Errata and addenda for Volume 5, 2008 printing

I collect here known errors and omissions, with their discoverers, in Volume 5 of my book *Measure Theory* (see my web page, `http://www1.essex.ac.uk/maths/people/fremlin/mt.htm`).

Part I

p 15 l 7 (511D) I have reformulated the definition of 'tight filtration' in 511Di, which now reads:

(i) Let $\mathfrak{A}$ be a Boolean algebra and κ a cardinal. A **tight κ-filtration** of $\mathfrak{A}$ is a family $\langle a_\xi\rangle_{\xi<\zeta}$ in $\mathfrak{A}$, where ζ is an ordinal, such that, writing $\mathfrak{A}_\alpha$ for the subalgebra of $\mathfrak{A}$ generated by $\{a_\xi : \xi < \alpha\}$, (α) $\mathfrak{A}_\zeta = \mathfrak{A}$ (β) for every $\alpha < \zeta$, the Freese-Nation index of $\mathfrak{A}_\alpha$ in $\mathfrak{A}$ is at most κ.

The connexion with the former definition is in the new exercise 518Xi.

p 16 l 6 (511G) Add new definition:

(b) Let (X, Σ, μ) be a measure space. The **π-weight** $\pi(\mu)$ of μ is the coinitiality of $\Sigma \setminus \mathcal{N}(\mu)$, where $\mathcal{N}(\mu)$ is the null ideal of μ.

p 16 l 32 Proposition 511I (on the cofinality of a product of partially ordered sets) has been moved to 513J. Consequently 511J-511K are now 511I-511J.

p 18 l 36 The exercises 511X have been rearranged: 511Xb-511Xc are now 511Xc-511Xd, 511Xd has been moved to 521F, 511Xe has been moved to 521Xe, 511Xf-511Xk are now 511Xe-511Xj, 511Xl is now 511Xb, 511Xm is now 511Xk.
Add new exercise:

(l) Let X be a set and $\mathcal{I}$ an ideal of subsets of X. Show that the coinitiality $\mathrm{ci}(\mathcal{P}X \setminus \mathcal{I})$ is at most $\#(X)^{\mathrm{shr}\,\mathcal{I}}$.

p 19 l 23 (511Y) Add new exercise:

(d) Show that, for a set I, $(\omega_1, \omega, \omega)$ is a precaliber triple of $\mathbb{N}^I$ iff I is countable.

p 21 l 43 The following remark has been added to 512Eb:

Note that if $\mathcal{M}(X)$ is the ideal of meager subsets of X, then $\mathrm{cov}(X, \mathcal{M}(X)) = n(X)$ unless $n(X) = \omega$, in which case $\mathrm{cov}(X, \mathcal{M}(X)) = 1$.

p 24 l 49 The exercises for §512 have been rearranged: 512Xc-512Xf are now 512Xe-512Xh, 512Xg is now 512Xd.
The former exercise 514Xa is now 512Xc.

p 25 l 14 Exercise 512Ya is covered by 513Yb (formerly 513Yh), and Exercise 512Yb has been moved to 513Yf.

p 30 l 23 Following the transfer of the former 511I to become 513I, 513J-513O are now 513K-513P.

p 33 l 3 (513Xe) Add new part:

(i) Show that if P, Q are partially ordered sets, P is Dedekind complete and $P \preccurlyeq_{\mathrm{T}} Q$, there is an order-preserving dual Tukey function from Q to P.

p 33 l 24 (513X) Add new exercise:

(q) Let $\langle P_i\rangle_{i\in I}$ be a family of partially ordered sets, and P their product. (i) Show that $\mathrm{cf}\,P$ is at most the cardinal product $\prod_{i\in I} \mathrm{cf}\,P_i$, with equality if I is finite. (ii) Show that if $P \neq \emptyset$ then $\sup_{i\in I} \mathrm{cf}\,P_i \le \mathrm{cf}\,P$. (iii) Show that if $P \neq \emptyset$ and for every $i \in I$ there is a $j \in I$ such that $\mathrm{cf}\,P_i < \mathrm{cf}\,P_j$, then $\sup_{i\in I} \mathrm{cf}\,P_i < \mathrm{cf}\,P$.

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p 33 l 28 (513Y) The former 512Yb is now 513Yf. There is a new exercise:

(d) For partially ordered sets P and Q, say that $P \approx Q$ if there is a partially ordered set R into which both P and Q can be embedded as cofinal subsets. (i) Show that if P, R and R' are partially ordered sets such that P can be embedded as a cofinal subset into both R and R', then $R \approx R'$. (ii) Show that $\approx$ is an equivalence relation on the class of partially ordered sets. (iii) Show that if $\mathcal{P}$ is a set of partially ordered sets, and $P \approx P'$ for all P, $P' \in \mathcal{P}$, then there is a partially ordered set R such that every member of $\mathcal{P}$ can be embedded into R as a cofinal set. (iv) Give an example of partially ordered sets P and Q such that $P \approx Q$ but $P \not\equiv_{\mathrm{T}} Q$.

Other exercises have been re-arranged: 513Yb is now 513Yc, 513Yc is now 513Ye, 513Yd-513Yg are now 513Yg-513Yj, 513Yh is now 513Yb.

p 40 l 27 Part (e) of Proposition 514H is now in 315H.

p 43 l 34 (Lemma 514M) Add new fragment:

(d)(ii) If $\mathcal{G}$ is a non-empty family of regular open subsets of P, then $\bigcap \mathcal{G}$ is a regular open subset of P, and is $\inf \mathcal{G}$ in the regular open algebra $\mathrm{RO}(P)$.

p 48 l 30 Exercise 514Xa has been moved to 512Xc. Add new exercises:

(f) Let X be a set. Show that $\tau(\mathcal{P}X)$ is the least cardinal λ such that $\#(X) \le 2^\lambda$.

(h) For a Boolean algebra $\mathfrak{A}$, write $\mathrm{hc}(\mathfrak{A}) = \min\{c(\mathfrak{B}) : \mathfrak{B}$ is a non-zero principal ideal of $\mathfrak{A}\}$, counting $\min\emptyset$ as ∞. (i) Show that if $\mathfrak{B}$ is a regularly embedded subalgebra of $\mathfrak{A}$, then $\mathrm{hc}(\mathfrak{B}) \le \mathrm{hc}(\mathfrak{A})$. (ii) Show that if $\mathfrak{B}$ is a Boolean algebra and there is a surjective order-continuous Boolean homomorphism from $\mathfrak{A}$ onto $\mathfrak{B}$, then $\mathrm{hc}(\mathfrak{B}) \le \mathrm{hc}(\mathfrak{A})$. (iii) Show that if $\mathfrak{B}$ is a principal ideal of $\mathfrak{A}$ then $\mathrm{hc}(\mathfrak{B}) \ge \mathrm{hc}(\mathfrak{A})$. (iv) Show that if $\mathfrak{B}$ is an order-dense subalgebra of $\mathfrak{A}$ then $\mathrm{hc}(\mathfrak{B}) = \mathrm{hc}(\mathfrak{A})$. (v) Show that if $\mathfrak{A}$ is the simple product of a family $\langle\mathfrak{A}_i\rangle_{i\in I}$ of Boolean algebras, then $\mathrm{hc}(\mathfrak{A}) = \min_{i\in I} \mathrm{hc}(\mathfrak{A}_i)$.

Other exercises for §514 have been rearranged: 514Xb-514Xe are now 514Xa-514Xd, 514Xf is now 514Xg, 514Xg-514Xl are now 514Xj-514Xo, 514Xm-514Xo are now 514Xq-514Xs, 514Xp is now 514Xp, 514Xq is now 514Xe.

p 50 l 3 (514Y) Add new exercise:

(g) Let $\mathfrak{A}$ be a ccc Dedekind complete Boolean algebra with Maharam type κ. Show that there is a σ-ideal $\mathcal{J}$ of the Baire σ-algebra $\mathcal{B}\mathfrak{a}(\{0,1\}^\kappa)$ such that $\mathfrak{A} \cong \mathcal{B}\mathfrak{a}(\{0,1\}^\kappa)/\mathcal{J}$.

p 51 l 44 (part (e-ii) of the proof of 515B): for '$\inf_{i\in I} b_i \ne 0$' read '$\inf_{i\in L} b_i \ne 0$'.

p 54 l 15 (part (d-ii) of the proof of 515H): there is a confusion here between the partitions of unity B_i and the Boolean-independent sets D_i, which I have I hope corrected.

p 55 l 20 In part (b) of the proof of 515K, we need to arrange for the sequence $\langle b_n\rangle_{n\in\mathbb{N}}$ to be disjoint; of course this is elementary.

p 56 l 6 (statement of Corollary 515M): for 'Dedekind complete' read 'Dedekind σ-complete'.

p 56 l 13 Proposition 515N has been revised, and new results added. The last part of this section now reads

515N Proposition Let I be a set. Write $\mathfrak{G}$ for the regular open algebra $\mathrm{RO}(\{0,1\}^I)$.

(a) $\mathfrak{G}$ is ccc and Dedekind complete and isomorphic to the category algebra of $\{0,1\}^I$. The algebra of open-and-closed subsets of $\{0,1\}^I$ is an order-dense subalgebra of $\mathfrak{G}$.

(b) Let $\mathfrak{A}$ be a Boolean algebra. Then $\mathfrak{A}$ is isomorphic to $\mathfrak{G}$ iff it is Dedekind complete and there is a Boolean-independent family $\langle a_i\rangle_{i\in I}$ in $\mathfrak{A}$ such that the subalgebra generated by $\{a_i : i \in I\}$ is order-dense in $\mathfrak{A}$.

(c) If I is infinite, $\mathfrak{G}$ is homogeneous.

515O Proposition (a) A Boolean algebra is isomorphic to $\mathfrak{G} = \mathrm{RO}(\{0,1\}^{\mathbb{N}})$ iff it is Dedekind complete, atomless, has countable π-weight and is not $\{0\}$. In particular, the regular open algebra $\mathrm{RO}(\mathbb{R})$ is isomorphic to $\mathfrak{G}$.

(b) Every atomless order-closed subalgebra of $\mathfrak{G}$ is isomorphic to $\mathfrak{G}$.

515P Proposition A Boolean algebra $\mathfrak{A}$ is isomorphic to $\mathrm{RO}(\{0,1\}^{\omega_1})$ iff
(α) it is non-zero, ccc and Dedekind complete,
(β) every non-zero principal ideal of $\mathfrak{A}$ has π-weight ω_1,
(γ) there is a non-decreasing family $\langle A_\xi\rangle_{\xi<\omega_1}$ of countable subsets of $\mathfrak{A}$ such that
each A_ξ is order-dense in the order-closed subalgebra of $\mathfrak{A}$ which it generates,
$A_\zeta=\bigcup_{\xi<\zeta}A_\xi$ for every non-zero countable limit ordinal ζ,
$\bigcup_{\xi<\omega_1}A_\xi$ is order-dense in $\mathfrak{A}$.

515Q Proposition Let $\mathfrak{A}$ be an atomless order-closed subalgebra of $\mathfrak{G}=\mathrm{RO}(\{0,1\}^{\omega_1})$. Then $\mathfrak{A}$ is isomorphic either to $\mathrm{RO}(\{0,1\}^{\omega})$ or to $\mathfrak{G}$ or to the simple product $\mathrm{RO}(\{0,1\}^{\omega})\times\mathfrak{G}$.

p 56 l 27 (515X) Add new exercise:

(b) Give an example of a Boolean algebra $\mathfrak{A}$ with Boolean-independent subalgebras $\mathfrak{B}$, $\mathfrak{C}$ such that the order-closed subalgebras generated by $\mathfrak{B}$ and $\mathfrak{C}$ are not Boolean-independent.

Other exercises have been renamed: 515Xb-515Xe are now 515Xc-515Xf.

p 56 l 42 (515Y) Add new exercise:

(b) Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra, and $\kappa\le\#(\mathfrak{A})$ a regular uncountable cardinal. Show that there is a strictly increasing family $\langle\mathfrak{A}_\xi\rangle_{\xi<\kappa}$ of subalgebras of $\mathfrak{A}$ with union $\mathfrak{A}$.

p 58 l 3 (proof of 516C): for '$(f(a_\xi),d)\in S$' and '$(a_\xi,g(d))\in R$' read '$(\phi(a_\xi),d)\in S$' and '$(a_\xi,\psi(d))\in R$'.

p 61 l 17 (statement of 516P): when I is infinite and $\theta\le\omega$ we need to suppose that κ is a regular uncountable cardinal, as in 516Ob. Similarly in 516Tb, 516Xj(ii) and 516Xk(ii) (now 516Xk-516Xl).

p 62 l 9 Proposition 516S has been rewritten, and now reads

516S Proposition Let $\mathfrak{A}$ be a Boolean algebra.

(a) If $\mathfrak{B}$ is a subalgebra of $\mathfrak{A}$ and $(\kappa,\lambda,{<}\theta)$ is a precaliber triple of $\mathfrak{A}$ such that $\theta\le\omega$, then $(\kappa,\lambda,{<}\theta)$ is a precaliber triple of $\mathfrak{B}$. In particular, every precaliber pair of $\mathfrak{A}$ is a precaliber pair of $\mathfrak{B}$ and $\mathfrak{B}$ will satisfy Knaster's condition if $\mathfrak{A}$ does.

(b) If $\mathfrak{B}$ is a regularly embedded subalgebra of $\mathfrak{A}$, then every precaliber triple of $\mathfrak{A}$ is a precaliber triple of $\mathfrak{B}$.

(c) If $\mathfrak{B}$ is a Boolean algebra and $\phi:\mathfrak{A}\to\mathfrak{B}$ is a surjective order-continuous Boolean homomorphism, then every precaliber triple of $\mathfrak{A}$ is a precaliber triple of $\mathfrak{B}$.

(d) If $\mathfrak{B}$ is a principal ideal of $\mathfrak{A}$ then every precaliber triple of $\mathfrak{A}$ is a precaliber triple of $\mathfrak{B}$.

(e) If $\mathfrak{A}$ is the simple product of a family $\langle\mathfrak{A}_i\rangle_{i\in I}$ of Boolean algebras, $(\kappa,\lambda,{<}\theta)$ is a precaliber triple of $\mathfrak{A}_i$ for every $i\in I$ and $\operatorname{cf}\kappa>\#(I)$, then $(\kappa,\lambda,{<}\theta)$ is a precaliber triple of $\mathfrak{A}$.

p 62 l 44 Add new result:

516V Proposition Let $\mathfrak{A}$ be an atomless Boolean algebra which satisfies Knaster's condition. Then $\mathfrak{A}$ has an atomless order-closed subalgebra with countable Maharam type.

p 62 l 47 (516X) Add new exercises:

(b) Let P and Q be partially ordered sets, and $f:P\to Q$ a surjection such that, for any finite set $I\subseteq P$, I is bounded above in P iff $f[I]$ is bounded above in Q. Show that P and Q have the same upwards precaliber pairs.

