $\mathbb{R}^{[0,1]}$ consisting of Lebesgue measurable functions. For $A \subseteq \mathbb{R}^{[0,1]}$ write $\overline{A}$ for its $\mathfrak{T}_p$-closure in $\mathbb{R}^{[0,1]}$. Can there be a countable set A such that (i) $\overline{A}$ is $\mathfrak{T}_p$-compact (ii) $\overline{A} \subseteq \mathcal{L}^0(\Sigma)$ (iii) there is a function in $\overline{A}$ which is not the limit almost everywhere of a sequence in A? (£21.) (If $\mathfrak{m} = \mathfrak{c}$, no; see TALAGRAND 84, 9-5-2. If $\overline{A}$ is countably tight, no; see FREMLIN 03, 463E.) What if $\overline{A}$ is convex? (£19.) (See SHELAH & FREMLIN 93.)

An equivalent form of the problem (if $\overline{A}$ is not assumed to be convex) is: Can there be a sequence $\langle E_n \rangle_{n \in \mathbb{N}}$ of measurable subsets of $[0,1]$ such that (i) $\lim_{n \to \mathcal{F}} E_n$ is measurable for every ultrafilter $\mathcal{F}$ on $\mathbb{N}$ (see **R** above) (ii) every finite subset of $[0,1]$ is included in some E_n (iii) $\lim_{n \to \infty} \mu_L E_n = 0$?

AM. (Sierpiński.) Is there a Borel subset of the plane which meets every straight line in exactly two points? (£34.) (See MAULDIN 98.)

AT. [Efimov] Must there be an infinite compact Hausdorff space K such that neither $\beta\mathbb{N}$ nor $\omega + 1$ (with its order topology) can be embedded homeomorphically as a subset of K? (£23.) (If $\mathrm{cf}[\mathfrak{s}]^{\le\omega} = \mathfrak{s}$ and $2^{\mathfrak{s}} < 2^{\mathfrak{c}}$, yes; see DOW 05. If $\mathfrak{b} = \mathfrak{c}$, yes; see DOW & SHELAH 13. In random real models, yes; see DOW & FREMLIN 07.)

BC. Let $F \subseteq \mathbb{R}$ be an F_σ set and $G \subseteq \mathbb{R}$ a G_δ set. Write $\mathcal{I}$ for the σ-ideal of subsets of $E = F \cup G$ generated by the compact subsets of E. Is $\mathcal{I}$ necessarily $\mathfrak{m}$-additive? (£9.) (See ENGELEN KUNEN & MILLER 89.)

BH. A Radon measure μ on a compact Hausdorff space X is **uniformly regular** if there are a compact metric space Z and a continuous function $f : X \to Z$ such that $\mu f^{-1}[f[F]] = \mu F$ for every closed $F \subseteq X$. Assuming that $\mathfrak{m} > \omega_1$, does every Radon measure on a first-countable compact Hausdorff space have to be uniformly regular? (£32.) (See FREMLIN 84, 32L-N, PLEBANEK 00 and FREMLIN 08, §533.)

BQ. Assume that $\mathfrak{m} = \mathfrak{c} = \omega_2$.

(a) Must there be a metric space X such that every separable subset of X is Borel, but not every subset of X is Borel? (If $\diamondsuit_{\mathfrak{c}}(E)$ is true (DANIELS & GRUENHAGE 87, B2Hb), yes.) (£2.)

(b) Can it be that whenever X is a metric space with Borel sets of every class $< \omega_1$, then X has a separable subset of cardinal $\mathfrak{c}$? (£1). (If $\diamondsuit_{\mathfrak{c}}(E)$ is true or $\square_{\omega_\omega}$ is true, no.)

(c) If X is a metric space in which every subset is Borel, does every subset have to be F_σ? (£9.) (See FREMLIN 84, §23, and FREMLIN HANSELL & JUNNILA 83.)

BU. Assume that $\mathfrak{m} > \omega_1$. Can there be an $\alpha < \omega_1$ such that $\omega_1 \not\rightarrow (\omega_1, \alpha)^2$? (£12.) (If *either* α is less than or equal to the ordinal power ω^2 *or* PFA is true, no; see Fremlin n99, S42A, and Fremlin n86a, 6E.)

BY. Let K be the statement 'any upwards-ccc partially ordered set satisfies Knaster's condition upwards' (see Fremlin 84, 41Ab, and Todorčević 89, §7). Does K imply that $\mathfrak{m} > \omega_1$? (£26.) Does K imply that $2^{\omega_1} = \mathfrak{c}$? (£9.)

CA. (A.H.Stone.) Can there be metric spaces X and Y and a Borel measurable function $f : X \to Y$ such that the Borel classes of $f^{-1}[H]$, as H runs over the open subsets of Y, are unbounded in ω_1? (£30.) (Under any of a large number of special axioms, no; see Fremlin Hansell & Junnila 83, Theorem 16.)

CC†. If X and Y are topological spaces, a relation $R \subseteq X \times Y$ is **lower Borel measurable** if $R^{-1}[H]$ is Borel measurable in X for every open set $H \subseteq Y$.

(a) Can there be metric spaces X and Y and a lower Borel measurable $R \subseteq X \times Y$, with compact vertical sections, which has no Borel measurable selector? (£8.) (A negative answer to **CA** would settle this.)

(b) Can there be complete metric spaces X and Y and a lower Borel measurable $R \subseteq X \times Y$, with closed separable vertical sections, which has no Borel measurable selector? (£8.) (Under certain special axioms, 'no'; see Fremlin 87, 3N.)

CE†. Can there be a cardinal κ such that (i) $\mathcal{P}\kappa$ is not countably generated as σ-algebra (ii) whenever J is a set and $\mathcal{A} \subseteq \mathcal{P}J$ has cardinal κ, there is a countable $\mathcal{B} \subseteq \mathcal{P}J$ such that $\mathcal{A}$ is included in the σ-algebra of subsets of J generated by $\mathcal{B}$? (£16.) What if $\kappa = \omega_1$? (£16.)

CG. Is it relatively consistent with ZFC that every universally measurable set has the Baire property? (Mauldin Preiss & Weizsäcker 83.) (£4.)

CJ. Let $A \subseteq \mathbb{R}^n$ be a random set chosen by a Poisson point process with parameter λ, so that

$$\Pr(\#(A \cap E) = k) = \frac{(\lambda\mu E)^k}{k!} e^{-\lambda\mu E}$$

whenever $k \in \mathbb{N}$ and $E \subseteq \mathbb{R}^n$ has Lebesgue measure μE. Let G be the random graph obtained by taking A for the set of vertices and joining all points of A which are within unit distance of each other. What is the probability that G has an infinite component?

If $n = 1$, it is zero. For $n \geq 2$, there is a critical value λ_n such that if $\lambda < \lambda_n$ then G almost surely has no infinite component, while if $\lambda > \lambda_n$ then G almost surely does have an infinite component. For an estimate of λ_2 see Mertens & Moore 12.