(n) Suppose that $\mathfrak{A}$ and $\mathfrak{B}$ are Boolean algebras and that there is a surjective Boolean homomorphism from $\mathfrak{A}$ to $\mathfrak{B}$. Show that if $(\kappa,\lambda,{<}\theta)$ is a precaliber triple of $\mathfrak{A}$ and θ is countable, then $(\kappa,\lambda,{<}\theta)$ is a precaliber triple of $\mathfrak{B}$.

Other exercises have been rearranged: 516Xb is now 516Xc, 516Xc-516Xd are now 516Xe-516Xf, 516Xe is now 516Xd, 516Xf-516Xl are now 516Xg-516Xm.

p 63 l 26 (516Y) Add new exercise:

(a) Let $\mathfrak{A}$ be an atomless Dedekind complete ccc Boolean algebra such that $(\kappa,\kappa,2)$ is a precaliber triple of $\mathfrak{A}$ for every regular uncountable cardinal κ. Then $\mathfrak{A}$ has an atomless closed subalgebra of countable Maharam type.

p 64 l 45 (part (b) of the proof of 517B) for '$\mathcal{D}$' read '$\mathcal{C}$' (twice).

p 66 l 10 (statement of 517Gb): we need to suppose that κ is uncountable. Similarly in 517Xe. See note on 516P above.

p 71 l 12 (Proposition 517R): add new part, formerly 555K:

(c) Suppose that X is a set and $\#(X) < \mathfrak{p}$. Then there is a countable set $\mathcal{A} \subseteq \mathcal{P}X$ such that $\mathcal{P}X$ is the σ-algebra generated by $\mathcal{A}$.

p l (517Y) Add new exercise:

(d) (i) Suppose that $\mathcal{A}$, $\mathcal{B} \in [[\mathbb{N}]^\omega]^{<\mathfrak{p}}$, $\mathcal{A}$ is downwards-directed and $A \cap B$ is infinite for all $A \in \mathcal{A}$, $B \in \mathcal{B}$. Show that there is a set $D \subseteq \mathbb{N}$ such that $D \setminus A$ is finite for every $A \in \mathcal{A}$ and $D \cap B$ is infinite for every $B \in \mathcal{B}$. (ii) Show that $\mathfrak{p}$ is regular.

p 73 l 23 (part (a-i) of the proof of 518B): for '$f(p) = [p_0, \infty[$ if $p \geq p_0$, $]-\infty, p]$ if $p \leq p_0$' read '$f(p) = [p_0, \infty[$ if $p \geq p_0$, $]-\infty, p_0]$ if $p < p_0$'.

p 73 l 43 (statement of part (b) of 518C): for 'Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra' read 'Let $\mathfrak{A}$ be an infinite Dedekind complete Boolean algebra'.

p 77 l 41 (part (d) of the proof of 518J): in fact the proof as written assumes that $\zeta \in A$, rather than that $\zeta \notin A$.

p 78 l 7 (part (d) of the proof of 518J): for '$\xi \in A \triangle (V \cap V')$' read '$\xi \in A \cap (V \triangle V')$'.

p l The propositions 518A and 518B have been put in reverse order.

p 79 l 29 Lemma 518O has been revised, and now reads

Lemma Let $\mathfrak{A}$ be a Boolean algebra, κ a cardinal and $\mathbb{G}$ a κ-Geschke system in $\mathfrak{A}$. Suppose that $\lambda \geq \kappa$ is a regular uncountable cardinal and that $f : [\mathfrak{A}]^{<\omega} \to [\mathfrak{A}]^{<\lambda}$ is a function. Then there is a $\mathfrak{B} \in \mathbb{G}$ such that $\#(\mathfrak{B}) < \lambda$ and $f(I) \subseteq \mathfrak{B}$ whenever $I \in [\mathfrak{B}]^{<\omega}$.

p 81 l 47 There is a serious muddle in part (b-ii) of the proof of 518R, which ought to have been

(ii) ? Suppose, if possible, that $\mathfrak{G}$ is tightly ω_1-filtered. Then it has an ω_1-Geschke system $\mathbb{B}$ say (518P). By 518O, with $\lambda = \omega_3$ and

$$f(\emptyset) = \{c''_{\xi t} : \xi < \omega_2, \, t \in T\} = C$$

say, there is a $\mathfrak{B}_1 \in \mathbb{B}$ such that $C \subseteq \mathfrak{B}_1$ and $\#(\mathfrak{B}_1) \leq \omega_2$; take $\xi \in \omega_3 \setminus \tilde{J}(\mathfrak{B}_1)$, and let $\mathfrak{B}_2 \in \mathbb{B}$ be such that $\mathfrak{B}_2$ is countable and $a_{\xi p} \in \mathfrak{B}_2$ for every $p \in \mathbb{Q}$. Then $\tilde{J}(\mathfrak{B}_2)$ is countable, so there is an $\eta \in \omega_2 \setminus \tilde{J}(\mathfrak{B}_2)$.

Set $w = \sup_{p,q \in \mathbb{Q}, p \leq q} a_{\xi p} \cap a_{\eta q}$, and for $t \in T$ set $w_t = c'_{\xi t} \cap c''_{\eta t}$. Then w belongs to a countable $\mathfrak{B}_0 \in \mathbb{B}$, while the subalgebra $\mathfrak{B}^*$ of $\mathfrak{G}$ generated by $\mathfrak{B}_1 \cup \mathfrak{B}_2$ belongs to $\mathbb{B}$. But if we set $J = \{\xi\} \times \mathbb{N}$, $K = \{\eta\} \times \mathbb{N}$ then we see that $\mathfrak{B}_1 \subseteq \mathfrak{G}_{(\omega_3 \times \mathbb{N}) \setminus J}$ and $\mathfrak{B}_2 \subseteq \mathfrak{G}_{(\omega_3 \times \mathbb{N}) \setminus K}$. So (a) tells us that any member of $\mathfrak{B}^*$ included in w can include only finitely many w_t, while $w_t \in \mathfrak{B}^* \cap [0, w]$. Thus $\mathrm{cf}(\mathfrak{B}^* \cap [0, w]) \geq \omega_1$. On the other hand, by (γ) of 518N, the countable set $\mathfrak{B}_0 \cap \mathfrak{B}^* \cap [0, w]$ is cofinal with $\mathfrak{B}^* \cap [0, w]$. **X**

This contradiction proves the result.

p 82 l 16 (518X) I do not know whether Exercise 518Xe is correct as written, and have restated it as

(f) Show that $\mathrm{FN}^*(\mathcal{P}\mathbb{N}/[\mathbb{N}]^{<\omega}) = \mathrm{FN}^*(\mathcal{P}\mathbb{N})$.

Exercise 518Xf is I think wrong, and should read

(g) Let P be a partially ordered set and Q a subset of P with Freese-Nation index κ in P. Show that if $\lambda \geq \max(\kappa, \mathrm{FN}(P))$ is a regular infinite cardinal then $\mathrm{FN}(Q) \leq \kappa$.

Exercise 518Xh has been revised in the light of the new formulation of the definition of tight filtration and is now

(i) Let $\mathfrak{A}$ be a Boolean algebra, κ a regular infinite cardinal and $\langle a_\xi \rangle_{\xi < \zeta}$ a family in $\mathfrak{A}$. For each $\alpha \leq \zeta$ let $\mathfrak{A}_\alpha$ be the subalgebra of $\mathfrak{A}$ generated by $\{a_\xi : \xi < \alpha\}$. Suppose that $\mathfrak{A}_\zeta = \mathfrak{A}$ and that the Freese-Nation index of $\mathfrak{A}_\alpha$ in $\mathfrak{A}_{\alpha+1}$ is at most κ for every $\alpha < \zeta$. Show that $\langle a_\xi \rangle_{\xi < \zeta}$ is a tight κ-filtration of $\mathfrak{A}$.

Add new exercises:

(c) Show that if P and Q are partially ordered sets, then $\mathrm{FN}(P \times Q)$ is at most the cardinal product $\mathrm{FN}(P) \cdot \mathrm{FN}(Q)$.

(h) Let P be a partially ordered set and $\langle P_\xi \rangle_{\xi<\zeta}$ a non-decreasing family of subsets of P such that $P_\xi = \bigcup_{\eta<\xi} P_\eta$ for every non-zero limit ordinal $\xi \le \zeta$. Suppose that κ is a regular infinite cardinal such that the Freese-Nation index of P_ξ in $P_{\xi+1}$ is at most κ for every $\xi < \zeta$. Show that the Freese-Nation index of P_0 in P_ζ is at most κ.

(k) Let $\mathfrak{A}$ be a Boolean algebra, κ a cardinal and $\mathbb{G}$ a κ-Geschke system in $\mathfrak{A}$. Show that $\mathbb{G}$ is a λ-Geschke system for every $\lambda \ge \kappa$.

Other exercises have been rearranged: 518Xc-518Xd are now 518Xd-518Xe, 518Xg is now 518Xj, 518Xi is now 518Xl.

p 82 l 27 Part (ii) of Exercise 518Yc has been revised, and now reads

(ii) Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra. Show that $\mathrm{FN}(\mathfrak{A}) \le \mathrm{FN}(L^0(\mathfrak{A})) \le \mathrm{FN}(\mathfrak{A}^{\mathbb{N}})$.

p 85 l 20 Add new result:

521D Proposition Let (X, Σ, μ) be a measure space and $(\mathfrak{A}, \bar{\mu})$ its measure algebra.

(a) $\pi(\mathfrak{A}) \le \pi(\mu) \le \max(\pi(\mathfrak{A}), \mathrm{cf}\,\mathcal{N}(\mu))$.

(b) If $\mu X > 0$, then $\mathrm{non}\,\mathcal{N}(\mu) \le \pi(\mu)$.

(c) If (X, Σ, μ) has locally determined negligible sets, then $\mathrm{shr}\,\mathcal{N}(\mu) \le \pi(\mu)$.

(d) Suppose that there is a topology $\mathfrak{T}$ on X such that $(X, \mathfrak{T}, \Sigma, \mu)$ is a quasi-Radon measure space. Then, writing $\mathfrak{A}^+$ for $\mathfrak{A} \setminus \{0\}$, the partially ordered sets $(\Sigma \setminus \mathcal{N}(\mu), \supseteq)$ and $(\mathfrak{A}^+, \supseteq)$ are Tukey equivalent and $\pi(\mu) = \pi(\mathfrak{A})$.

Other paragraphs have been rearranged: 521D-521N are now 521F-521P, 521O is now 521E, 521P-521S are now 521Q-521T.

p 85 l 23 In Proposition 521D (now 521F), add new parts:

(e) If either $A \in \Sigma$ or (X, Σ, μ) has locally determined negligible sets, $\pi(\mu_A) \le \pi(\mu)$.

(f) If μ_A is semi-finite, then $\tau(\mu_A) \le \tau(\mu)$.

p 85 l 40 In Proposition 521E (now 521G), add

$\tau(\mu) \le \max(\omega, \sup_{i\in I} \tau(\mu_i), \min\{\lambda : \#(I) \le 2^\lambda\})$,

$\pi(\mu)$ is the cardinal sum $\sum_{i\in I} \pi(\mu_i)$.

p 86 l 3 In Proposition 521F (now 521H), add new fragments:

(a)(ii) If there is a topology on Y such that ν is a topological measure inner regular with respect to the closed sets, then $\pi(\nu) \le \pi(\mu)$.

(iii) If ν is σ-finite, then $\tau(\nu) \le \tau(\mu)$.

p 86 l 37 In part (a) of Proposition 521H, now 521J, add a new item:

$\pi(\mu) \ge \sup_{i\in I} \pi(\mu_i)$.

p 87 l 40 (part (c-i) of the proof of 521H, now 521J): for '$f_i(x) \in \psi_i(C_{Enmi})$' read '$f_i(x(i)) \in \psi_i(C_{Enmi})$'.

p 88 l 22 In Proposition 521I (now 521K), add

$\pi(\mu) \ge \pi(\mu_L)$, where μ_L is Lebesgue measure on $\mathbb{R}$.

p 89 l 10 Proposition 521K, now 521M, has been revised, and now reads

521M Proposition Let (X, Σ, μ) be a complete locally determined measure space of magnitude at most $\mathrm{add}\,\mu$. Then it is strictly localizable.

p l (Proposition 521M, now 521O) Add new part:

(d) If $\langle A_i \rangle_{i\in I}$ is a disjoint family of subsets of X and $\#(I) > \max(\omega, \mathrm{mag}(\mu))$ then there is an $i \in I$ such that $X \setminus A_i$ has full outer measure.

p 92 l 4 Part (d) of 521R (now 521S) has been strengthened to

(d) There is a countably separated semi-finite measure space with magnitude $2^{\mathfrak{c}}$.

p 92 l 42 Proposition 521S (now 521T) has been re-formulated, and now reads

521T Proposition Let I be a set, and suppose that a non-zero $\theta \in (M_{\mathrm{m}} \cap M_\tau^\perp)^+$, as defined in §464, corresponds to the Radon measure μ_θ on βI. Let ν be the usual measure on $\mathcal{P}I$. Then the Maharam type of μ_θ is at least $\operatorname{cov}\mathcal{N}(\nu)$.

p 92 l 49 (proof of 521S, now 521T): for '$\nu\times\nu^{n+1}$-negligible', 'ν^{n+1}-negligible' read '$\nu\times\nu^{n+1}$-conegligible', 'ν^{n+1}-conegligible'.

p 93 l 13 The exercises 521X have been substantially changed. 521Xb-521Xd are now 521Xg-521Xi, 521Xe has been dropped, 521Xf-521Xi are now 521Xj-521Xm, 521Xj is now 521Xe, 521Xk is now 521Xb. There are new exercises:

(c) Let (X,Σ,μ) be a complete locally determined measure space, and κ a cardinal such that $\kappa < \operatorname{cov}\mathcal{N}(\mu_E)$ for every non-negligible measurable set $E \subseteq X$, writing μ_E for the subspace measure. Suppose that $A \subseteq X$ is such that both A and $X \setminus A$ are expressible as the union of at most κ members of Σ. Show that $A \in \Sigma$.

(d)(i) Find a probability space (X,Σ,μ), with measure algebra $\mathfrak{A}$, such that $\pi(\mathfrak{A}) < \pi(\mu)$. (ii) Find a probability space (X,Σ,μ), with null ideal $\mathcal{N}(\mu)$, such that $\operatorname{cf}\mathcal{N}(\mu) < \pi(\mu)$. (iii) Find a probability space (X,Σ,μ) such that $\pi(\mu) < \operatorname{cf}\mathcal{N}(\mu)$.

(f) Let (X,Σ,μ) be a semi-finite measure space which is not purely atomic. Show that $\pi(\mu) \ge \pi(\mu_L)$, where μ_L is Lebesgue measure on $\mathbb{R}$.

(n) For a measure space (X,Σ,μ) with null ideal $\mathcal{N}(\mu)$, write $\operatorname{hcov}(\mu)$ for $\inf_{E\in\Sigma\setminus\mathcal{N}(\mu)} \operatorname{cov}(E,\mathcal{N}(\mu))$. (Count $\inf\emptyset$ as ∞, as usual.) Show that if (X,Σ,μ) and (Y,T,ν) are semi-finite measure spaces, neither having zero measure, with c.l.d. product $(X\times Y,\Lambda,\lambda)$, then $\operatorname{hcov}(\lambda) = \min(\operatorname{hcov}(\mu),\operatorname{hcov}(\nu))$.

p 93 l 23 521Xc (now 521Xh) has a new fragment:

$$\pi(\mu\times\nu) \ge \max(\pi(\mu),\pi(\nu)).$$

p 93 l 36 521Xj (now 521Xe) has been expanded, and now reads:

Let (X,Σ,μ) be a measure space and ν an indefinite-integral measure over μ. Show that $\operatorname{add}\mathcal{N}(\nu) \ge \operatorname{add}\mathcal{N}(\mu)$, $\operatorname{cf}\mathcal{N}(\nu) \le \operatorname{cf}\mathcal{N}(\mu)$, $\operatorname{non}\mathcal{N}(\nu) \ge \operatorname{non}\mathcal{N}(\mu)$, $\operatorname{cov}\mathcal{N}(\nu) \le \operatorname{cov}\mathcal{N}(\mu)$, $\operatorname{shr}\mathcal{N}(\nu) \le \operatorname{shr}\mathcal{N}(\mu)$, $\operatorname{shr}^+\mathcal{N}(\nu) \le \operatorname{shr}^*\mathcal{N}(\mu)$, $\pi(\nu) \le \pi(\mu)$ and $\tau(\nu) \le \tau(\mu)$.

p 94 l 1 (521Y) Add new exercise:

(b) Find a strictly localizable measure space (X,Σ,μ), a set Y, and a function $f : X \to Y$ such that, setting $\nu = \mu f^{-1}$, ν is semi-finite and $\tau(\mu) < \tau(\nu)$.

521Yb-521Yd are now 521Yc-521Ye.

p 95 l 17 In (ii) of the statement of Lemma 522C, the definition of $\preccurlyeq$ should read '$f \preccurlyeq g$ if either $f \le g$ or $\{n : g(n) \le f(n)\}$ is finite'. Add new part:

(iii) $(\mathbb{N}^{\mathbb{N}},\le^*) \equiv_{\mathrm{T}} (\mathbb{N}^{\mathbb{N}},\preccurlyeq)$.

p l (part (e) of the proof of 522C): for '$(\mathbb{N}^{\mathbb{N}},\le', [\mathbb{N}^{\mathbb{N}}]^{\le\omega}) \equiv_{\mathrm{GT}} (\mathbb{N}^{\mathbb{N}},\le^*,\mathbb{N}^{\mathbb{N}}) \equiv_{\mathrm{GT}} (\mathbb{N}^{\mathbb{N}},\le^*,\mathbb{N}^{\mathbb{N}})$' read '$(\mathbb{N}^{\mathbb{N}},\le', [\mathbb{N}^{\mathbb{N}}]^{\le\omega}) \equiv_{\mathrm{GT}} (\mathbb{N}^{\mathbb{N}},\le^*,\mathbb{N}^{\mathbb{N}}) \equiv_{\mathrm{GT}} (\mathbb{N}^{\mathbb{N}},\le^*,\mathbb{N}^{\mathbb{N}})$'.

p 98 l 39 In the statement of Lemma 522L, for '$(\mathbb{N}^{\mathbb{N}},\subseteq^*,\mathcal{S}_I^{(\alpha)}) \equiv_{\mathrm{GT}} (\mathbb{N}^{\mathbb{N}},\subseteq^*,\mathcal{S}_I^{(\beta)})$' read '$(I^{\mathbb{N}},\subseteq^*,\mathcal{S}_I^{(\alpha)}) \equiv_{\mathrm{GT}} (I^{\mathbb{N}},\subseteq^*,\mathcal{S}_I^{(\beta)})$'.

p 111 l 5 (Theorem 522R, now 522S) Add new part:

(c) $\operatorname{non}\mathcal{M}$ is the least cardinal of any set $A \subseteq \mathbb{N}^{\mathbb{N}}$ such that for every $g \in \mathbb{N}^{\mathbb{N}}$ there is an $f \in A$ such that $\{n : f(n) = g(n)\}$ is infinite.

p 102 l 38 (part (b-i) of the proof of 522S): for '$i \in \operatorname{dom}\sigma'' \setminus \operatorname{dom}\sigma$ and $\sigma''(i)$ must be disjoint from $F' \supseteq F$' read '$i \in \operatorname{dom}\sigma'' \setminus \operatorname{dom}\sigma'$ and $\sigma''(i)$ must be disjoint from $F' \supseteq F$'.

p l (part (b-ii-α) of the proof of 522S): for '$f^*(j+1) = \tilde f(\tilde f(f^*(j))) \ge \tilde f(\tilde f(g(i))) \ge \tilde f(g(i+1)) \ge g(i+2)$' read '$f^*(j+1) \ge \tilde f(\tilde f(f^*(j))) \ge \tilde f(\tilde f(g(i))) \ge \tilde f(g(i+1)) \ge g(i+2)$'.

p l Exercise 522Xc is wrong, and has been deleted. 522Xe is a copy of 521Xc, and has been dropped. 522Xb is now 522Xc, 522Xd is now 522Xe, 522Xf-522Xg are now 522Xg-522Xh.

p 108 l 13 There are rather a lot of blunders in the exercises 522Y.
522Yc is wrong, and has been revised as 522Yi.
In 522Yd (now 522Yc), we need to add the hypothesis that Q has no greatest member.
In 522Yf (now 522Ye), for 'Show that $\kappa = \mathfrak{c}$' read 'Show that $\kappa \ge \mathfrak{c}$'.
522Yg seems to be wrong, and I have deleted it.
522Yj (now 522Yh) should be

> Suppose that we have supported relations (A, R, B) and (A, S, A) such that $(a', b) \in R$ whenever $(a, b) \in R$ and $(a', a) \in S$. Show that if $\omega \le \operatorname{cov}(A, R, B) < \infty$ then $\operatorname{cf}(\operatorname{cov}(A, R, B)) \ge \operatorname{add}(A, S, A)$.

522Yk got upstaged by 522Xe-522Xf, so should be dropped.
The reshuffle means that 522Ye is now 522Yd, 522Yh is now 522Yf and 522Yi is now 522Yg.

p l Add new problem:

> **522Z** Is it the case that $(\mathbb{R}, \in, \mathcal{M}) \equiv_{\mathrm{GT}} (\mathbb{N}^{\mathbb{N}}, \mathtt{finint}, \mathbb{N}^{\mathbb{N}})$?

p 111 l 21 (proof of 523G): for 'since $\bigcup \mathcal{E}_J = \mathcal{E}$ covers $\{0,1\}^\kappa$, $\{H_J : J \in \mathcal{J}\}$ covers κ' read 'since $\bigcup_{J \in \mathcal{J}} \mathcal{E}_J = \mathcal{E}$ covers $\{0,1\}^\kappa$, $\{H_J : J \in \mathcal{J}\}$ covers $\{0,1\}^\kappa$'.

p 111 l 37 Theorem 523I has been reorganized and fractionally strengthened, and now reads

> **523I Theorem** (a) For any cardinal κ,
> (i) $\operatorname{non} \mathcal{N}_\kappa \le \max(\operatorname{non} \mathcal{N}, \operatorname{cf}[\kappa]^{\le\omega})$,
> (ii) $\operatorname{non} \mathcal{N}_{\kappa^+} \le \max(\kappa^+, \operatorname{non} \mathcal{N}_\kappa)$,
> (iii) $\operatorname{non} \mathcal{N}_{2^\kappa} \le \max(\mathfrak{c}, \operatorname{cf}[\kappa]^{\le\omega})$,
> (iv) $\operatorname{non} \mathcal{N}_{2^{\kappa^+}} \le \max(\kappa^+, \operatorname{non} \mathcal{N}_{2^\kappa})$.
> (b) If $\operatorname{cf} \kappa > \omega$, then $\operatorname{non} \mathcal{N}_{\kappa^+} \le \max(\operatorname{cf} \kappa, \sup_{\lambda<\kappa} \operatorname{non} \mathcal{N}_\lambda)$.

p 113 l 10 (statement of Corollary 523K): for '$\#(K) < \max(\omega, \#(I))$' read '$\#(K) < \min(\omega, \#(I))$'.

p l (part (c) of the proof of 523N): for '$\operatorname{cov}(\kappa, [\kappa]^{\le\omega})$' read '$\operatorname{cov}(\kappa, \in, [\kappa]^{\le\omega})$'.

p 115 l 34 I have been unable to decide whether the result in Exercise 523Xb ('$\operatorname{non} \mathcal{N}_{\operatorname{Tr}_{\mathcal{I}}(\kappa;\lambda)} \le \max(\kappa, \operatorname{non} \mathcal{N}_\lambda)$ whenever $\mathcal{I}$ is a proper σ-ideal of subsets of κ' is true, and have therefore dropped this exercise for the time being.
523Xc-523Xe are now 523Xb-523Xd.

p l Part (ii) of Exercise 523Yf is superseded by 524Na below, so has been dropped.

p 132 l 6 Add new result:

> **524U Lemma** Let $(\mathfrak{A}, \bar{\mu})$ be a probability algebra. Then there is a Radon probability measure on $\{0,1\}^{\tau(\mathfrak{A})}$ with measure algebra isomorphic to $(\mathfrak{A}, \bar{\mu})$.

p 133 l 32 Proposition 525B is now 521Dd. 525C-525U are now 525B-525T.

p 140 l 37 (Corollary 525U, now 525T, part (d)): add new sentence 'So if $\omega \le \kappa < \mathfrak{m}_K$, κ is a measure-precaliber of every probability algebra.'

p 141 l 5 (525X) Add new exercises:

> **(c)**(ii) Suppose that $\operatorname{non} \mathcal{N}_\omega = \mathfrak{c}$. Show that $\mathfrak{c}$ is not a precaliber of $\mathfrak{B}_\omega$.
>
> **(f)** Suppose that $\lambda \le \kappa$ are infinite cardinals, $(\mathfrak{A}, \bar{\mu})$ is a homogeneous probability algebra, and that $\gamma < 1$ is such that whenever $\langle a_\xi \rangle_{\xi<\kappa}$ is a family in $\mathfrak{A}$ such that $\bar{\mu} a_\xi \ge \gamma$ for every $\xi < \kappa$, there is a $\Gamma \in [\kappa]^\lambda$ such that $\{a_\xi : \xi \in \Gamma\}$ is centered. Show that (κ, λ) is a measure-precaliber pair of $(\mathfrak{A}, \bar{\mu})$.
>
> **(g)** Let $\mathfrak{A}$ be a Boolean algebra, and λ, κ cardinals such that (κ, λ) is a measure-precaliber pair of every probability algebra. Suppose that $A \subseteq \mathfrak{A} \setminus \{0\}$ has positive intersection number. Show that if $\langle a_\xi \rangle_{\xi<\kappa}$ is a family in A, then there is a $\Gamma \in [\kappa]^\lambda$ such that $\{a_\xi : \xi \in \Gamma\}$ is centered.
>
> **(i)** Show that if $(X, \mathfrak{T}, \Sigma, \mu)$ is a Radon measure space and $\mu X > 0$, then $\operatorname{cov} \mathcal{N}(\mu) \ge \mathfrak{m}_{\mathrm{K}}$.

525Xf is now 525Xh.

p 141 l 37 (Proposition 526A) Add new fragment:

If $\mathcal{I} \subseteq \mathcal{Z}$ is such that $\sum_{I\in\mathcal{I}} \nu I$ is finite, then $\bigcup\mathcal{I} \in \mathcal{Z}$.

p 149 l 27 (526L) Add new result:

526L Proposition $\mathcal{N}\mathrm{wd} \not\preccurlyeq_{\mathrm{T}} \mathcal{Z}$, so $\mathcal{Z} \not\preccurlyeq_{\mathrm{T}} \mathbb{N}^{\mathbb{N}}$ and $\ell^1 \not\preccurlyeq_{\mathrm{T}} \mathcal{N}\mathrm{wd}$.

526L is now 526M.

p l (Exercise 526Xc) For '$\mathrm{non}\,\mathcal{N}\mathrm{wd} = \mathrm{non}\,\mathcal{M}$' and '$\mathrm{cf}\,\mathcal{N}\mathrm{wd} = \mathrm{cf}\,\mathcal{M}$' read '$\mathrm{non}\,\mathcal{N}\mathrm{wd} = \omega$' and '$\mathrm{cf}\,\mathcal{N}\mathrm{wd} \le \mathrm{cf}\,\mathcal{M}$'.

p 150 l 30 526Z has been deleted, following the solution of the problems there.

p 152 l 8 In Theorem 527C, we have to assume that μ and ν are inner regular with respect to the Borel sets.

p 152 l 23 (part (a-i) of the proof of 527D): for '$W \in \mathcal{M}^* \setminus \mathcal{M}(X \times Y)$' read '$W \in \mathcal{M}_1^* \setminus \mathcal{M}(X \times Y)$'. Similarly, on lines 29-30, each $\mathcal{M}^*$ should be $\mathcal{M}_1^*$.

p 153 l 19 (part (a) of the proof of 527F) for 'algebra of subsets of $\mathcal{P}(\mathbb{N}^{\mathbb{N}})$' read 'subalgebra of $\mathcal{P}(\mathbb{N}^{\mathbb{N}})$'.

p 153 l 25 (part (a) of the proof of 527F): following '$\tau \subseteq \sigma$ and $U_\tau \setminus E \supseteq U_{\sigma^\frown<0>} \notin \mathcal{I}$, or $U_\tau \cap U_\sigma = \emptyset$ and $U_\tau \setminus E \supseteq U_\tau \notin \mathcal{I}$' add the remaining possibility 'or there is an $i \in \mathbb{N}$ such that $\tau \supseteq \sigma^\frown<i>$ and again $U_\tau \setminus E \subseteq U_\tau \notin \mathcal{I}$'.

p 156 l 32 (part (b-iv) of the proof of 527J). There is a slip in the proof that $\mathcal{D}$ is cofinal with $\mathcal{D}'$. The closure $\overline{D}$ of the arbitrary member D of $\mathcal{D}$ is closed, and $\overline{D}[\{x\}]$ is always nowhere dense, but this is not quite enough to put it into $\mathcal{D}_0$ as described in 527I. Instead, choose for each $H \in \mathcal{H} \setminus \{\emptyset\}$ a $U_H \in \mathcal{U}$ such that $U_H \subseteq (X \times H) \setminus D$, and try $D_1 = (X \setminus Y) \setminus \bigcup_{H\in\mathcal{H}} U_H$. Then D_1 is a closed set including D, and its vertical sections are nowhere dense, and its complement is a countable union of sets of the form $G \times H$ where G is open and $H \in \mathcal{H}$; so $D_1 \in \mathcal{D}_0$ and $D_1 \in \mathcal{D}$, as required.