An equivalent problem is the following. Imagine a random set $C \subseteq \mathbb{R}^n$ generated by the following process. Set $C_0 = \{0\}$. Given C_r, set $V_r = \{y : \rho(y, C_r) < 1\} \setminus \bigcup_{i<r} V_i$. Let C_{r+1} be a random subset of V_r chosen by a Poisson point process with parameter λ, and continue. Set $C = \bigcup_{r\in\mathbb{N}} C_r$. Then

$$\Pr(C \text{ is infinite}) > 0 \iff \Pr(G \text{ has an infinite component})=1,$$

and

$$\mathrm{Exp}^{\mathrm{n}}(\#(C)) < \infty \iff \exists\ r \in \mathbb{N},\ \mathrm{Exp}^{\mathrm{n}}(\#(C_r)) < 1 \iff \lambda < \lambda_n$$

(Meester & Roy 96, §3.5). But if $\lambda = \lambda_n$, what is $\Pr(C$ is infinite$)$? (£22).

CL. A topological space X has the property 'AF_∞' (see Fremlin 87) if no non-meager open set in X can be covered by a totally ordered family of meager sets. Can $\{0,1\}^\kappa$ have this property for every κ? (£3.) (If $\mathfrak{m} = \mathfrak{c}$, no.)

Compare **CQ**.

CN. For each cardinal κ give $\mathbb{R}^\kappa$ the box product topology, and let S_κ be the set of injective functions in $\mathbb{R}^\kappa$.

(a) Is S_{ω_1} comeager in $\mathbb{R}^{\omega_1}$? (£8.)

(b) Is $S_{\mathfrak{c}}$ meager in $\mathbb{R}^{\mathfrak{c}}$? (£8.)

(See Fremlin n83. If $\kappa < \mathfrak{c}$ then S_κ is not meager in $\mathbb{R}^\kappa$; see Kolman & Shelah 98.)

CQ. A measure space (X, Σ, μ) has the property 'AF_∞' (see Fremlin 87) if no non-negligible measurable set can be covered by a totally ordered family of negligible sets. Can every Radon measure space have this property? (£21). (If $\mathfrak{m} = \mathfrak{c}$, no.)

Recall that an infinite cardinal κ is a **precaliber** of a Boolean algebra $\mathfrak{A}$ if for every family $\langle a_\xi \rangle_{\xi<\kappa}$ in $\mathfrak{A} \setminus \{\mathbf{0}\}$ there is a set $\Gamma \in [\kappa]^\kappa$ such that $\langle a_\xi \rangle_{\xi\in\Gamma}$ is centered, that is, $\inf_{\xi\in I} a_i \neq \mathbf{0}$ for every non-empty finite set $I \subseteq \Gamma$. Now an equivalent formulation of this problem is: can every regular uncountable cardinal be a precaliber of every probability algebra? (See FREMLIN 08, §525.)

Compare **CL**, **DW**.

CR. (ERDŐS 74.) Let $A \subseteq \mathbb{R}$ be an infinite set. Is there necessarily a set $E \subseteq \mathbb{R}$, with strictly positive Lebesgue measure, not including any set similar to A? (£19.) What if $A = \{2^{-n} : n \in \mathbb{N}\}$? (£18.) (See SVETIC 01 and the references there.)

CT. A topological space X has the property 'Borel-AF$_\omega$' if $\bigcup \mathcal{E}$ is meager whenever $\mathcal{E}$ is a point-countable family of meager subsets of X and $\bigcup \mathcal{E}'$ is Borel in X for every $\mathcal{E}' \subseteq \mathcal{E}$. (See FREMLIN 87, §8.) Can there be a metric space without this property? (£4.)

CV. If κ, λ and θ are cardinals, with $1 \leq \theta \leq \lambda \leq \kappa$, and $t \in [0,1]$, say that $(\kappa, t) \Rightarrow [\lambda]^\theta$ if whenever $\langle E_I \rangle_{I\in[\kappa]^\theta}$ is a family of Lebesgue measurable subsets of $[0,1]$ and the Lebesgue measure of every E_I is at least t, then there is a $K \in [\kappa]^\lambda$ such that $\bigcap_{I\in[K]^\theta} E_I \neq \emptyset$. Write $t_0(\kappa, [\lambda]^\theta)$ for $\inf\{t : (\kappa, t) \Rightarrow [\lambda]^\theta\}$. It is known that $\frac{3+\sqrt{17}}{12} \geq t_0(\omega, [4]^3) \geq \frac{5}{9}$, $t_0(\omega_1, [4]^3) = \frac{1}{2}$, $t_0(\mathfrak{c}, [4]^3) \geq \frac{1}{4}$, $t_0(\mathfrak{c}^+, [4]^3) = 0$; see FREMLIN N85, CHUNG & LU 99. The calculation of $t_0(\omega, [4]^3) = \lim_{n\to\infty} t_0(n, [4]^3)$ is an old problem of P.Turán.

(a) Is $t_0(\omega, [4]^3)$ exactly $\frac{5}{9}$? (£20.)

(b) Can $t_0(\mathfrak{c}, [4]^3)$ be $\frac{1}{4}$? (£20.)

(c) Can $t_0(\omega_2, [4]^3)$ be $\frac{1}{2}$? (£20.)

DC. Let $\mathcal{F}$ be an ultrafilter on $\mathbb{N}$, and κ a cardinal. $\mathcal{F}$ is a **p(κ)-point** ultrafilter if it is non-principal and whenever $\mathcal{A} \subseteq \mathcal{F}$ and $\#(\mathcal{A}) < \kappa$ then there is an $I \in \mathcal{F}$ such that $I \setminus A$ is finite for every $A \in \mathcal{A}$. $\mathcal{F}$ is **(ω, κ)-saturating** if whenever $\mathcal{B} \subseteq \mathcal{P}(\mathbb{N}\times\mathbb{N})$ is such that $\#(\mathcal{B}) < \kappa$ and the first projection $\pi_1[\bigcap \mathcal{B}_0]$ belongs to $\mathcal{F}$ for every non-empty finite $\mathcal{B}_0 \subseteq \mathcal{B}$, then there is a function $f : \mathbb{N} \to \mathbb{N}$ such that $\{n : (n, f(n)) \in B\} \in \mathcal{F}$ for every $B \in \mathcal{B}$. (See FREMLIN 84, A3D, and FREMLIN & NYIKOS 89; also STEPRĀNS 92.) Can it be that $\mathfrak{p} = \mathfrak{c}$ and at the same time there is a p($\mathfrak{c}$)-point ultrafilter which is not $(\omega, \mathfrak{c})$-saturating? (£8.) (If $\mathfrak{m} = \mathfrak{c}$, no.)

DM. A topological space $(X, \mathfrak{T})$ is **copolish** if there is a Polish topology $\mathfrak{S}$ on X, coarser than $\mathfrak{T}$, such that every point of X has a $\mathfrak{T}$-neighbourhood base consisting of $\mathfrak{S}$-closed sets. $(X, \mathfrak{T})$ is **pseudo-complete** if there is a sequence $\langle \mathcal{U}_n \rangle_{n\in\mathbb{N}}$ of π-bases for $\mathfrak{T}$ such that $\bigcap_{n\in\mathbb{N}} U_n \neq \emptyset$ whenever $\langle U_n \rangle_{n\in\mathbb{N}}$ is a sequence such that $\overline{U}_{n+1} \subseteq U_n \in \mathcal{U}_n$ for every $n \in \mathbb{N}$. (See FREMLIN 84, §43.) Is every copolish space pseudo-complete? (£25.)