p 156 l 36 (part (b-v) of the proof of 527J): for '527Gc' read '526Hc'.

p 157 l 30 (part (c-iii) of the proof of 527J) for '$f^{-1}[F] \subseteq N_V \cup \{x : \phi_0[\{x\}] \notin \mathcal{M}(Y)\}$' read '$f^{-1}[F] \subseteq N_V \cup \{x : \phi_0(V)[\{x\}] \notin \mathcal{M}(Y)\}$'.

p 157 l 37 (part (c-iii) of the proof of 527J) for '$V_0 = (N^* \times Y) \cup \tilde{W}$' and '$x \in X \setminus N^*$' read '$V_0 = (\tilde{N} \times Y) \cup \tilde{W}$' and '$x \in X \setminus \tilde{N}$'.

p 158 l 23 (part (c) of the proof of 527L): for 'because A is uncountable' read 'because A_n is uncountable'.

p 160 l 37 (part (d) of the proof of 527O): for '$F \in \mathcal{B}(Y) \setminus \mathcal{M}(X)$' read '$F \in \mathcal{B}(Y) \setminus \mathcal{M}(Y)$'.

p 161 l 1 (527X) Exercise 527Xg has been moved to 513Xr. Exercise 527Xh has been moved to 527Ye, and corrected, replacing '$\mathcal{N}(\mu) \ltimes_{\mathcal{B}(X\times Y)} \mathcal{M}(Y) \preccurlyeq_{\mathrm{T}} \mathcal{N}(\mu) \times \mathcal{M}$' with '$\mathcal{N}(\mu) \ltimes_{\mathcal{B}(X\times Y)} \mathcal{M}(Y) \preccurlyeq_{\mathrm{T}} \mathcal{N}(\mu) \times \mathcal{N}$'. Add new exercises:

(d) Let X be a set, Σ a σ-algebra of subsets of X, and $\mathcal{I}$ an ideal of subsets of X; let Y be a topological space, $\mathcal{B}$ its Borel σ-algebra, $\widehat{\mathcal{B}}$ its Baire-property algebra, and $\mathcal{M}$ its meager ideal. Show that $\mathcal{I} \ltimes_{\Sigma\widehat{\otimes}\mathcal{B}} \mathcal{M} = \mathcal{I} \ltimes_{\Sigma\widehat{\otimes}\widehat{\mathcal{B}}} \mathcal{M}$.

(h) Show that a measurable algebra is harmless iff it is purely atomic.

527Xd-527Xf are now 527Xe-527Xg.

p 161 l 20 (527Y) Exercise 527Yd is wrong, and has been dropped. Add new exercises:

(b) Extend the idea of 527Xb to define an ideal $\ltimes_{\xi<\zeta}\mathcal{I}_\xi$ of subsets of $\prod_{\xi<\zeta} X_\zeta$ when ζ is any ordinal and I_ξ is an ideal of subsets of X_ξ for every $\xi < \zeta$.

(d) Let $(Y, \mathfrak{T})$ be a topological space. Show that there is a topology $\mathfrak{S}$ on Y, coarser than $\mathfrak{T}$, such that the weight of $(Y, \mathfrak{S})$ is equal to the π-weight of $(Y, \mathfrak{T})$, and the two topologies have the same nowhere dense sets, the same meager ideal and the same Baire-property algebras.

527Yc has been moved to 527Yf, and amended, as follows:

(f) Let $\langle\mathfrak{A}_i\rangle_{i\in I}$ be any family of harmless Boolean algebras all satisfying Knaster's condition, and $\mathfrak{A}$ their free product. Show that $\mathfrak{A}$ is harmless.

527Yb is now 527Yc.

p 162 l 8 Add new result:

528B Lemma Let $(\mathfrak{A},\bar\mu)$ be a measure algebra and $0<\gamma\le\bar\mu 1$. Set $P=\{a:a\in\mathfrak{A},\ \bar\mu a<\gamma\}$.

(a) Two elements a, $b\in P$ are compatible upwards in P iff $\bar\mu(a\cup b)<\gamma$.

(b) Suppose that $(\mathfrak{A},\bar\mu)$ is semi-finite and atomless.

(i) P is separative upwards, so $[a,\infty[\,\in\mathrm{RO}^{\uparrow}(P)$ for every $a\in P$.

(ii) If $A\subseteq P$ is non-empty, then the infimum $\inf_{a\in A}[a,\infty[$ is empty unless $\sup A$ is defined in $\mathfrak{A}$ and belongs to P, and in this case $\inf_{a\in A}[a,\infty[\,=[\sup A,\infty[$.

528B is now 528C, and 528C is now the second part of 528D. 528E-528F have been reorganised. An initial fragment of the proof of the old 528E has been extracted as a lemma:

528E Lemma Let $(\mathfrak{A},\bar\mu)$ be an atomless semi-finite measure algebra. Then there is a family $\langle c_\alpha\rangle_{\alpha\in[0,\bar\mu 1]}$ in $\mathfrak{A}$ such that $c_\alpha\subseteq c_\beta$ and $\bar\mu c_\alpha=\alpha$ whenever $0\le\alpha\le\beta\le\bar\mu 1$, and $\alpha\mapsto c_\alpha$ is continuous for the measure-algebra topology of $\mathfrak{A}$.

The former 528E as stated is now 528Fc; the old 528F is now 528Fa, and in addition there is a new part in 528F, being

(b) Suppose that $\mathfrak{A}$ is atomless and semi-finite, and that $0<\gamma<\bar\mu 1$. Let $\langle e_k\rangle_{k\in\mathbb{N}}$ be a non-decreasing sequence in $\mathfrak{A}$ with supremum 1, and suppose that $\bar\mu e_k\ge\gamma$ for every $k\in\mathbb{N}$. Then we have a sequence $\langle\pi_k\rangle_{k\in\mathbb{N}}$ such that $\pi_k:\mathrm{AM}(\mathfrak{A}_{e_k},\bar\mu\restriction\mathfrak{A}_{e_k},\gamma)\to\mathrm{AM}(\mathfrak{A},\bar\mu,\gamma)$ is a regular embedding for every $k\in\mathbb{N}$, and $\bigcup_{k\in\mathbb{N}}\pi_k[\mathrm{AM}(\mathfrak{A}_{e_k},\bar\mu\restriction\mathfrak{A}_{e_k},\gamma)]$ τ-generates $\mathrm{AM}(\mathfrak{A},\bar\mu,\gamma)$.

p 173 l 22 Part (b) of Theorem 528R, on Maharam types of amoeba algebras, has been moved to 528V below in a stronger form.

p 176 l 10 The following material has been added:

528S Definition Let $(\mathfrak{A},\bar\mu)$ be a measure algebra. A **well-spread basis** for $\mathfrak{A}$ is a non-decreasing sequence $\langle D_n\rangle_{n\in\mathbb{N}}$ of subsets of $\mathfrak{A}$ such that

(i) setting $D=\bigcup_{n\in\mathbb{N}}D_n$, $\#(D)\le\max(\omega,c(\mathfrak{A}),\tau(\mathfrak{A}))$;

(ii) if $a\in\mathfrak{A}$, $\gamma\in\mathbb{R}$ and $\bar\mu a<\gamma$, there is a set $D\subseteq\bigcup_{n\in\mathbb{N}}D_n$ such that $a\subseteq\sup D$ and $\bar\mu(\sup D)<\gamma$;

(iii) if $n\in\mathbb{N}$ and $\langle d_i\rangle_{i\in\mathbb{N}}$ is a sequence in D_n such that $\bar\mu(\sup_{i\in\mathbb{N}}d_i)<\infty$, there is an infinite set $J\subseteq\mathbb{N}$ such that $d=\sup_{i\in J}d_i$ belongs to D_n;

(iv) whenever $n\in\mathbb{N}$, $a\in\mathfrak{A}$ and $\bar\mu a\le\gamma'<\gamma<\bar\mu 1$, there is a $b\in\mathfrak{A}$ such that $a\subseteq b$ and $\gamma'\le\bar\mu b<\gamma$ and $\bar\mu(b\cup d)\ge\gamma$ whenever $d\in D_n$ and $d\not\subseteq a$.

528T Lemma (a) Let κ be an infinite cardinal, and $\langle e_\xi\rangle_{\xi<\kappa}$ the standard generating family in $\mathfrak{B}_\kappa$. For $n\in\mathbb{N}$ let C_n be the set of elements of $\mathfrak{B}_\kappa$ expressible as $\inf_{\xi\in I}e_\xi\cap\inf_{\xi\in J}(1\setminus e_\xi)$ where I, $J\subseteq\kappa$ are disjoint and $\#(I\cup J)\le n$. Then $\langle C_n\rangle_{n\in\mathbb{N}}$ is a well-spread basis for $(\mathfrak{B}_\kappa,\bar\nu_\kappa)$. Moreover,

(*) for each $n\ge 1$, there is a set $C'_n\subseteq C_n$, with cardinal κ, such that $\bar\nu_\kappa c=2^{-n}$ for every $c\in C'_n$, and whenever $a\in\mathfrak{B}_\kappa\setminus\{1\}$ and $I\subseteq C'_n$ is infinite, there is a $c\in I$ such that $c'\not\subseteq a\cup c$ whenever $c\subset c'\in C_n$.

(b) Let $(\mathfrak{A},\bar\mu)$ be a measure algebra and $e\in\mathfrak{A}$. If $\langle C_n\rangle_{n\in\mathbb{N}}$ is a well-spread basis for $(\mathfrak{A}_e,\bar\mu\restriction\mathfrak{A}_e)$ and $\langle D_n\rangle_{n\in\mathbb{N}}$ is a well-spread basis for $(\mathfrak{A}_{1\setminus e},\bar\mu\restriction\mathfrak{A}_{1\setminus e})$, then $\langle C_n\cup D_n\rangle_{n\in\mathbb{N}}$ is a well-spread basis for $(\mathfrak{A},\bar\mu)$.

528U Lemma Let $(\mathfrak{A},\bar\mu)$ be an atomless semi-finite measure algebra and $0<\gamma<\bar\mu 1$. Let E, ϵ, $\preccurlyeq$ and $\mathcal{F}$ be such that

E is a partition of unity in $\mathfrak{A}$ such that $\mathfrak{A}_e$ is homogeneous and $0<\epsilon\le\bar\mu e<\infty$ for every $e\in E$;

$\preccurlyeq$ is a well-ordering of E such that $\tau(\mathfrak{A}_e)\le\tau(\mathfrak{A}_{e'})$ whenever $e\preccurlyeq e'$ in E;

$\mathcal{F}$ is a partition of E such that each member of $\mathcal{F}$ is either a singleton or a countable set with no $\preccurlyeq$-greatest member.

Let P_0 be

$$\{a:a\in\mathfrak{A},\ \bar\mu a<\gamma,\ \gamma\le\bar\mu(a\cup e)\text{ whenever }\{e\}\in\mathcal{F}\},$$

ordered by $\subseteq$. Then $\mathrm{RO}^{\uparrow}(P_0)$ has countable Maharam type.

528V Theorem Let $(\mathfrak{A},\bar\mu)$ be an atomless semi-finite measure algebra and $0<\gamma<\bar\mu 1$. Then $\mathrm{AM}(\mathfrak{A},\bar\mu,\gamma)$ has countable Maharam type.

p 176 l 17 Exercise 528Xc (now 528Xd) has been strengthened to

(d) Show that if $(\mathfrak{A},\bar\mu)$ is a probability algebra, $0<\gamma\le 1$ and $\kappa\ge\max(\omega,\tau(\mathfrak{A}))$ then $\mathrm{AM}(\mathfrak{A},\bar\mu,\gamma)$ can be regularly embedded in $\mathrm{AM}(\mathfrak{B}_\kappa,\bar\nu_\kappa,\gamma)$.

p 176 l 18 Exercise 528Xd (now 528Xf) is wrong as stated, and should read

Let $\mathfrak{A},\bar\mu)$ be an atomless σ-finite measure algebra and $0<\gamma<\bar\mu 1$. Show that $\mathfrak{m}(\mathrm{AM}(\mathfrak{A},\bar\mu,\gamma))=\mathrm{wdistr}(\mathfrak{A})$.

p 176 l 19 Another slip: in exercise 528Xe (now 528Xg), for 'any integer $m\ge 1$' read 'any integer $m\ge 2$'.

p 176 l 23 And another: in exercise 528Xf (now 528Xe), we must assume that $\gamma>0$.

p 176 l 28 Exercise 528Xh has been replaced by

(h) Show that for any cardinal κ there is a probability algebra $(\mathfrak{A},\bar\mu)$ such that $\mathrm{AM}(\mathfrak{A},\bar\mu,\frac{1}{2})$ has Maharam type κ.

The former 528Xh has been moved to 528Yh, and strengthened to

(h) Show that if $(\mathfrak{A},\bar\mu)$ is a measure algebra with at most $\mathfrak{c}$ atoms, then $\tau(\mathrm{AM}^*(\mathfrak{A},\bar\mu))\le\omega$.

Other exercises have been rearranged: 528Xa-528Xb are now 528Xb-528Xc, 528Xg is now 528Xa.

p 176 l 33 Exercise 528Yc, as stated, is wrong. Perhaps something can be salvaged, but for the time being I have deleted it.

p 176 l 39 Exercise 528Ye (now 528Yf) is wrong as stated, and now reads

(f) Let $(\mathfrak{A},\bar\mu)$ be a purely atomic semi-finite measure algebra of magnitude at most $\mathfrak{c}$, and $0<\gamma<\bar\mu 1$. Show that $\mathrm{AM}(\mathfrak{A},\bar\mu,\gamma)$ has countable Maharam type.

p 176 l 40 (528Y) Add new exercises:

(b) Let $(\mathfrak{A},\bar\mu)$ be a totally finite measure algebra, and suppose that $\mathrm{AM}(\mathfrak{A},\bar\mu,\gamma)$ can be regularly embedded in $\mathrm{AM}^*(\mathfrak{A},\bar\mu)$ for every $\gamma\in\,]0,\bar\mu 1[$. Show that $\mathfrak{A}$ is homogeneous.

(c) Show that $\mathfrak{B}_{\omega_1}$ cannot be regularly embedded in $\mathrm{AM}(\mathfrak{B}_\omega,\bar\nu_\omega,\frac{1}{2})$.

(g) Let $(\mathfrak{A},\bar\mu)$ be an atomless semi-finite measure algebra and $0<\gamma<\bar\mu 1$. Set $\kappa=\max(\omega,c(\mathfrak{A}),\tau(\mathfrak{A}))$ and $P=\{a: a\in\mathfrak{A},\ \bar\mu a<\gamma\}$; let $\mathbb{P}$ be the forcing notion $(P,\subseteq,0,\uparrow)$. Show that $\Vdash_{\mathbb{P}}\check\kappa<\omega_1$.

Other exercises have been rearranged: 528Ya is now 528Yd, 528Yb is now 528Ya, 528Yd is now 528Ye, 528Yf is now 528Yb, 528Yh is now 528Yc.

p 177 l 5 Problem 528Zc has been solved (see the new Exercise 528Yc).

p 179 l 29 for '$\#(K_n)\le 2^{2n+1}\|v\|^2$' and '$9\cdot 8^{-n}\#(K_n)\le 18\cdot 2^{-n}\|v\|^2$ read '$\#(K_n)\le 2^{2n+4}\|v\|^2$' and '$9\cdot 8^{-n}\#(K_n)\le 36\cdot 2^{-n}\|v\|^2$.

p 183 l 3 (Exercise 529Xd, now 529Xg) Add new part:

(i) Show that $\mathfrak{r}(\omega,\omega)\ge\operatorname{cov}\mathcal{E}\ge\max(\operatorname{cov}\mathcal{N},\operatorname{cov}\mathcal{M})$, where $\mathcal{E}$ is the ideal of subsets of $\mathbb{R}$ with Lebesgue negligible closures, and $\mathcal{M}$ the ideal of meager subsets of $\mathbb{R}$.

p 183 l 8 (529X) Add new exercise:

(f) Show that if $\theta\le\theta'\le\lambda'\le\lambda$ are cardinals, then $\mathfrak{r}(\theta,\lambda)\le\mathfrak{r}(\theta',\lambda')$.

Other exercises have been rearranged: 529Xc is now 529Xe, 529Xd is now 529Xg, 529Xe-529Xf are now 529Xc-529Xd.

p 183 l 9 The exercises 529Y have been rearranged: 529Ya is now 529Yd, 529Yb is now 529Ya, 529Yc is now 529Ye, 529Yd-529Ye are now 529Yb-529Yc.

p 183 l 13 Exercise 529Yc (now 529Ye) now reads

(e) Show that $\mathfrak{b}\le\mathfrak{r}(\omega,\omega)\le\pi(\mathcal{P}\mathbb{N}/[\mathbb{N}]^{<\omega})$.

p 183 l 18 (Exercise 529Ye, now 529Yc): for '479Xe' read '479Xi'.

p 185 l 10 (part (c) of the proof of 531A): for '$F = \bigcup_{m\in\mathbb{N}} \bigcap_{n\in\mathbb{N}} H_n$' read '$F = \bigcup_{m\in\mathbb{N}} \bigcap_{n\ge m} H_n$'.

p 186 l 25 (part (d) of the proof of 531E): for 'μ' witnesses that $\kappa \in \mathrm{Mah}_{\mathrm{R}}(Y)$' read '$\mu'$ witnesses that $\kappa \in \mathrm{Mah}_{\mathrm{R}}(X)$'.

p 186 l 38 (part (f-i) of the proof of 531E): for '$\langle w(\xi, n)\rangle_{n\in\mathbb{N}}$ is Cauchy' read '$\langle w(\xi, n)\rangle_{n\in\mathbb{N}} \to 1$'.

p 186 l 8 Part (d) of Proposition 531E has been strengthened, and now reads

(d) If X is K-analytic (in particular, if X is compact) and Y is a continuous image of X, $\mathrm{Mah}_{\mathrm{R}}(Y) \subseteq \mathrm{Mah}_{\mathrm{R}}(X)$.

p 188 l 32 (case 5 in part (b) of the proof of 531G): for 'becomes $\kappa = \omega$' read 'becomes $\kappa \le \omega$'.

p 189 l 34 Theorem 531L has been restated, and now reads

Theorem Let X be a Hausdorff space.

(a) If $\omega \in \mathrm{Mah}_{\mathrm{R}}(X)$ then $\{0, 1\}^\omega$ is a continuous image of a compact subset of X.

(b) If $\kappa \ge \omega_2$ belongs to $\mathrm{Mah}_{\mathrm{R}}(X)$ and $\lambda \le \kappa$ is an infinite cardinal such that (κ, λ) is a measure-precaliber pair of every probability algebra, then $\{0, 1\}^\lambda$ is a continuous image of a compact subset of X.

p 190 l 1 (part (b) of the proof of 531L): for

$$\bar\nu_\kappa(\inf_{\xi\in I} c_\xi \cap e_\xi \cap \inf_{\eta\in J} c_\eta \setminus e_\eta) = \frac{1}{2^{\#(I\cup J)}}\bar\nu_\kappa(\inf_{\xi\in I\cup J} c_\xi)$$

read

$$\bar\mu(\inf_{\xi\in I} c_\xi \cap e_\xi \cap \inf_{\eta\in J} c_\eta \setminus e_\eta) = \frac{1}{2^{\#(I\cup J)}}\bar\mu(\inf_{\xi\in I\cup J} c_\xi).$$

p 190 l 17 Proposition 531M has been restated, and now reads

Proposition If κ is an infinite cardinal and $\{0, 1\}^\kappa$ is a continuous image of a closed subset of X whenever X is a compact Hausdorff space such that $\kappa \in \mathrm{Mah}_{\mathrm{R}}(X)$, then κ is a measure-precaliber of every probability algebra.

p 190 l 44 531N has been moved to 531V. 531O-53P are now 531N-531O. A second part has been added to Proposition 531N (now 531Vb):

(a) Suppose that the continuum hypothesis is true. Then there is a compact Hausdorff space X such that $\omega_1 \in \mathrm{Mah}_{\mathrm{R}}(X)$ but $[0, 1]^{\omega_1}$ is not a continuous image of $P_{\mathrm{R}}(X)$.

p 192 l 47 The last part of the proof of Proposition 531P (now 531O) is hopelessly wrong. Following Kunen & Mill 95, I offer an alternative argument to show that if ω_1 is not a precaliber of $\mathfrak{B}_{\omega_1}$, there is a first-countable compact Hausdorff space X with $\omega_1 \in \mathrm{Mah}_{\mathrm{R}}(X)$.

(α) By 525J, $\mathrm{cov}\,\mathcal{N}_{\omega_1} = \omega_1$ and there is a family $\langle A_\xi\rangle_{\xi<\omega_1}$ of negligible subsets of $\{0, 1\}^{\omega_1}$ covering $\{0, 1\}^{\omega_1}$. For each $\xi < \omega_1$, let $A'_\xi \supseteq A_\xi$ be a negligible set determined by coordinates in a countable set $J_\xi \subseteq \omega_1$; set $\tilde A_\xi = \bigcup\{A'_\eta : \eta < \xi,\ J_\eta \subseteq \xi\}$; then $\tilde A_\xi$ is determined by coordinates less than ξ. Set $H_\xi = \{y{\restriction}\xi : y \in \tilde A_\xi\}$, so that H_ξ is a ν_ξ-negligible subset of $\{0, 1\}^\xi$.

We see that $\langle \tilde A_\xi\rangle_{\xi<\omega_1}$ is non-decreasing, and

$$\bigcup_{\xi<\omega_1} \tilde A_\xi = \bigcup_{\xi<\omega_1} A'_\xi = \{0, 1\}^{\omega_1}.$$

Consequently $y{\restriction}\xi \in H_\xi$ whenever $\eta \le \xi < \omega_1$, $y \in \{0, 1\}^{\omega_1}$ and $y{\restriction}\eta \in H_\eta$, while for every $y \in \{0, 1\}^{\omega_1}$ there is a $\xi < \omega_1$ such that $y{\restriction}\xi \in H_\xi$.

(β) Set $Y = \{0\} \cup \{2^{-n} : n \in \mathbb{N}\} \subseteq [0, 1]$. For $\xi \le \omega_1$ define $\phi_\xi : Y^\xi \to \{0, 1\}^\xi$ by setting

$$\begin{aligned}\phi_\xi(x)(\eta) &= 0 \text{ if } \eta < \xi \text{ and } x(\eta) = 0,\\ &= 1 \text{ for other } \eta < \xi.\end{aligned}$$

Observe that ϕ_ξ is Borel measurable for every $\xi < \omega_1$. Choose $\langle X_\xi\rangle_{\xi<\omega_1}$, and $\langle K_{\xi n}\rangle_{\xi<\omega_1,n\in\mathbb{N}}$ inductively, as follows. The inductive hypothesis will be that X_ξ is a compact subset of Y^ξ, $\phi_\xi[X_\xi]$ is conegligible in $\{0,1\}^\xi$, $\phi_\xi\restriction X_\xi$ is injective and $x\restriction\eta \in X_\eta$ whenever $\eta \le \xi < \omega_1$.

Start with $X_0 = Y^0 = \{\emptyset\}$ and $\phi_0 : X_0 \to \{0,1\}^0$ the identity map.

Given $\xi < \omega_1$ and $X_\xi \subseteq Y^\xi$, then 433D tells us that there is a Radon measure μ_ξ on X_ξ such that ν_ξ is the image measure $\mu_\xi\phi_\xi^{-1}$. Let $\langle K_{\xi n}\rangle_{n\in\mathbb{N}}$ be a disjoint sequence of compact subsets of $X_\xi \setminus \phi_\xi^{-1}[H_\xi]$ with μ_ξ-conegligible union. Set

$$X_{\xi+1} = \{x : x \in Y^{\xi+1},\, x\restriction\xi \in X_\xi,\, x(\xi) = 0\} \\ \cup \bigcup_{n\in\mathbb{N}} \{x : x \in Y^{\xi+1},\, x\restriction\xi \in K_{\xi n},\, x(\xi) = 2^{-n}\}.$$

It is easy to see that $X_{\xi+1}$ is compact and $\phi_{\xi+1}\restriction X_{\xi+1}$ is injective, while surely $x\restriction\eta \in X_\eta$ whenever $x \in X_{\xi+1}$ and $\eta \le \xi + 1$, just because $x\restriction\xi \in X_\xi$. Also

$$\phi_{\xi+1}[X_{\xi+1}] \supseteq \{y : y \in \{0,1\}^{\xi+1},\, y\restriction\xi \in \bigcup_{n\in\mathbb{N}} \phi_\xi[K_{\xi n}]\}$$

is conegligible for $\nu_{\xi+1}$ because $\phi_\xi[K_{\xi n}]$ must be analytic for every n and

$$\nu_\xi(\bigcup_{n\in\mathbb{N}} \phi_\xi[K_{\xi n}]) = \mu_\xi(\bigcup_{n\in\mathbb{N}} K_{\xi n}) = 1$$

because ϕ_ξ is injective.

Given that X_η has been defined for $\eta < \xi$, where $\xi < \omega_1$ is a non-zero limit ordinal, set

$$X_\xi = \{x : x \in Y^\xi,\, x\restriction\eta \in X_\eta \text{ for every } \eta < \xi\}.$$

Of course X_ξ is compact and $\phi_\xi\restriction X_\xi$ is injective. To see that $\phi_\xi[X_\xi]$ is conegligible, observe that

$$W = \bigcap_{\eta<\xi}\{y : y \in \{0,1\}^\xi,\, y\restriction\eta \in \phi_\eta[X_\eta]\}$$

is conegligible. But if $y \in W$ and we choose $x_\eta \in X_\eta$ such that $\phi_\eta(x_\eta) = y\restriction\eta$ for each $\eta < \xi$, then we must have $x_\zeta = x_\eta\restriction\zeta$ whenever $\zeta \le \eta < \xi$, because $\phi_\zeta\restriction X_\zeta$ is injective; so there is an $x \in Y^\xi$ such that $x_\eta = x\restriction\eta$ for every $\eta < \xi$, in which case $x \in X_\xi$ and $\phi_\xi(x) = y$. Thus $\phi_\xi[X_\xi] \supseteq W$ is conegligible.

(γ) At the end of the induction, set

$$X = \{x : x \in Y^{\omega_1},\, x\restriction\xi \in X_\xi \text{ for every } \xi < \omega_1\}, \quad \phi = \phi_{\omega_1}\restriction X.$$

As in the limit stage of the construction in (β), we see that X is a closed subset of Y^{ω_1}, so with the subspace topology is a zero-dimensional compact Hausdorff space. This time, we do not expect that $\phi[X]$ should be conegligible in $\{0,1\}^{\omega_1}$, but we find that it has full outer measure. **P** If $K \subseteq \{0,1\}^{\omega_1}$ is a non-negligible closed G_δ set, there is a $\xi < \omega_1$ such that K is determined by coordinates less than ξ. Set $K' = \{y\restriction\xi : y \in K\}$; then $\nu_\xi K' = \nu_{\omega_1} K > 0$, so there is an $x_0 \in X_\xi$ such that $\phi_\xi(x_0) \in K'$. Extending x_0 to $x \in Y^{\omega_1}$ by setting $x(\eta) = 0$ for $\xi \le \eta < \omega_1$, we see by induction on ζ that $x\restriction\zeta \in X_\zeta$ for $\xi \le \zeta < \omega_1$, so $x \in X$; also $\phi(x)\restriction\xi = \phi_\xi(x_0) \in K'$, so $\phi(x) \in K$ and K meets $\phi[X]$. As ν_{ω_1} is completion regular, $\phi[X]$ has full outer measure. **Q**

(δ) X is first-countable. **P** If $x \in X$, $\xi < \omega_1$ and $x(\xi) \ne 0$, then $x\restriction(\xi+1)$ belongs to $X_{\xi+1}$, and there must be some $n \in \mathbb{N}$ such that $x(\xi) = 2^{-n}$ and $x\restriction\xi \in K_{\xi n}$; in which case $\phi_\xi(x\restriction\xi) \notin H_\xi$. Now take any $x \in X$. Then there is a $\xi < \omega_1$ such that $\phi(x) \in \tilde{A}_\xi$ and $\phi_\xi(x) = \phi(x)\restriction\xi$ belongs to H_ξ. In this case, $V = \{x' : x' \in X,\, x'\restriction\xi = x\restriction\xi\}$ is a G_δ subset of X containing x. But if $x' \in V$ then, for any $\eta \ge \xi$, $\phi_\eta(x'\restriction\eta) \in H_\eta$ and $x'(\eta) = 0$. Thus $V = \{x\}$. By 4A2Gd, as usual, x has a countable base of neighbourhoods in X; as x is arbitrary, X is first-countable. **Q**

(ϵ) By 234F[1], there is a measure λ on X such that ϕ is inverse-measure-preserving for λ and ν_{ω_1}. Of course λ is a probability measure. Now for any $\xi < \omega_1$ and $n \in \mathbb{N}$,

$$\{x : x \in X,\, x(\xi) = 0\} = \{x : \phi(x)(\xi) = 0\},$$

[1]Formerly 132G.

$$\begin{aligned}\{x : x \in X,\, x(\xi) = 2^{-n}\} &= \{x : \phi(x)(\xi) = 1,\, x{\restriction}\xi \in K_{\xi n}\}\\ &= \{x : \phi(x)(\xi) = 1,\, \phi_\xi(x{\restriction}\xi) \in \phi_\xi[K_{\xi n}]\}\\ &= \{x : \phi(x)(\xi) = 1,\, \phi(x){\restriction}\xi \in \phi_\xi[K_{\xi n}]\}\end{aligned}$$

are measured by λ. So the domain of λ includes a base for the topology of the zero-dimensional compact Hausdorff space X. By 416Qa, there is a Radon measure μ on X agreeing with λ on the open-and-closed subsets of X; by the Monotone Class Theorem (136C), μ and λ agree on the σ-algebra generated by the open-and-closed sets, that is, the Baire σ-algebra of X (4A3Od). In particular, setting $E_\xi = \{x : x \in X,\, x(\xi) = 0\}$ for $\xi < \omega_1$,

$$\begin{aligned}\mu(E_\xi \cap E_\eta) = \lambda(E_\xi \cap E_\eta) &= \nu_{\omega_1}\{y : y \in \{0,1\}^{\omega_1},\, y(\xi) = y(\eta) = 0\}\\ &= \frac{1}{2} \text{ if } \xi = \eta < \omega_1,\\ &= \frac{1}{4} \text{ if } \xi,\, \eta < \omega_1 \text{ are different.}\end{aligned}$$

It follows that $\mu(E_\xi \triangle E_\eta) = \frac{1}{2}$ for all distinct ξ, $\eta < \omega_1$, so μ has uncountable Maharam type and $\omega_1 \in \mathrm{Mah}_{\mathrm{R}}(X)$.

p 193 l 23 A further property of the measure constructed in 531Q (now 531Q) is declared, so that the statement of the proposition now reads

Proposition Suppose that $\mathrm{cf}\,\mathcal{N}_\omega = \omega_1$. Then there are a hereditarily separable perfectly normal compact Hausdorff space X, of weight ω_1, with a Radon probability measure of Maharam type ω_1 such that every negligible set is metrizable.