DN. Can it be that for every hereditarily Lindelöf compact Hausdorff space X there are a metric space Z and a continuous function $f : X \to Z$ such that $\#(f^{-1}[\{z\}]) \leq 2$ for every $z \in Z$? (£26.) (See FREMLIN 84, 44Qc, and FREMLIN N86A, 9C.)

DO. Let $\mathcal{B}$ be the algebra of Borel subsets of $[0,1]$ and μ the restriction of Lebesgue linear measure to $\mathcal{B}$.

(a) If $\mathfrak{c} \geq \omega_3$ can μ have a lifting? (£28.) (If we add ω_2 Cohen reals to a model of GCH, then μ has a lifting; see FREMLIN 08, 554I.)

(b) Assume that $\mathfrak{m} = \mathfrak{c} > \omega_1$. Does μ have a lifting? (£28.)

(c) (A.H.Stone.) Can there be a lifting θ of μ such that the Borel classes of θE, as E runs over $\mathcal{B}$, are bounded in ω_1? (£26.)

(d) Can there be a lifting θ of μ_2, the restriction of Lebesgue planar measure to the Borel σ-algebra of $[0,1]^2$ which **respects coordinates**, i.e., is such that if E, $F \in \mathcal{B}$ then $\theta(E \times F)$ is of the form $E' \times F'$? (£22.) (Compare TALAGRAND 82 and FREMLIN 02, §346.) A negative answer might contribute to a solution of **D** above.)

(e) If we add ω_2 random reals to a model of GCH, does μ have a lifting? (£12.)

DP. For any $\epsilon > 0$, let $\mathbb{P}_\epsilon$ be the set of open subsets of $[0,1]$ of Lebesgue measure strictly less than ϵ, and let $\mathfrak{A}_\epsilon$ be the regular open algebra of $\mathbb{P}_\epsilon$ when $\mathbb{P}_\epsilon$ is given its up-topology (see FREMLIN 84, 12B-C). Let $\mathbb{P}$ be the set of open subsets of $\mathbb{R}$ of Lebesgue measure strictly less than 1, and $\mathfrak{A}$ the regular open algebra of $\mathbb{P}$ for its up-topology. It is known that any of these Boolean algebras can be embedded into all the others, and that the $\mathfrak{A}_\epsilon$, for $0 < \epsilon < 1$, are isomorphic (Truss 88, or Fremlin 08, §528). But are the $\mathfrak{A}_\epsilon$ actually isomorphic to $\mathfrak{A}$? (£15.)

DS. Assume that $\mathfrak{m} = \mathfrak{c} = \omega_3$. Can there be a set $A \subseteq \mathbb{R}$ of cardinal ω_2 such that $\mathbb{R} \setminus A$ is not expressible as the union of ω_1 Borel sets? (£5.) (See Fremlin & Jasiński 86.)

DU†. Suppose that $S \subseteq [\omega_1]^{<\omega}$ is such that for every finite $I \subseteq \omega_1$ there is a $J \subseteq I$ with $\#(J) \geq \frac{1}{2}\#(I)$ and $\mathcal{P}J \subseteq S$.

(a) Must there be an infinite $A \subseteq \omega_1$ such that $[A]^{<\omega} \subseteq S$? (£17.)

(b) Assume that $\mathfrak{m} > \omega_1$. Must there be an uncountable $A \subseteq \omega_1$ such that $[A]^{<\omega} \subseteq S$? (£16.)

DW. Can it be that whenever κ is an infinite cardinal, (X, μ) is a probability space, $\langle E_\xi \rangle_{\xi<\kappa}$ is a family of measurable subsets of X, and $0 \leq \gamma < \inf_{\xi<\kappa} \mu E_\xi$, there is a set $\Gamma \in [\kappa]^\kappa$ such that $\mu(\bigcap_{\xi \in I} E_\xi) \geq \gamma^{\#(I)}$ for every non-empty finite $I \subseteq \Gamma$? (£8.) (If **CQ** has a negative answer, no. See Fremlin 88.)

DX. Can there be a countably-generated σ-subalgebra Σ of $\mathcal{P}\omega_1$ such that (i) $\mathcal{I} = \Sigma \cap \mathrm{NS}_{\omega_1}$ is cofinal with NS_{ω_1}, the ideal of non-stationary subsets of ω_1 (ii) the quotient algebra $\Sigma/\mathcal{I}$ is ccc? (£7.) (See Fremlin n86b.)

DY. For partially ordered sets P, Q say that $P \preccurlyeq_{\mathrm{T}} Q$ if there is a function $f : P \to Q$ such that $f[A]$ is unbounded above in Q whenever $A \subseteq P$ is non-empty and unbounded above in P (f is a 'Tukey function'; see Fremlin 93b).

(a) Suppose that (X, μ) is a Radon probability space of Maharam type κ. Write $\mathcal{N}_\mu$ for the ideal of μ-negligible sets, $\mathcal{N}$ for the ideal of Lebesgue negligible subsets of $\mathbb{R}$. Is it necessarily true that $\mathcal{N}_\mu \preccurlyeq_{\mathrm{T}} \mathcal{N} \times [\kappa]^{\leq\omega}$? (£11.) (If V=L, yes; see Fremlin 93b.)

DZ.(a) [P.J.Nyikos] Can there be a Radon compact Hausdorff space (see **W** above) which is not sequentially compact? (£24.) (If $2^{\mathfrak{t}} > \mathfrak{c}$, no.)

(b) Is the continuous image of a Radon compact Hausdorff space always Radon? (£24.)

EB. [V.Bergelson] Let $(\mathfrak{A}, \mu)$ be a probability algebra and $\pi : \mathfrak{A} \to \mathfrak{A}$ a measure-preserving automorphism. Suppose that $a \in \mathfrak{A}$ and that $\delta < (\mu a)^2$. Is there necessarily a set $I \subseteq \mathbb{N}$, not of zero asymptotic density, such that $\mu(\pi^i(a) \cap \pi^j(a)) \geq \delta$ for all $i, j \in I$? (£22.)

EC. For any topological space X, write $\mathcal{K}(X)$ for the family of compact subsets of X. Can there be is an analytic, non-Borel set $X \subseteq \mathbb{R}$ such that $\mathcal{K}(X)$ is a PCA ($= \Sigma^1_2$) subset of $\mathcal{K}(\mathbb{R})$, where $\mathcal{K}(\mathbb{R})$ is given its usual Polish topology? (If the axiom of projective determinacy is true, no; see Fremlin 90.)

ED. Can there be κ, $\mathcal{I}$, $\mathcal{J}$, B such that (i) $\kappa > \omega$ is a cardinal (ii) $\mathcal{I}$ is a κ-additive ideal of $\mathcal{P}\kappa$ (iii) $\mathcal{J}$ is a σ-ideal of the algebra $\mathcal{B}$ of Borel subsets of $\mathbb{R}$ (iv) B is a Borel subset of $\mathbb{R}^2$ (v) $\mathcal{J}$ consists precisely of those Borel subsets of $\mathbb{R}$ which are included in vertical sections of B (vi) $\mathcal{P}\kappa/\mathcal{I} \cong \mathcal{B}/\mathcal{J}$ as Boolean algebras (vii) $\mathcal{P}\kappa/\mathcal{I}$ is atomless, ccc and not $\{0\}$? (£3.) (If we omit (v), yes, it seems; see Główczyński 91. $\mathcal{J}$ cannot be the Lebesgue null ideal nor the ideal of meager subsets of $\mathbb{R}$; see Gitik & Shelah 89 and Fremlin 08, §547.)

EG. Assume that κ is an atomlessly-measurable cardinal, that is, that there is an atomless κ-additive probability ν with domain $\mathcal{P}\kappa$.

(b) Can we be sure that there is a set $A \subseteq \mathbb{R}$, of cardinal κ, such that no uncountable subset of A is Lebesgue negligible? (£14.) (Such a set can be found of any cardinal less than κ; see Fremlin 08, 544G.)

(d) Write $\mathfrak{d}$ for $\mathrm{cf}(\mathbb{N}^\mathbb{N})$. Can κ be less than $\mathfrak{d}$? (£24.) (It is not possible to have $\kappa = \mathrm{cf}(\mathfrak{d})$; see Fremlin 08, 544Nc.)

(e) Can there be an atomless κ-additive probability ν with domain $\mathcal{P}\kappa$ such that the measure algebra $\mathcal{P}\kappa/\mathcal{N}_\nu$ is not homogeneous? (£19.) What if $\kappa = \mathfrak{c}$? (£8.)

(f) Can it be that $\kappa < \mathfrak{c}$ and that κ is not weakly Π^1_1-indescribable? (£3.) (See Fremlin 93a, 4Lb.)

(g) Can it be that κ is weakly Π^1_1-indescribable, but that there is some $n \in \mathbb{N}$ such that κ is not weakly Π^1_n-indescribable? (See Fremlin 93a, 4R.) (£1.)

(h) Suppose that ν is **normal**, that is, that whenever $\langle E_\xi \rangle_{\xi<\kappa}$ is a family of subsets of κ such that $\nu E_\xi = 1$ for every ξ, then $\nu\{\xi : \xi < \kappa, \xi \in E_\eta \ \forall \ \eta < \xi\} = 1$. Let $\langle A_\zeta \rangle_{\zeta<\omega_1}$ be a family of subsets of κ

such that $\nu A_\zeta > 0$ for every $\zeta < \omega_1$. Must there be a countable set $N \subseteq \kappa$ such that $N \cap A_\zeta \neq \emptyset$ for every $\zeta < \omega_1$? (See FREMLIN 08, 555F.)(£15.)

(i) Can it be that every infinite compact Hausdorff space of weight less than κ has a non-trivial convergent sequence? (£6. See DOW & FREMLIN 07.)

EI. (T.Bartoszyński.) For Boolean algebras $\mathfrak{A}$, $\mathfrak{B}$ write $\mathfrak{A} \otimes \mathfrak{B}$ for their free product (see FREMLIN 02, §315), and $\mathfrak{A}\hat{\otimes}\mathfrak{B}$ for the Dedekind completion of $\mathfrak{A} \otimes \mathfrak{B}$. Now take $\mathfrak{A}$ to be the Lebesgue measure algebra. Is $\mathfrak{A}\hat{\otimes}\mathfrak{A}$ isomorphic to $\mathfrak{A}\hat{\otimes}\mathfrak{A}\hat{\otimes}\mathfrak{A}$? (£11.) Is there an order-continuous Boolean homomorphism from $\mathfrak{A}\hat{\otimes}\mathfrak{A}\hat{\otimes}\mathfrak{A}$ to $\mathfrak{A}\hat{\otimes}\mathfrak{A}$? (£11.)

EJ. A **measurable space with negligibles** is a triple $(X, \Sigma, \mathcal{I})$ where X is a set, Σ is a σ-algebra of subsets of X, and $\mathcal{I}$ is a σ-ideal of $\mathcal{P}X$ generated by $\Sigma \cap \mathcal{I}$. (See FREMLIN 87.) In this case, $(X, \Sigma, \mathcal{I})$ is ω_1**-saturated** if $\Sigma/\Sigma \cap \mathcal{I}$ is ccc, and **complete** if $\mathcal{I} \subseteq \Sigma$; a **lifting** of $(X, \Sigma, \mathcal{I})$ is a Boolean homomorphism $\theta : \Sigma \to \Sigma$ such that $\theta E = \emptyset$ whenever $E \in \Sigma \cap \mathcal{I}$ and $E \triangle \theta E \in \mathcal{I}$ for every $E \in \Sigma$. (See **D** above.)

(b) (A. H. Stone) Taking $X = \mathbb{R}$, $\mathcal{B}$ the algebra of Borel subsets of $\mathbb{R}$ and $\mathcal{M}$ the ideal of meager subsets of $\mathbb{R}$, can there be a lifting θ of $(\mathbb{R}, \mathcal{B}, \mathcal{M})$ such that the Baire classes of θE, as E runs over $\mathcal{B}$, are bounded in ω_1? (£7.) (See **DO(c)**.)

EK. The following questions seem to be left over from TALAGRAND 84 and SHELAH & FREMLIN 93, in addition to **R** and **AD** above.

(a) Write $\mathcal{L}^0(\Sigma)$ for the space of real-valued Lebesgue measurable functions on $[0, 1]$ and $\mathfrak{T}_p$ for the pointwise topology of $\mathbb{R}^{[0,1]}$. Can there be a $\mathfrak{T}_p$-separable $\mathfrak{T}_p$-compact set $K \subseteq \mathcal{L}^0(\Sigma)$ such that the closed convex hull of K in $\mathbb{R}^{[0,1]}$ does not lie within $\mathcal{L}^0(\Sigma)$? (£12.)

(b) Can there be a separable compact Radon measure space $(X, \mathfrak{T}, \Sigma, \mu)$ and a function $f : [0, 1] \times X \to \mathbb{R}$ which is measurable in the first variable, continuous in the second, but not jointly measurable with respect to the completed product measure on $[0, 1] \times X$? (£6.) (A positive answer to (a) would imply the same answer to (b).)

(c) Can there be a set $A \subseteq [0, 1]$, of outer Lebesgue measure 1, and a $\mathfrak{T}_p$-compact subset $K \subseteq \mathbb{R}^A$ such that every function in K is measurable for the subspace measure on A, but not every sequence in K has a subsequence convergent almost everywhere? (£8.) (If $\mathfrak{m} = \mathfrak{c}$, no; see TALAGRAND 84, 9-3-3.)

EL. (b) For which cardinals κ is it always true that whenever $\mathcal{A}$ is a family of subsets of $[0, 1]$ and $\#(\mathcal{A}) \leq \kappa$ then there is a set $B \subseteq [0, 1]$ such that $\mu_L^*(A \cap B) = \mu_L^*(A \setminus B) = \mu_L^* A$ for every $A \in \mathcal{A}$, writing μ_L for Lebesgue measure? This is true for $\kappa \leq \omega$ and possibly false for $\kappa = \mathfrak{c}$ (see FREMLIN 08, §547). What about $\kappa = \omega_1$? (£22.)

(c) Repeat (b) for category in the place of measure.