The proof of 531Q has been rewritten, with the first part now presented as follows:

531P Lemma Let Y be a zero-dimensional compact metrizable space, μ an atomless Radon probability measure on Y, $A \subseteq Y$ a μ-negligible set and $\mathcal{Q}$ a countable family of closed subsets of Y. Then there are closed sets K, $L \subseteq Y$, with union Y, such that

$K \cup L = Y$, $\quad K \cap L \cap A = \emptyset$, $\quad \mu(K \cap L) \ge \frac{1}{2}$,

$K \cap Q = \overline{Q \setminus L}$ and $L \cap Q = \overline{Q \setminus K}$ for every $Q \in \mathcal{Q}$.

p 195 l 31 (part (g) of the proof of 531Q): for '$\sum_{\delta \in I} \mu_\eta Q'_{\delta\eta} = \mu\pi_\eta[H]$' read '$\sum_{\delta \in I} \mu_\eta Q'_{\delta\eta} = \mu_\eta\pi_\eta[H]$'.

p 195 l 41 (part (g) of the proof of 531Q): for '$Q_{\delta\eta} \subseteq \pi_{\delta\eta}[Q'_{\delta\eta}]$' read '$Q_{\delta\eta} \subseteq \pi_{\delta\eta}^{-1}[Q'_{\delta\eta}]$'.

p 197 l 25 *et seq.* The undefined set 'C' in parts (a) and (c) of the proof of 531S (now 531S) is to be taken to be ω_1.

p 198 l 2 (part (c) of the proof of 531S): for '$I \cup J$ does not meet K' read '$I \cup J \subseteq K$ does not meet $J^*(c)$'.

p 198 l 7 Theorem 531T (now 531T) has been restated, and now reads

Theorem Suppose that $\omega \le \kappa < \mathfrak{m}_{\mathrm{K}}$. If X is a Hausdorff space and $\kappa \in \mathrm{Mah}_{\mathrm{R}}(X)$, then $\{0,1\}^\kappa$ is a continuous image of a compact subset of X.

p 198 l 18 Add new results:

531U Proposition Let X be a Hausdorff space.

(a) Give the space $P_{\mathrm{R}}(X)$ of Radon probability measures on X its narrow topology. If $\kappa \ge \omega_2$ belongs to $\mathrm{Mah}_{\mathrm{R}}(X)$, then $\{0,1\}^\kappa$ is a continuous image of a compact subset of $P_{\mathrm{R}}(X)$.

(b) Give the space $P_{\mathrm{R}}(X \times X)$ its narrow topology. Then its tightness $t(P_{\mathrm{R}}(X \times X))$ is at least $\sup \mathrm{Mah}_{\mathrm{R}}(X)$.

531V Proposition (a) Suppose that the continuum hypothesis is true. Then there is a compact Hausdorff space X such that $\omega_1 \in \mathrm{Mah}_{\mathrm{R}}(X)$ but $\{0,1\}^{\omega_1}$ is not a continuous image of a closed subset of $P_{\mathrm{R}}(X)$.

(b) Suppose that there is a family $\langle W_\xi \rangle_{\xi<\omega_1}$ in $\mathcal{N}_{\omega_1}$ such that every closed subset of $\{0,1\}^{\omega_1} \setminus \bigcup_{\xi<\omega_1} W_\xi$ is scattered. Then there is a compact Hausdorff space X such that $\omega_1 \in \mathrm{Mah}_{\mathrm{R}}(X)$ but $\{0,1\}^{\omega_1}$ is not a continuous image of a closed subset of X.

p 198 l 32 (531X) Exercises 531Xg-531Xi are now 531Xh-531Xj, 531Xj is now 531Xq, 531Xk-531Xn are now 531Xm-531Xp.

p 198 l 40 (Exercise 531Xk, now 531Xm): for 'at most $2^{\chi(X)}$' read 'at most $\max(\omega, 2^{\chi(X)})$'.

p 199 l 3 Add new exercises:

(g) Let X be a Hausdorff space such that $\mathrm{Mah}_{\mathrm{R}}(X) \subseteq \{0, \omega\}$, and $\mathcal{N}$ the null ideal of Lebesgue measure on $\mathbb{R}$. Show that the union of fewer than $\operatorname{add}\mathcal{N}$ universally Radon-measurable subsets of X is universally Radon-measurable.

>(k) Let X be a Hausdorff space and κ a cardinal. Show that there is a Radon probability measure on X with Maharam type κ iff *either* κ is finite and $2^\kappa \le \#(X)$ *or* $\kappa = \omega \le \#(X)$ *or* $\kappa \in \mathrm{Mah}_{\mathrm{R}}(X)$ *or* $\operatorname{cf}\kappa = \omega$ and $\kappa = \sup \mathrm{Mah}_{\mathrm{R}}(X)$.

(l) Let X be a Hausdorff space and κ an infinite cardinal. (i) Show that $\{0,1\}^\kappa$ is a continuous image of a compact subset of X iff $[0,1]^\kappa$ is a continuous image of a compact subset of X, and that in this case $\{0,1\}^\kappa$ is a continuous image of a compact subset of $P_{\mathrm{R}}(X)$. (ii) Show that if X is normal and $\{0,1\}^\kappa$ is a continuous image of a closed subset of X then $[0,1]^\kappa$ is a continuous image of X. (iii) Show that if X is completely regular and $\{0,1\}^\kappa$ is a continuous image of a compact subset of X then $[0,1]^\kappa$ is a continuous image of X.

p 199 l 11 (531Y) Add new exercises:

(d) Let X be a completely regular Hausdorff space and $\kappa \ge \omega_2$ a cardinal. Show that if $\kappa \in \mathrm{Mah}_{\mathrm{R}}(X)$ then the Banach space $\ell^1(\kappa)$ is isomorphic, as linear topological space, to a subspace of the Banach space $C_b(X)$.

(e) Let X be a locally compact Hausdorff space and κ an infinite cardinal such that $\ell^1(\kappa)$ is isomorphic, as linear topological space, to a subspace of $C_0(X)$ (definition: 436I). Show that $\kappa \in \mathrm{Mah}_{\mathrm{R}}(X)$.

p 199 l 12 An affirmative answer to the main question in 531Z is already provided by combining 531O, 554Dc and 554E. So I have split the supplementaries as

(a) Can there be a perfectly normal compact Hausdorff space X such that $\omega_2 \in \mathrm{Mah}_{\mathrm{R}}(X)$?

(b) Can there be a hereditarily separable compact Hausdorff space X such that $\omega_2 \in \mathrm{Mah}_{\mathrm{R}}(X)$?

p 202 l 34 (statement of 532Ec): here we need to suppose that every μ_i is inner regular with respect to the Borel sets.

p 204 l 37 (proof of 532M): for '$\{b_i : i < f_a(n)^2\}$' read '$\{e_i : i < f_a(n)^2\}$' (and generally, 'e_i' for 'b_i' elsewhere in this proof).

p 205 l 24 (part (a) of the proof of 532N): for '$I_x = \{n : x(i) = 0 \text{ for } n \le i < 2n\}$' read '$I_x = \{n : x(i) = 0 \text{ for } 2^n \le i < 2^{n+1}\}$'.

p 206 l 41 (statement of 532Qa): we have to suppose that κ is uncountable to exclude the trivial case $\omega = \lambda = \kappa$.

p 206 l 47 (proof of 532Qa): for '$\{\xi : \xi < \omega_1,\ c \subseteq b_\xi\}$' read '$\{\xi : \xi < \omega_1,\ a \subseteq b_\xi\}$'.

p 215 l 12 (533Y) Add new exercise:

(d) Let $(X, \mathfrak{T}, \Sigma, \mu)$ be a Radon measure space with countable Maharam type, $\mathcal{A} \subseteq \Sigma$ a set with cardinal less than $\operatorname{add}\mathcal{N}_\omega$, and $\mathfrak{S}$ the topology on X generated by $\mathfrak{T} \cup \mathcal{A}$. Show that μ is $\mathfrak{S}$-Radon.

p 215 l 26534 has been thoroughly reorganised, with a few corrections and some supplementary results. In particular, the Galvin-Mycielski-Solovay characterization of strong measure zero in $\mathbb{R}$ (534H) is now 534K.

p 217 l 21 (part (c-ii) of the proof of 534B): for '$\mu^*_{Hr}T^{-1}[E + v]$' read '$\mu^*_{Hr}T^{-1}[E - v]$'.

p 219 l 34 (part (d-ii) of the proof of 534F, now 534E): for '$\{W[\{x\}] : x \in K\}$' read '$\{W_n[\{x\}] : x \in K\}$'.

p 220 l 24 (part (ii)⇒(iv)(β) of the proof of 534H, now 534K): for '$\langle V_nK\rangle_{n\in\mathbb{N}}$ is a non-decreasing sequence' read '$\langle V_nK\rangle_{n\in\mathbb{N}}$ is a non-increasing sequence'.

p 223 l 23-24 (part (b) of the proof of 534M, now 534O): for 'Σ' read '$\mathcal{P}X$', and for 'T' read '$\mathcal{P}Y$'.

p 226 l 5 *et seq.* (part (c) of the proof of 534P, now 534S): for 'let $f:\mathbb{R}\setminus\mathbb{Q}\to\{0,1\}^{\mathbb{N}}$ be the continuous function defined by setting $f(a)=h\psi\phi^{-1}(a-n)$ if $a\in\,]n,n+1[\,\setminus\mathbb{Q}$' read 'let $f:\mathbb{R}\setminus\mathbb{Q}\to\mathbb{R}$ be the continuous function defined by setting $f(a)=h\psi\phi^{-1}(a-n)+n$ if $a\in\,]n,n+1[\,\setminus\mathbb{Q}$'. Later, for '$f$ is uniformly continuous for ρ and the usual metric of $[0,1]$' read 'f is uniformly continuous for ρ and the usual metric of $\mathbb{R}$'.

p 226 l 14 (Exercise 534Xa, now 534Ya): we need to suppose that $\mu_{Hr}X>0$.

p 227 l 9 (Exercise 534Xr): I think we need more than '$\mathfrak{b}=\omega_1$; '$\mathfrak{d}=\omega_1$' works.

p 227 l 27 Part (v) of Exercise 534Yc is I think wrong, and has been deleted.

p 230 l 22 In part (b) of Proposition 535E, we are to suppose that $\mu X>0$.

p 230 l 38 (Proposition 535F) for '$\theta E^\bullet\supseteq\underline{\theta}E$ for every $E\in\Sigma$' read '$\theta a\supseteq\underline{\theta}a$ for every $a\in\mathfrak{A}$'.
The result can be proved with a slightly weaker hypothesis: instead of '$\#(\mathfrak{A})\le\omega_1$' we can use '$\#(\mathfrak{A})\le\operatorname{add}\mu$' and a trifling modification to the proof.

p 233 l 19 (part (b) of the proof of 535J): for '$\phi E=\psi_1'E=\emptyset$' read '$\phi E=\emptyset$, $\psi_1'E=0$'.

p 234 l 5 (part (b) of the proof of 535K): for 'x and y must belong to different members of $\mathcal{L}_{n+1}$' read '$f(x)\restriction m(n+1)\ne f(y)\restriction m(n+1)$'.

p 234 l 26 (part (a) of the proof of 535L): for '$\mathfrak{S}'$ is zero-dimensional' read '$\mathfrak{S}$ is zero-dimensional'.

p 237 l 32 (proof of 535R): for '$S_1f(x,y)=\int S_0Vf(x,y',z)\nu_L(dz)=\int S_0Vf(x,y',z)\nu_L(dz)=S_1f(x,y')$' read '$S_1f(x,y)=\int S_0Vf(x,y,z)\nu_L(dz)=\int S_0Vf(x,y',z)\nu_L(dz)=S_1f(x,y')$'.

p 237 l 37 (Exercise 535Xa) for 'A any subset of X' read 'A a non-negligible subset of X'.

p 238 l 4 Exercise 535Xf now reads

> **(f)** Let $\mathfrak{A}$, $\mathfrak{B}$ be Boolean algebras such that $\sup A$ is defined in $\mathfrak{A}$ whenever $A\subseteq\mathfrak{A}$ and $\#(A)<\#(\mathfrak{B})$, and $\pi:\mathfrak{A}\to\mathfrak{B}$ a surjective Boolean homomorphism. Suppose that $\phi:\mathfrak{B}\to\mathfrak{A}$ is such that $\phi0=0$, $\pi\phi b\subseteq b$ for every $b\in\mathfrak{B}$ and $\phi(b\cap c)=\phi b\cap\phi c$ for all b, $c\in\mathfrak{B}$. Show that there is a Boolean homomorphism $\theta:\mathfrak{B}\to\mathfrak{A}$ such that $\phi b\subseteq\theta b$ and $\pi\theta b=b$ for every $b\in\mathfrak{B}$.

p 238 l 21 In Exercise 535Xl, we need to assume that the marginal measure of μ on Z is countably compact.

p 238 l 28 Exercise 535Ya has been moved to 565Yb. 535Yb-535Yc are now 535Ya-535Yb.

p 238 l 38 (535Y) Add new exercise:

> **(c)** Let (X,Σ,μ) be a countably separated perfect complete strictly localizable measure space, $\mathfrak{A}$ its measure algebra and G a subgroup of $\operatorname{Aut}\mathfrak{A}$ of cardinal at most $\min(\operatorname{add}\mathcal{N},\mathfrak{p})$, where $\mathcal{N}$ is the null ideal of Lebesgue measure on $\mathbb{R}$. Show that there is an action $\bullet$ of G on x such that $\pi\bullet E=\{\pi\bullet x:x\in E\}$ belongs to Σ and $(\pi\bullet E)^\bullet=\pi(E^\bullet)$ whenever $\pi\in G$ and $E\in\Sigma$.

p 240 l 1 (536C) Add new result:

> **536C Proposition** Let (X,Σ,μ) be a probability space such that the π-weight $\pi(\mu)$ of μ is at most $\mathfrak{p}$. If $K\subseteq\mathcal{L}^0$ is $\mathfrak{T}_p$-compact then it is $\mathfrak{T}_m$-compact.