EN. Let (X, μ) be a probability space and $E_0, \ldots, E_n$ stochastically independent measurable subsets of X with $\sup_{i \leq n} \mu E_i = \epsilon$. Suppose that $E \subseteq X$ is a measurable set such that $E \times E \subseteq \bigcup_{i \leq n} E_i \times E_i$, and that $\epsilon \leq \frac{1}{2}$. Then $\mu E \leq \epsilon$ (FREMLIN 91). But suppose that $E \times E \times E \subseteq \bigcup_{i \leq n} E_i \times E_i \times E_i$ and that $\epsilon \leq \frac{2}{3}$. Does it now follow that $\mu E \leq \epsilon$? (£16.)

ER. Let ν be **Talagrand's measure** on $\mathcal{P}\mathbb{N}$, that is, for $E \subseteq \mathcal{P}\mathbb{N}$ and $\alpha \in [0, 1]$, νE is defined and equal to α iff there are $H \subseteq \mathcal{P}\mathbb{N}$, $\langle \mathcal{F}_n \rangle_{n \in \mathbb{N}}$ such that (i) μH exists and is equal to α, where μ is the usual Radon measure on $\mathcal{P}\mathbb{N}$ (ii) each $\mathcal{F}_n$ is a filter on $\mathbb{N}$ with $\mu^* \mathcal{F}_n = 1$ (iii) $E \cap \bigcap_{n \in \mathbb{N}} \mathcal{F}_n = H \cap \bigcap_{n \in \mathbb{N}} \mathcal{F}_n$. Then the map $a \mapsto \chi a : \mathcal{P}\mathbb{N} \to \ell^\infty$ is scalarly measurable, that is, $a \mapsto h(\chi a)$ is $\operatorname{dom} \nu$-measurable for every $h \in (\ell^\infty)^*$. (See FREMLIN & TALAGRAND 79, §1, TALAGRAND 84, 13-3-2 or FREMLIN 03, §464.) But is it measurable for the weak topology $\mathfrak{T}_s(\ell^\infty, (\ell^\infty)^*)$, in the sense that for any weakly open set $G \subseteq \ell^\infty$ the set $\{a : \chi a \in G\}$ is measured by ν? (£19.)

ET†. Let $\langle G_x \rangle_{x \in [0,1]}$ be a family of open subsets of $\mathbb{R}$ such that $\mu(A \setminus \bigcup_{x \in A} G_x) = 0$ for every $A \subseteq [0, 1]$, where μ is Lebesgue measure on $\mathbb{R}$. Let $\epsilon > 0$. Is there necessarily a family $\langle H_x \rangle_{x \in [0,1]}$ of open sets such that $x \in H_x$ for every $x \in [0, 1]$ and $\mu(\bigcup_{x \in A} H_x \setminus \bigcup_{x \in A} G_x) \leq \epsilon$ for every $A \subseteq [0, 1]$? (£17.) (This is relevant to Problem 4Ga in FREMLIN 95.)

EU. Let $\langle (X_n, \mu_n) \rangle_{n \in \mathbb{N}}$ be a sequence of probability spaces, with (completed) product (X, Σ, μ). Is there necessarily a lifting θ for μ which respects coordinates in the sense that θE is determined by coordinates in J

whenever $E \in \Sigma$ is determined by coordinates in $J \subseteq \mathbb{N}$? (£10.) (If every μ_n is Maharam-type-homogeneous, yes; see FREMLIN 02, §346.) What if $X_n = \{0,1\}$, $\mu_n\{1\} = \frac{2}{3}$ for every n? (£9.)

EV. If (X, Σ, μ) is a measure space, a **lower density** for μ is a function $\phi : \Sigma \to \Sigma$ such that (i) $\phi\emptyset = \emptyset$ (ii) $\phi(E \cap F) = \phi E \cap \phi F$ for all E, $F \in \Sigma$ (iii) $\phi E = \phi F$ whenever $\mu(E \triangle F) = 0$ (iv) $\mu(E \triangle \phi E) = 0$ for every $E \in \Sigma$. Let μ be the usual measure on $X = \{0,1\}^{\mathbb{N}}$. Is there a lower density for μ which is invariant under permutations of coordinates? (£16.)

EY. Take $p \in]1, \infty[$ and f, $g : [0,1] \to \mathbb{R}$ measurable functions such that $\alpha = (\int |f|^p)^{1/p}$, $\beta = (\int |g|^p)^{1/p}$ are finite and the conditional expectation of g on f is zero. For given α, β what is the greatest possible value of $(\int |f+g|^p)^{1/p}$? (£1.) The case of principal interest is $\beta \le \alpha$.

FA. Can it be that every totally finite completion regular Borel measure on a completely regular Hausdorff space is τ-additive? (£14.) (If *either* there is a real-valued-measurable cardinal *or* $\mathfrak{c} = \omega_1$, no; see FREMLIN 03, 439I, and JUHÁSZ KUNEN & RUDIN 76.)

FB. (WAJCH 91) Let $\mathfrak{A}$ be a Boolean algebra. A **Maharam submeasure** on $\mathfrak{A}$ is a functional $\nu : \mathfrak{A} \to [0, \infty[$ such that $\nu 0 = 0$, $\nu a \le \nu(a \cup b) \le \nu a + \nu b$ for all a, $b \in \mathfrak{A}$, and $\lim_{n\to\infty} \nu a_n = 0$ whenever $\langle a_n \rangle_{n\in\mathbb{N}}$ is a non-increasing sequence in $\mathfrak{A}$ with infimum 0. If ν is a Maharam submeasure with domain $\mathcal{P}X$, must there be a measure μ, with domain $\mathcal{P}X$, such that $\nu E = 0$ whenever $\mu E = 0$? (£13.) (If there is no 'quasi-measurable cardinal' (FREMLIN 08, §542), yes.)

FC. Let μ be Lebesgue measure on $\mathbb{R}$, and ν 'Cantor measure', that is, the image of the usual measure on $\{0,1\}^{\mathbb{N}}$ under the map $x \mapsto 2\sum_{i=0}^{n} 3^{-i-1} x(i)$. Must there be a set $A \subseteq \mathbb{R}$ such that $\mu^* A > 0$ but $\nu(x + A) = 0$ for every $x \in \mathbb{R}$? (£11.) (If the uniformity of Lebesgue measure is $\mathfrak{c}$, or the covering number of Lebesgue measure is equal to the cofinality of Lebesgue measure, yes; and if we replace ordinary Cantor measure by 'middle three-fifths' Cantor measure, yes; see FREMLIN N09. There is no such Borel set; see FREMLIN 03, 444L.)

FD. Let $(X, \mathfrak{T}, \Sigma, \mu)$ be a Radon probability space, of countable Maharam type, such that $\mu G > 0$ for every non-empty open set $G \subseteq X$. Does it necessarily have a lifting ϕ which is **strong**, in the sense that $\phi G \supseteq G$ for every open set G? (£11.) (If $\mathfrak{c} = \omega_1$, yes; see MOKOBODZKI 75 or FREMLIN 08, §535. For a Radon measure with no strong lifting, see LOSERT 79 or FREMLIN 03, 453N.)

FF. (a) Let X and Y be topological spaces with Borel probability measures μ and ν. Is there necessarily a Borel measure λ on $X \times Y$ such that $\lambda(E \times F) = \mu E \cdot \nu F$ for all Borel sets $E \subseteq X$, $F \subseteq Y$? (£12.) (See FREMLIN 03, 435O.)

(b) A **Baire measure** on a topological space is a measure with domain the Baire σ-algebra (see **D** above). Let X and Y be completely regular Hausdorff spaces with Baire probability measures μ and ν. Is there necessarily a Baire measure λ on $X \times Y$ such that $\lambda(E \times F) = \mu E \cdot \nu F$ for all Baire sets $E \subseteq X$, $F \subseteq Y$? (£11.)

FG. Can it be that whenever μ is a Baire probability measure on a normal Hausdorff space X, then μ has an extension to a Borel measure? (£13.) (If $\clubsuit$, no. Only Dowker spaces X need be considered. See FREMLIN 03, 435C and 439O, and KOJMAN & MICHALEWSKI P06.)

FJ. (M.Foreman) **(a)** Let us say that a filter $\mathcal{F}$ on $\mathbb{N}$ is **measure-converging** if whenever $\langle f_n \rangle_{n\in\mathbb{N}}$ is a sequence of measurable functions on a probability space (equivalently, on $[0,1]$) converging to 0 in measure, then $\lim_{n\to\mathcal{F}} f_n(x) = 0$ for almost every x. (See FREMLIN 08, §538.) Must there be a measure-converging filter on $\mathbb{N}$? (£14.) (If $\mathfrak{p} = \mathfrak{c}$, yes.)

(b) Every rapid filter is measure-converging. (A filter $\mathcal{F}$ is **rapid** if for every sequence $\langle x_n \rangle_{n\in\mathbb{N}}$ of positive numbers converging to 0 there is an $F \in \mathcal{F}$ such that $\sum_{n\in F} x_n$ is finite.) If there is a measure-converging filter, must there be a rapid filter? (£6.)

FK. Are there a Hausdorff space X and a transitive group G of autohomeomorphisms of X and two non-zero G-invariant Radon measures on X which are not scalar multiples of each other? (If X is locally compact and metrizable, 'no'; see FREMLIN N00.) (£9.) What if X is compact? (£9.)

FL. Let $\mathfrak{A}$ be the Lebesgue measure algebra, $\mathrm{Aut}(\mathfrak{A})$ the group of all Boolean algebra automorphisms of $\mathfrak{A}$, $\mathrm{Aut}_\mu(\mathfrak{A})$ the group of measure-preserving automorphisms of $\mathfrak{A}$. Can either $\mathrm{Aut}(\mathfrak{A})$ or $\mathrm{Aut}_\mu(\mathfrak{A})$ be generated (as a group) by finitely many abelian subgroups? (£6.)

FM. Assume that $\mathfrak{m} > \omega_1$. Let X be a Banach space of weight ω_1. Must X be a Radon space when given its weak topology? (£10.)

FN†. A family $\mathcal{K}$ of sets is countably compact if any sequence in $\mathcal{K}$ with the finite intersection property has non-empty intersection. A measure space (X, Σ, μ) is countably compact if μ is inner regular with respect to some countably compact family of sets. Suppose now that μ is a probability measure defined on a σ-subalgebra of the Borel algebra of $[0, 1]$. Must μ be countably compact? (£10.) (If $\mathfrak{c} = \omega_1$, yes.)

FO. For a family $\mathcal{K}$ of sets, write $\Gamma(\mathcal{K})$ for the infinite game in which players choose alternately elements of $\mathcal{K}$, each a subset of the preceding one, and the second player wins if the intersection of the sequence is non-empty. $\mathcal{K}$ is **weakly α-favourable** if the second player has a winning strategy in $\Gamma(\mathcal{K})$, and **α-favourable** if he has a winning tactic, that is, a winning strategy depending only on the first player's previous move. Is there a probability space (X, Σ, μ) such that $\Sigma \setminus \mathcal{N}_\mu$ is weakly α-favourable but not α-favourable, where $\mathcal{N}_\mu$ is the ideal of μ-negligible sets? (£11.) (See Fremlin 00.)

FP. Let P be a partially ordered set. A **Freese-Nation function** on P is a function $f : P \to \mathcal{P}P$ such that $f(p) \cap f(q) \cap [p, q] \neq \emptyset$ whenever $p \leq q$ in P. The **Freese-Nation number** $\mathrm{FN}(P)$ of P is the smallest cardinal κ such that there is a Freese-Nation function from P to $[P]^{<\kappa}$.

(a) For finite n, how large is the smallest P such that $\mathrm{FN}(P) = n$? (£3.)(For $n \geq 4$, it has between $2n-4$ and $4n - 11$ elements.)

(b) Set $f(n) = \mathrm{FN}(\mathcal{P}n) - 1$. Then $f(3) = 4$ and $\lim_{n\to\infty} f(n)^{1/n} = \inf_{n\geq 1} f(n)^{1/n}$ lies between $2/\sqrt{3}$ and $\sqrt[3]{4}$ (Fremlin & Penman p01). What is its exact value? (£2.)

FR†. Can it be that whenever X is a compact Hausdorff space and $\langle \mu_i \rangle_{i\in I}$ is a family of mutually singular Radon probability measures on X, then $\#(I) \leq \#(X)$? (£3.)

FU. Let G and H be Polish groups (that is, topological groups for which the topologies are Polish) and $f : G \to H$ a homomorphism which is universally measurable (that is, the inverse image of any open set in H is a universally measurable subset of G). Does it follow that f is continuous? (If either group is abelian, yes; see Christensen 74, chap. 7.) (£9.)

FV. (See Fremlin 08, §534.) (a) Let $\mathcal{S}\mathfrak{mz}$ be the family of subsets of $\mathbb{R}$ with strong measure zero. Can we prove in ZFC that $\mathrm{cf}\, \mathcal{S}\mathfrak{mz} \geq \omega_2$? (£4.)

(b) Two metric spaces (X, ρ) and (Y, σ) are **$\mathcal{S}\mathfrak{mz}$-equivalent** if there is a bijection $f : X \to Y$ such that a set $A \subseteq X$ has strong measure zero for ρ iff $f[A]$ has strong measure zero for σ. How many types of Polish spaces under $\mathcal{S}\mathfrak{mz}$-equivalence can there be? If we give $\mathbb{Z}^{\mathbb{N}}$ and $\mathbb{R}^{\mathbb{N}}$ right-translation-invariant metrics inducing their topologies, are they (in ZFC) $\mathcal{S}\mathfrak{mz}$-equivalent? (£7.)

(c) Suppose that there is a separable metric space (X, ρ) of cardinal $\mathfrak{c}$ with strong measure zero. Is there necessarily a subset of $\mathbb{R}$, of cardinal $\mathfrak{c}$, with strong measure zero? (£5.)

FX. A filter $\mathcal{F}$ on $\mathbb{N}$ is **measurable** if it is measured by the usual measure on $\mathcal{P}\mathbb{N}$ (in which case it must be negligible, by the zero-one law). Must there be a measurable filter which is the intersection of ω_1 ultrafilters on $\mathbb{N}$? (£5). (If $\mathfrak{c} = \omega_1$, yes. See Fremlin 03, §464.)

FY. Let X be the set of all partial orders on $\mathbb{N}$, regarded as a closed subspace of the compact Hausdorff space $\mathcal{P}(\mathbb{N} \times \mathbb{N})$. Let G be the group of all permutations of $\mathbb{N}$, and for $a \in G$, $x \in X$ write $a{\bullet}x = \{(i, j) : (a^{-1}(i), a^{-1}(j)) \in x\}$, so that $\bullet$ is a continuous action of G on X if G is given its topology of pointwise convergence. Let Q be the set of G-invariant Radon probability measures on X. What are the extreme points of Q? (£8.)