The former 536C is now 536D. A new part has been added to this theorem:

> **(g)** $\pi(\mu)>\mathfrak{p}$.

So the former 536Cg is now 536Dh.

p 242 l 2 Add new result:

> **536E Proposition** Let (X,Σ,μ) be a semi-finite measure space, with null ideal $\mathcal{N}(\mu)$. Suppose that $\pi(\mu)\le\operatorname{cov}(E,\mathcal{N}(\mu))$ whenever $E\in\Sigma\setminus\mathcal{N}(\mu)$. Then every $\mathfrak{T}_p$-separable $\mathfrak{T}_p$-compact subset of $\mathcal{L}^0=\mathcal{L}^0(\Sigma)$ is stable.

p 242 l 8 (563X) Add new exercise:

(c) Suppose that $\mathfrak{m}_K = \mathfrak{c}$. Let $(X, \mathfrak{T}, \Sigma, \mu)$ be a Radon measure space such that $\tau(\mu) \le \mathfrak{c}$. Show that every pointwise compact subset of $L^0(\Sigma)$ is stable.

p 254 l 4 (part (c) of the proof of 537S): for 'the union of any sequence in $\mathcal{K}$ belongs to $\mathcal{K}$' read 'the union of any sequence in $\mathcal{K}$ is included in a member of $\mathcal{K}$'.

p 256 l 16 (538A) I now take the formulation in the old exercise 538Xf as the definition of 'measure-centering' filter, which now reads

(f) $\mathcal{F}$ is **measure-centering** or has **property M** if whenever $\mathfrak{A}$ is a Boolean algebra, $\nu : \mathfrak{A} \to [0, \infty[$ is an additive functional, and $\langle a_n \rangle_{n\in\mathbb{N}}$ is a sequence in $\mathfrak{A}$ such that $\inf_{n\in\mathbb{N}} \nu a_n > 0$, there is an $A \in \mathcal{F}$ such that $\{a_n : n \in A\}$ is centered.

p 265 l 5 (part (b) of the proof of 538J): for '$\lim_{n\to\mathcal{F}} \nu F_n$' read '$\lim_{n\to\mathcal{F}} \mu' F_n$'.

p 265 l 8 (part (c) of the proof of 538J): for '$\mathcal{A}' = \{\lim_{n\to\mathcal{F}'} E_n : E_n \in \Sigma' \ \forall\ n \in \mathbb{N}\}$' read '$\mathcal{A}' = \{\lim_{n\to\mathcal{F}'} E_n : E_n \in \Sigma \ \forall\ n \in \mathbb{N}\}$'.

p 267 l 15 (part (d) of the proof of 538L): for '$\psi(\langle c_{\tau^\frown<k>} \rangle_{k\in\mathbb{N}}) \in \mathfrak{C}_\xi$' read '$\psi_\xi(\langle c_{\tau^\frown<k>} \rangle_{k\in\mathbb{N}}) \in \mathfrak{C}_\xi$'.

p 267 l 46 (part (g) of the proof of 538L): for '$I = J \cup \{k\}$' read '$I = J \cup \{j\}$'.

p 268 l 11 (part (a) of the proof of 538M): Since the construction of 538E-538L demands a top element of the family $\langle \mathcal{F}_\xi \rangle_{1\le\xi\le\zeta}$, we must in the present proof pick a well-ordering of $\mathfrak{F}$ with greatest element $\mathcal{F}^*$ say. Now the countable sets D of part (b) of the proof can be taken to have greatest members.

p 268 l 29 Proposition 538N has been elaborated, and now reads

Proposition (a) Let $\mathcal{F}$ be a free filter on $\mathbb{N}$. Let ν_ω be the usual measure on $\{0,1\}^{\mathbb{N}}$, and T_ω its domain. Then the following are equiveridical:

(i) $\mathcal{F}$ is measure-converging;

(ii) whenever $\langle F_n \rangle_{n\in\mathbb{N}}$ is a sequence in T_ω and $\lim_{n\to\infty} \nu_\omega F_n = 1$, then $\bigcup_{A\in\mathcal{F}} \bigcap_{n\in A} F_n$ is conegligible;

(iii) whenever (X, Σ, μ) is a measure space with locally determined negligible sets, and $\langle f_n \rangle_{n\in\mathbb{N}}$ is a sequence in $\mathcal{L}^0$ which converges in measure to $f \in \mathcal{L}^0$, then $\lim_{n\to\mathcal{F}} f_n = f$ a.e.;

(iv) whenever μ is a Radon measure on $\mathcal{P}\mathbb{N}$ such that $\lim_{n\to\infty} \mu E_n = 1$, where $E_n = \{a : n \in a \subseteq \mathbb{N}\}$ for each n, then $\mu\mathcal{F} = 1$.

(b) Every measure-converging filter is free.

(c) Suppose that $\mathcal{F}$ is a measure-converging filter.

(i) If $\mathcal{G}$ is a filter on $\mathbb{N}$ including $\mathcal{F}$, then $\mathcal{G}$ is measure-converging.

(ii) If $f : \mathbb{N} \to \mathbb{N}$ is finite-to-one, then $f[[\mathcal{F}]]$ is measure-converging.

(d) Every rapid filter is measure-converging.

(e) If there is a rapid filter, there is a measure-converging filter which is not rapid.

(f) Let $\mathcal{F}$ be a measure-converging filter on $\mathbb{N}$ and $\mathcal{G}$ any filter on $\mathbb{N}$. Then $\mathcal{G} \ltimes \mathcal{F}$ is measure-converging.

(g) If $\operatorname{cov} \mathcal{M} = \mathfrak{d}$, where $\mathcal{M}$ is the ideal of meager subsets of $\mathbb{R}$, there is a rapid filter.

p 271 l 10 (part (a) not(iii)$\Rightarrow$not-(i) of the proof of 538O): in the formula $\{(x, \alpha) : x \in G,\ 0 \le \alpha < g(x)\}$ we had better replace α by a different dummy variable.

p 274 l 15 (part (iv)$\Rightarrow$(i) of the proof of 538P): as we have no assurance that the sets $F_n = \phi^{-1}[E_n]$ are Borel, the proof has to be revised. The simplest approach seems to be to take Borel sets F'_n which differ from the F_n on a negligible set, and show that

$$\int \nu_\omega F_n \nu(dn) = \int \nu_\omega F'_n \nu(dn) = \iint \chi F'_n(x) \nu(dn) \nu_\omega(dx)$$
$$= \iint \chi F_n(x) \nu(dn) \nu_\omega(dx) = \int \nu(a)\, \mu(da).$$

p 274 l 45 (part (e) of the statement of 538R): for '$\mathcal{L}^\infty(\nu)$' read '$\mathcal{L}^\infty(\lambda)$'.

p 276 l 37 (part (e-i) of the proof of 538R): for 'equal almost everywhere to f'_n' read 'equal almost everywhere to f_n'.

p 277 l 24 A complementary result has been added to Theorem 538S:

(b) Suppose that the filter dichotomy is true. If I is any set and ν is a finitely additive real-valued functional on $\mathcal{P}I$ which is universally measurable for the usual topology on $\mathcal{P}I$, then ν is completely additive. Consequently there is no medial limit.

p 280 l 19 (Exercise 538Yg) Add new part:

(iv) Let T be a σ-subalgebra of Σ. Let $\langle f_n\rangle_{n\in\mathbb{N}}$ be a μ-uniformly integrable sequence of real-valued functions, and for each $n\in\mathbb{N}$ let g_n be a conditional expectation of f_n on T; set $f(x)=⨍ f_n(x)\nu(dn)$ and $g(x)=⨍ g_n(x)\nu(dn)$ whenever these are defined. Show that g is a conditional expectation of f on T.

p 278 l 36 (538X) Exercise 538Xh is now the definition of 'measure-centering filter', and 538Xt has been dropped. Exercise 538Xr is now built into the proof of 538Sb, so has been deleted. The following exercises have been added:

(c) Let I be a set and $\mathcal{F}$ a filter on I which, regarded as a subset of $\mathcal{P}I$ with its usual topology, is universally measurable, and $\mathcal{G}$ another filter such that $\mathcal{G}\le_{\mathrm{RK}}\mathcal{F}$. Show that $\mathcal{G}$ is universally measurable.

(i) Let (X,Σ,μ) be a complete perfect probability space, $(Y,\mathfrak{S})$ a perfectly normal compact Hausdorff space, $\langle f_n\rangle_{n\in\mathbb{N}}$ a sequence of measurable functions from X to Y, $\mathcal{F}$ a measure-centering ultrafilter on $\mathbb{N}$ and λ the $\mathcal{F}$- extension of μ. (i) Setting $f(x)=\lim_{n\to\mathcal{F}}f_n(x)$ for $x\in X$, show that f is $\operatorname{dom}\lambda$-measurable. (ii) For each $n\in\mathbb{N}$, show that there is a unique Radon measure ν_n on Y such that f_n is inverse-measure-preserving for μ and ν_n. (iii) Let ν be the limit $\lim_{n\to\mathcal{F}}\nu_n$ for the narrow topology on the space of Radon probability measures on Y. Show that f is inverse-measure-preserving for λ and ν.

(n) Let X be a locally compact Hausdorff topological group, and μ a left Haar measure on X. Show that there is a complete locally determined left-translation-invariant measure λ on X such that $\lambda(\lim_{n\to\mathcal{F}}E_n)$ is defined and equal to $\sup_{K\subseteq X\text{ is compact}}\lim_{n\to\mathcal{F}}\mu(E_n\cap K)$ whenever $\mathcal{F}$ is a Ramsey ultrafilter on $\mathbb{N}$ and $\langle E_n\rangle_{n\in\mathbb{N}}$ is a sequence of Haar measurable subsets of X.

(p) Suppose that $\langle\mathcal{F}_\xi\rangle_{\xi<\kappa}$ is a family of measure-converging filters, where κ is non-zero and less than the additivity $\operatorname{add}\mathcal{N}$ of Lebesgue measure. Show that $\bigcap_{\xi<\kappa}\mathcal{F}_\xi$ is measure-converging.

(w) Let $\mathfrak{A}$ be a Boolean algebra, ν a non-negative finitely additive functional on $\mathfrak{A}$ such that $\nu 1=1$ and $\langle a_n\rangle_{n\in\mathbb{N}}$ a sequence in $\mathfrak{A}\setminus\{0\}$. Show that there is a set $A\subseteq\mathbb{N}$ such that $\{a_n:n\in A\}$ is centered and A has upper asymptotic density at least $\inf_{n\in\mathbb{N}}\nu a_n$.

538Xc-538Xg are now 538Xd-538Xh, 538Xi-538Xl are now 538Xj-538Xm, 538Xm is now 538Xo, 538Xn-538Xq are now 538Xq-538Xt, 538Xs is now 538Xu, 538Xu is now 538Xv.

p 278 l 43 (Exercise 538Xe, now 538Xf) Add new parts:

(ii) Show that a p-point ultrafilter is nowhere dense. (iii) In 538E, show that if every $\mathcal{F}_\xi$ is a nowhere dense ultrafilter, then $\mathcal{G}_\zeta$ is a nowhere dense ultrafilter.

p 279 l 11 (Exercise 538Xj, now 538Xk) Add new part:

(ii) Show that if $(\mathfrak{A},\bar\mu)$ is any probability algebra and $\mathcal{F}$ and $\mathcal{G}$ are non-principal ultrafilters on $\mathbb{N}$, then the probability algebra reduced powers $(\mathfrak{A},\bar\mu)^{\mathbb{N}}|\mathcal{F}$ and $(\mathfrak{A},\bar\mu)^{\mathbb{N}}|\mathcal{G}$ are isomorphic.

p 279 l 19 Part (i) of Exercise 538Xm (now 538Xo) now reads

Let $\langle\mathcal{F}_n\rangle_{n\in\mathbb{N}}$ be a sequence of measure-converging filters on $\mathbb{N}$. Show that $\bigcap_{n\in\mathbb{N}}\mathcal{F}_n$ is measure-converging, so that $\lim_{n\to\mathcal{F}}\mathcal{F}_n$ is measure-converging for any filter $\mathcal{F}$ on $\mathbb{N}$.

p 279 l 22 Exercise 538Xn (now 538Xq) has a new part:

(i) Let $\mathcal{F}$ be a filter on $\mathbb{N}$. Show that $\mathcal{F}$ has the Fatou property iff $\int f d\mu$ and $\lim_{n\to\mathcal{F}}\int f_n d\mu$ are defined and equal whenever (X,Σ,μ) is a measure space, $g:X\to[0,\infty[$ is an integrable function and $\langle f_n\rangle_{n\in\mathbb{N}}$ is a sequence of measurable functions on X such that $|f_n|\le_{\text{a.e.}}g$ for every n and $\lim_{n\to\mathcal{F}}f_n=_{\text{a.e.}}f$.

p 279 l 39 (538Y) Add new exercises:

(a) Show that if $\mathcal{F}$ and $\mathcal{G}$ are filters and $\mathcal{F} \le_{\mathrm{RK}} \mathcal{G}$, then, in the language of 512A, $(\mathcal{F}, \supseteq, \mathcal{F}) \preccurlyeq_{\mathrm{GT}} (\mathcal{G}, \supseteq, \mathcal{G})$, so that $\operatorname{ci}\mathcal{F} \le \operatorname{ci}\mathcal{G}$ and $\mathcal{F}$ is κ-complete whenever κ is a cardinal and $\mathcal{G}$ is κ-complete.

(h)(i) Show that if $f : \mathbb{N} \to \mathbb{N}$ is finite-to-one and $\mathcal{F}$ is a rapid filter on $\mathbb{N}$, then $f[[\mathcal{F}]]$ is rapid. (ii) Show that if $f : \mathbb{N} \to \mathbb{N}$ is finite-to-one and $\mathcal{F}$ is a filter on $\mathbb{N}$ with the Fatou property, then $f[[\mathcal{F}]]$ has the Fatou property.