FZ. Can $\mathrm{FN}(\mathcal{P}\mathbb{N})$ (definition: **FP** above) have countable cofinality? (£7.)

GA. Can there be a hereditarily Lindelöf compact Hausdorff space X with a Radon probability measure of Maharam type ω_2? (£7.)(If *either* $\mathfrak{c} = \omega_1$ *or* $\mathbb{R}$ is not covered by ω_1 Lebesgue negligible sets, no. See Fremlin 08, §531.)

GB. If $\mathcal{I}$ is a proper ideal of subsets of a set X, its **shrinking number** $\mathrm{shr}\, \mathcal{I}$ is the least cardinal κ such that for every $A \in \mathcal{P}X \setminus \mathcal{I}$ there is a $B \in [A]^{\leq\kappa} \setminus \mathcal{I}$. Now suppose that μ is Lebesgue measure on $[0, 1]$ and that (Z, ν) is the Stone space of the measure algebra of $([0, 1], \mu)$ (Fremlin 02, 321K). Let $\mathcal{N}(\mu)$, $\mathcal{N}(\nu)$ be the respective null ideals. Can $\mathrm{shr}\, \mathcal{N}(\mu)$ and $\mathrm{shr}\, \mathcal{N}(\nu)$ be different? (£7.)

GC. (S.Zeberski) Let $\mathcal{A}$ be a partition of $[0,1]$ into Lebesgue negligible sets. Must there be a $\mathcal{C} \subseteq \mathcal{A}$ such that $\bigcup \mathcal{C}$ has inner measure 0 and outer measure 1? (See FREMLIN & TODORČEVIĆ N04.) (£7.) If every member of $\mathcal{A}$ is countable, yes (KUMAR & SHELAH P15).

GD. If $\mathcal{I}$ is a proper ideal of subsets of a set X with $\bigcup \mathcal{I} = X$, then $\operatorname{add} \mathcal{I}$ is the least cardinal of any subset of $\mathcal{I}$ with union not in I, and $\operatorname{cov} \mathcal{I}$ is the least cardinal of any subset of $\mathcal{I}$ with union X. For a cardinal κ, write $\mathcal{N}_\kappa$ for the null ideal of the usual measure on $\{0,1\}^\kappa$. (See FREMLIN 08, §521.) Can $\operatorname{add} \mathcal{N}_\omega$ be greater than $\operatorname{cov} \mathcal{N}_{\omega_1}$? (£6.)

GE†. A Boolean algebra $\mathfrak{A}$ is **weakly σ-distributive** or **ω^ω-bounding** if for every sequence $\langle A_n \rangle_{n \in \mathbb{N}}$ of countable partitions of unity in $\mathfrak{A}$ there is a partition C of unity such that $\{a : a \in A_n,\ a \cap c \neq 0\}$ is finite for every $n \in \mathbb{N}$, $c \in \mathfrak{C}$. Let $\mathcal{B}$ be the Borel σ-algebra of $[0,1]$, and $\mathcal{I}$ a σ-ideal of $\mathcal{B}$ such that for every $E \in \mathcal{B} \setminus \mathcal{I}$ there is a compact set $K \subseteq E$ such that $K \notin \mathcal{I}$. Is the quotient algebra $\mathfrak{A} = \mathcal{B}/\mathcal{I}$ necessarily weakly σ-distributive? (£4.) What if $\mathfrak{A}$ is ccc? (£4.)

GF. A Boolean algebra $\mathfrak{A}$ is **σ-finite-cc** if $\mathfrak{A} \setminus \{0\}$ is expressible as $\bigcup_{n \in \mathbb{N}} A_n$ where no A_n includes any infinite disjoint family. $\mathfrak{A}$ is **σ-bounded-cc** if $\mathfrak{A} \setminus \{0\}$ is expressible as $\bigcup_{n \in \mathbb{N}} A_n$ where no A_n includes any disjoint family of size greater than n members. Is there a σ-finite-cc Boolean algebra which is not σ-bounded-cc? (£5.)

GG. A **Maharam algebra** is a Dedekind complete Boolean algebra $\mathfrak{A}$ such that there is a Maharam submeasure $\nu : \mathfrak{A} \to [0, \infty[$ (definition: **FB** above) which is **strictly positive**, that is, $\nu a > 0$ whenever $a \neq 0$. A **measurable algebra** is a Dedekind complete Boolean algebra $\mathfrak{A}$ such that there is a strictly positive countably additive functional $\mu : \mathfrak{A} \to [0, \infty[$ (see FREMLIN 02, chap. 39). Now that we know that we have non-trivial Maharam algebras (TALAGRAND 06, or FREMLIN 02, §394), we can ask the following questions.

(a) If $\mathfrak{A}$ is a non-zero atomless Maharam algebra, must it have an atomless order-closed subalgebra $\mathfrak{C}$ which is a measurable algebra? (£6.)

(b) A Boolean algebra $\mathfrak{A}$ is **σ-bounded-cc** if $\mathfrak{A} \setminus \{0\}$ is expressible as $\bigcup_{n \in \mathbb{N}} A_n$ where no A_n includes any disjoint family of size greater than n members. Is every Maharam algebra σ-bounded-cc? (See FREMLIN N06, §4, and TODORČEVIĆ 14.) (£6.)

(c) Let $\mathfrak{A}$ be a Maharam algebra and ν a strictly positive Maharam submeasure on $\mathfrak{A}$. Then for every $\epsilon > 0$ we have a rank function $r_\epsilon : \mathfrak{A} \to \mathrm{On}$ defined by saying that

$$r_\epsilon(a) = \sup\{r_\epsilon(b) : b \subseteq a,\ \nu(a \setminus b) \geq \epsilon\}$$

for every $a \in \mathfrak{A}$ (FREMLIN N06, §6). Is it possible to have $r_\epsilon(1) \geq \omega_1$? (£6.) that $\mathfrak{A}$ is measurable iff $r_\epsilon(1) \leq \omega$ for every $\epsilon > 0$; see FREMLIN N06, §7.

(d) A Boolean algebra $\mathfrak{A}$ is **σ-linked** if $\mathfrak{A} \setminus \{0\}$ is expressible as $\bigcup_{n \in \mathbb{N}} A_n$ where no A_n contains any pair of disjoint elements. A measurable algebra is σ-linked iff it has cardinal at most $\mathfrak{c}$ (FREMLIN 08, 524Mf). But does a Maharam algebra of cardinal $\mathfrak{c}$ have to be σ-linked? (£5.)

(e) Let $\mathfrak{A}$ be a Boolean algebra. Its **π-weight** $\pi(\mathfrak{A})$ is the least cardinal of any coinitial set in $\mathfrak{A} \setminus \{0\}$; its **centering number** $d(\mathfrak{A})$ is the least cardinal of any cover of $\mathfrak{A} \setminus \{0\}$ by downwards-directed sets not containing 0. Let $\mathfrak{A}$ be an atomless Maharam algebra of countable Maharam type, not $\{0\}$. Must we have $\pi(\mathfrak{A}) = \pi(\mathfrak{B}_\omega)$, where $\mathfrak{B}_\omega$ is the measure algebra of Lebesgue measure? (£5.) Must we have $d(\mathfrak{A}) = d(\mathfrak{B}_\omega)$? (£5.) (See FREMLIN 08, chap. 52, and FREMLIN N06, §9.)

(f) Is there a rigid Maharam algebra other than $\{0\}$ and $\{0,1\}$? (£3.)

GH. A measure space (X, Σ, μ) has the **measurable envelope property** if every subset A of X has a measurable envelope, that is, a set $E \in \Sigma$ such that $A \subseteq E$ and any measurable subset of $E \setminus A$ is negligible (FREMLIN 01, 213Xl). Is it possible that Hausdorff 1-dimensional measure on $\mathbb{R}^2$ has the measurable envelope property? (£6.) (If $\mathfrak{m} = \mathfrak{c}$, no; see FREMLIN N05.)

GI†. A Boolean algebra $\mathfrak{A}$ is **σ-n-linked** if $\mathfrak{A} \setminus \{0\}$ is expressible as $\bigcup_{k \in \mathbb{N}} A_k$ where $\inf_{i<n} a_i \neq 0$ for any family $\langle a_i \rangle_{i \in n}$ in any A_k. Is it possible that every Boolean algebra which is σ-n-linked for every n has centering number less than $\mathfrak{c}$? (£5.)

GK. Let X be a Hausdorff space. A **Radon submeasure** on X is a Maharam submeasure ν with domain $\Sigma \subseteq \mathcal{P}X$ such that (i) every open set belongs to Σ (ii) $\nu E = \sup\{\nu K : K \subseteq E \text{ is compact}\}$ for every

$E \in \Sigma$ (iii) $E \in \Sigma$ whenever $E \subseteq F \in \Sigma$ and $\nu F = 0$. A **lifting** for a submeasure ν on a Boolean algebra $\mathfrak{A}$ is a Boolean homomorphism $\phi : \operatorname{dom}\nu \to \operatorname{dom}\nu$ such that (i) $\phi a = 0$ whenever $\nu a = 0$ (ii) $\nu(a \triangle \phi a) = 0$ for every $a \in \operatorname{dom}\nu$. Which Radon submeasures have liftings? (£2.) (See Fremlin n06.)

GL. (R.W.Kaye) Let $\mathcal{N}$ be the Lebesgue null ideal. Is there a non-trivial automorphism π of the quotient Boolean algebra $\mathcal{P}\mathbb{R}/\mathcal{N}$ such that $\pi E^\bullet = E^\bullet$ for every Lebesgue measurable set E? (£7.)

GM. (Bogachev & Lukintsova p07) If X is a topological space and μ is a probability measure on X, I say that a sequence $\langle x_n\rangle_{n\in\mathbb{N}}$ in X is **equidistributed** if $\mu F \ge \limsup_{n\to\infty} \frac{1}{n}\#(\{i : i < n,\, x_i \in F\}$ for every measurable closed set $F \subseteq X$ (Fremlin 03, §491). Suppose that X and Y are compact Hausdorff spaces such that every Radon probability measure on either X or Y has an equidistributed sequence. Does it follow that every Radon probability measure on $X \times Y$ has an equidistributed sequence? (£6.)

GO†. (J.Mycielski) Let μ be a Radon probability measure on $\mathbb{R}^r$, where $r \ge 1$, and $F \subseteq \mathbb{R}^r$ a closed set. For $\boldsymbol{x} \in (\mathbb{R}^r)^{\mathbb{N}}$, $n \in \mathbb{N}$ and $y \in \mathbb{R}^r$, set $\delta_{\boldsymbol{x}yn} = \min_{i\le n} \|\boldsymbol{x}(i) - y\|$, $k_{\boldsymbol{x}yn} = \min\{i : i \le n,\, \|\boldsymbol{x}(i) - y\| = \delta_{\boldsymbol{x}yn}\}$; set $F_{\boldsymbol{x}n} = \{y : \boldsymbol{x}(k_{\boldsymbol{x}yn}) \in F\}$. Is it always the case that $\lim_{n\to\infty} \mu(F \triangle F_{\boldsymbol{x}n}) = 0$ for $\mu^{\mathbb{N}}$-almost every $\boldsymbol{x}$? (£4.) (If $r = 1$, yes; if μ is the uniform measure on $[0,1]^r$, yes.)

GQ. For $k \in \mathbb{N}$, say that a topological group G is k**-Steinhaus** if whenever $V \subseteq G$ is a symmetric set, containing the identity, such that countably many left translates of V cover G, then V^k is a neighbourhood of the identity. (See Rosendal & Solecki 07.) Let G be the group of measure-preserving automorphisms of the measure algebra $(\mathfrak{A}, \bar\mu)$ of Lebesgue measure on $[0,1]$; give it the topology generated by the metric ρ, where $\rho(\pi, \phi) = \sup_{a\in\mathfrak{A}} \bar\mu(\pi a \triangle \phi a)$. Then G is 38-Steinhaus (Kittrell & Tsankov 09, or Fremlin 03, 494O). Find the smallest k for which G is k-Steinhaus. (£2.)

GR†. For $k \ge 1$, let G_k be the following k-double-move game for two players. I chooses $n \in \mathbb{N}$ and sets $X_0 = [2n]^n$. When I has chosen X_i, where $i < k$, II chooses $S_i \subseteq [X_i]^2$ and I chooses $X_{i+1} \subseteq X_i$ such that $[X_i]^2$ is either included in S_i or disjoint from S_i. I wins if $\#(X_k) \ge 3$ and $\bigcap X_k = \emptyset$. Which player has a winning strategy? (£2.) What if $k = 1$? (£1.)

GS. Set $X = \mathbb{R}^2$; let $\mathcal{C}$ be the family of rectangles $I \times J$ where I, $J \subseteq \mathbb{R}$ are connected sets, and $\mathcal{E}$ the algebra of subsets of X generated by $\mathcal{C}$. Suppose that $\phi : \mathcal{C} \to [0,1]$ is such that $\sum_{C\in\mathcal{C}_0} \phi C \le 1$ for every disjoint collection $\mathcal{C}_0 \subseteq \mathcal{C}$. Must there be an additive functional $\lambda : \mathcal{E} \to [0,\infty[$ such that $\lambda C \ge \phi C$ for every $C \in \mathcal{C}$? (£1.) Indeed, will there always be such a functional $\lambda : \mathcal{E} \to [0,2]$? (£1.) (See Fremlin n11, 2D.)

GT. Let X be a compact Hausdorff space such that the space $P(X)$ of Radon probability measures on X with its narrow topology (the topology induced by the weak* topology of $C(X)^*$) is countably tight. Does it follow that every member of $P(X)$ has countable Maharam type? (If $\mathfrak{m} > \omega_1$, yes. See Plebanek & Sobota p14.) (£2.)

GU. Let $\mu^{(2)}_{H1}$, $\mu^{(1)}_{H,1/2}$ be one-dimensional Hausdorff measure on $\mathbb{R}^2$ and $\frac{1}{2}$-dimensional Hausdorff measure on $\mathbb{R}$ respectively, for their usual metrics. Are the measure spaces $(\mathbb{R}^2, \mu^{(2)}_{H1})$ and $(\mathbb{R}, \mu^{(1)}_{H,1/2})$ isomorphic?

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