(k) Let (X, Σ, μ) be a measure space and $\langle f_m \rangle_{m \in \mathbb{N}}$, $\langle g_n \rangle_{n \in \mathbb{N}}$ two sequences of real-valued functions on X which are both uniformly integrable. Let ν, $\nu' : \mathcal{P}\mathbb{N} \to \mathbb{R}$ be bounded additive functionals. Show that $\iiint f_m \times g_n d\mu\, \nu(dm)\nu'(dn) = \iiint f_m \times g_n d\mu\, \nu'(dn)\nu(dm)$.

(m) Suppose that $\mathcal{F}$ is a filter on $\mathbb{N}$ with the Fatou property, and $\langle \nu_n \rangle_{n \in \mathbb{N}}$ a sequence of medial limits. Set $\mathcal{G} = \{A : A \subseteq \mathbb{N}, \lim_{n \to \mathcal{F}} \nu_n A = 1$. Show that $\mathcal{G}$ is a filter with the Fatou property.

(n) Show that $\mathfrak{u} \ge \mathfrak{r}(\omega, \omega)$.

(o)(i) Show that if $\mathcal{F}$ is a rapid filter on $\mathbb{N}$, then $\operatorname{ci}\mathcal{F} \ge \mathfrak{d}$. (ii) Show that $\mathfrak{g} \ge \mathfrak{g}$. (iii) Show that if $\mathfrak{u} < \mathfrak{g}$ then there are no rapid ultrafilters on $\mathbb{N}$.

(p) Suppose that the filter dichotomy is true. (i) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra. Show that if $\nu : \mathfrak{A} \to \mathbb{R}$ is an additive functional which is universally measurable for the order-sequential topology of $\mathfrak{A}$, then ν is countably additive. (ii) Let $(\mathfrak{A}, \bar{\mu})$ be a localizable measure algebra. Show that if $\nu : \mathfrak{A} \to \mathbb{R}$ is an additive functional which is universally measurable for the measure-algebra topology on $\mathfrak{A}$, then it is continuous.

Other exercises have been rearranged: 538Ya-538Yb are now 538Yb-538Yc, 538Yc-538Yd are now 538Yf-538Yg, 538Ye-538Yf are now 538Yi-538Yj, 538Yg is now 538Yl, 538Yh-538Yi are now 538Yd-538Ye, 538Yj-538Yk are now 538Yq-538Yr.

p 284 l 28 (statement of Proposition 539E): we must of course suppose that $\mathfrak{A} \ne \{0\}$.

p 284 l 29 *et seq.* (part (a) of the proof of 539G): for '$\mathcal{B}(X)$' read 'Σ' (six times).

p 286 l 12 Add new result, formerly 555L:

539I Corollary Suppose that $\#(X) < \max(\mathfrak{s}, \mathfrak{m}_{\text{countable}})$, where $\mathfrak{s}$ is the splitting number. Let Σ be a σ-algebra of subsets of X such that (X, Σ) is countably separated, and $\mathcal{I}$ a σ-ideal of Σ containing singletons. Then there is no non-zero Maharam submeasure on $\Sigma/\mathcal{I}$.

539I-539T are now 539J-539U.

p 288 l 6 Part (d) of the proof of 539K (now 539L) should begin 'Let $\langle I_n \rangle_{n \in \mathbb{N}}$ be a sequence in $\mathcal{I}$'.

p 289 l 47 539Pc (now 539Qc) now reads

(c) If every countable subset of $\mathfrak{A}$ is included in a subalgebra of $\mathfrak{A}$ with the σ-interpolation property, then $\mathfrak{A}$ has the σ-interpolation property.

p 291 l 36 (539X) Add new exercise:

(d) Let X be a set, Σ a σ-algebra of subsets of X, and $\nu : \Sigma \to [0, \infty[$ a non-zero Maharam submeasure; set $\mathcal{I} = \{E : E \in \Sigma, \nu E = 0\}$ and $\mathfrak{A} = \Sigma/\mathcal{I}$. Suppose that $\#(\mathfrak{A}) \le \omega_2$ and $\mathrm{FN}(\mathcal{P}\mathbb{N}) = \omega_1$. Show that there is a lifting for ν, that is, a Boolean homomorphism $\theta : \mathfrak{A} \to \Sigma$ such that $(\theta a)^\bullet = a$ for every $a \in \mathfrak{A}$.

539Xd-539Xe are now 539Xe-539Xf.

p 292 l 4 Exercise 539Yb has been revised, and now reads

(b) Let X be a set, Σ a σ-algebra of subsets of X, and $\mathcal{I}$ a proper σ-ideal of subsets of X generated by $\Sigma \cap \mathcal{I}$; let Σ_L be the algebra of Lebesgue measurable subsets of $\mathbb{R}$. Write $\mathfrak{A}$ for $\Sigma/\Sigma \cap \mathcal{I}$, $\mathcal{L}$ for $(\Sigma \widehat{\otimes} \Sigma_L) \cap (\mathcal{I} \ltimes \mathcal{N})$ and $\mathfrak{C}$ for $\Sigma \widehat{\otimes} \Sigma_L/\mathcal{L}$. (i) Show that $c(\mathfrak{C}) = \max(\omega, c(\mathfrak{A}))$ and $\tau(\mathfrak{C}) = \max(\omega, \tau(\mathfrak{A}))$. (ii) Show that $\mathfrak{C}$ is weakly (σ, ∞)-distributive iff $\mathfrak{A}$ is. (iii) Show that $\mathfrak{C}$ is measurable iff $\mathfrak{A}$ is. (iv) Show that $\mathfrak{C}$ is a Maharam algebra iff $\mathfrak{A}$ is.

p 292 l 9 (Exercise 539Yc): for '$r_\epsilon : \mathfrak{B} \to \omega_1$ and $\hat{r}_\epsilon : \mathfrak{A} \to \omega_1$' read '$r_\epsilon : \mathfrak{B} \to \mathrm{On}$ and $\hat{r}_\epsilon : \mathfrak{A} \to \mathrm{On}$'.

p 292 l 18 (539Y) Add new exercise:

(g) Suppose that $\mathfrak{A}$ is a non-measurable Maharam algebra. Show that $\mathrm{Mhsr}(\mathfrak{A}) = \omega \cdot \mathrm{Mhsr}(\mathfrak{A})$.

Part II

p 10 l 35 (part (a-i) of the proof of 541I): for '$A = \{\xi : \xi < \kappa,\ \xi \in \bigcup_{\eta<\xi} I_\eta\}$' read '$I = \{\xi : \xi < \kappa,\ \xi \in \bigcup_{\eta<\xi} I_\eta\}$',

p 11 l 27 (part (a) of the statement of 541K): for 'there is an $\alpha \le \kappa$ such that $\{\xi : \xi \in S,\ f(\xi) \ge \alpha\} \in \mathcal{I}$' read 'there is an $\alpha < \kappa$ such that $\{\xi : \xi \in S,\ f(\xi) \ge \alpha\} \in \mathcal{I}$'.

p 13 l 16 (proof of 541O): for '$\mathcal{A}_\xi = \{C_{\xi x} : \zeta \in E_\xi\} \setminus \mathcal{I}$' read '$\mathcal{A}_\xi = \{C_{\xi x} : x \in E_\xi\} \setminus \mathcal{I}$'

p 15 l 2 In Lemma 541S, we must add '$\kappa \le 2^\delta$' to the hypotheses.

p 21 l 44 Add new result:

542K Proposition Let κ be a quasi-measurable cardinal.

(a) For every cardinal $\theta < \kappa$ there is a family $\mathcal{D}_\theta$ of countable sets, with cardinal less than κ, which is stationary over θ.

(b) There is a family $\mathcal{A}$ of countable sets, with cardinal at most κ, which is stationary over κ.

p 24 l 29 (part (b) of the proof of 543D): for '$\mu^*C^{-1}[\{\xi\}] = \mu^*\{f(\eta) : \eta < \xi\}$' read '$\mu^*C^{-1}[\{\xi\}] = \mu^*\{f(\eta) : \eta \le \xi\}$'.

p 30 l 8 (543X) Add new exercise:

(d) Let μ be Lebesgue measure on $\mathbb{R}$, and $\theta = \frac{1}{2}(\mu^* + \mu_*)$ the outer measure described in 413Xd. Show that μ is the measure defined from θ by Carathéodory's method.

p 26 l 9 (part (d) of the proof of 543E): for '$\pi_M = \tilde{g}_\alpha$' read '$g^*_\alpha \pi_M = \tilde{g}_\alpha$'.

p 37 l 28 (Exercise 544Xb): for for 'show that $\mathbb{R}^\lambda$ is strongly measure-compact for every $\lambda < \kappa$' read 'show that $\mathbb{R}^\lambda$ is measure-compact for every $\lambda < \kappa$'.

p 37 l 33 (Exercise 544Xd) I think we need to suppose that the augmented shrinking $\mathrm{shr}^+(\mu)$ is at most κ, as well as that the magnitude of μ is less than κ.

p 37 l 37 (Exercise 544Xf): for for '$\mathrm{non}\,\mathcal{N}_{2^{(\kappa^+)}} \le \kappa^{(+n)}$' read '$\mathrm{non}\,\mathcal{N}_{2^{(\kappa^+)}} \le \kappa^+$'.

p 38 l 1 Exercise 544Xi has been elaborated, and now reads

(i) Let κ be an atomlessly-measurable cardinal, and G a group of permutations of κ such that $\#(G) < \kappa$. Show that there is a non-zero strictly localizable atomless G-invariant measure with domain $\mathcal{P}\kappa$ and magnitude at most $\#(G)$.

p 38 l 3 (Problem 544Za) In order to apply 544C in the way suggested, we need to suppose that the witnessing probability ν here is normal.

p 41 l 1 Sections §§546-547 have been rewritten as §§546-548, incorporating work of A.Kumar and S.Shelah.

p 43 l 47 (part (e) of the proof of 546E): for '$C_n[\{x\}] \subseteq W'[\{x\}]$' read '$C_n[\{x\}] \setminus V \subseteq W'[\{x\}]$'.

p 49 l 20 (part (c-iii) of the proof of 546L): for

$$E_0^* = \{x : x \in X,\ V_n[\{x\}] \cap W_n[\{x\}] \ne \emptyset\} \\ \cup \{x : x \in X,\ V_n[\{x\}] \cup W_n[\{x\}] \text{ is not dense}\}$$

read

$$E_0^* = \bigcup_{n\in\mathbb{N}} \big(\{x : x \in X,\ V_n[\{x\}] \cap W_n[\{x\}] \ne \emptyset\} \\ \cup \{x : x \in X,\ V_n[\{x\}] \cup W_n[\{x\}] \text{ is not dense}\big)\}.$$

p 49 l 26 (part (c-iii) of the proof of 546L): for '$b = (X \times G)^\bullet$' read '$b = G^\bullet$'.

p 49 l 28 (part (c-iii) of the proof of 546L): for '$\mathrm{upr}_1(\varepsilon_1(b) \cap d_n) \cap \mathrm{upr}_1(\varepsilon_2(b) \setminus d_n)$' read

$$\text{`}\mathrm{upr}_1(\varepsilon_2(b) \cap d_n) \cap \mathrm{upr}_1(\varepsilon_2(b) \setminus d_n)\text{'.}$$

p 49 l 40 (part (c-iv) of the proof of 546L): for '$h^{-1}[H]$ belongs to the σ-algebra generated by $\{W_n : n \in \mathbb{N}\}$' read '$h^{-1}[H]$ belongs to the σ-algebra generated by $\{V_n : n \in \mathbb{N}\}$'.

p 54 l 26 (part (a) of the proof of 546P): for '$\mathcal{L} = (\mathcal{B}\mathfrak{a}(X)\widehat{\otimes}\mathcal{B}(Z)) \cap (\mathcal{N}(\nu_\Gamma) \ltimes \mathcal{M}(Y))$' read '$\mathcal{L} = (\mathcal{B}\mathfrak{a}(X)\widehat{\otimes}\mathcal{B}(Z)) \cap (\mathcal{N}(\nu_\Gamma) \ltimes \mathcal{M}(Z))$'.

p 65 l 15 (551Ca): for

$$\{(i, f^{-1}[\{i\}]^\bullet) : i \in \{0,1\},\ f^{-1}[\{i\}] \notin \mathcal{I}\}$$

read

$$\{(\check{\imath}, f^{-1}[\{i\}]^\bullet) : i \in \{0,1\},\ f^{-1}[\{i\}] \notin \mathcal{I}\}.$$

p 69 l 36 (part (b-iii) of the proof of 551F): for

$$\Vdash_{\mathbb{P}} \vec{W}_n = \textstyle\bigcup_{\beta<\alpha}(E_{n\beta} \times \{0,1\}^I)^{\rightharpoonup} \cap \vec{W}_{n\beta}$$

read

$$\Vdash_{\mathbb{P}} \vec{W}_n = \textstyle\bigcup_{\beta<\check{\alpha}}(E_{n\beta} \times \{0,1\}^I)^{\rightharpoonup} \cap \vec{W}_{n\beta}.$$

p 71 l 38 In Proposition 551K, the argument assumes that $(\Omega, \Sigma, \mathcal{I})$ has a Dedekind complete quotient algebra, so this ought to be included in the hypotheses.

p 73 l 13 (statement of (c) in Proposition 551N): for

$$\Vdash_{\mathbb{P}} \psi(x) = \lim_{n\to\infty} \vec{\psi}_n(x) \text{ for every } x \in \{0,1\}^{\check{I}}$$

read

$$\Vdash_{\mathbb{P}} \vec{\psi}(x) = \lim_{n\to\infty} \vec{\psi}_n(x) \text{ for every } x \in \{0,1\}^{\check{I}}.$$

p 74 l 44 In Theorem 551P, I think we need to assume that $(\Omega, \Sigma, \mathcal{I})$ is ω_1-saturated.

p 75 l 40 Part (a-ii) of the proof of 551P is seriously defective. I have changed it to the following.

(ii) Now $\Vdash_{\mathbb{P}} \dot{\pi}$ is injective. **P** I aim to use the condition in 5A3Eb. I take the argument in two bites.

(α)? Suppose, if possible, that V, $W \in \Lambda$ and $p \in P$ are such that $p \Vdash_{\mathbb{P}} \vec{V}^\bullet = \vec{W}^\bullet$ but $p \not\Vdash_{\mathbb{P}} (V^\bullet)^{\rightharpoonup} \subseteq (W^\bullet)^{\rightharpoonup}$. Then there are a $q \in P$, stronger than p, and a $\mathbb{P}$-name $\dot{x}$ such that

$$q \Vdash_{\mathbb{P}} \dot{x} \in (V^\bullet)^{\rightharpoonup} \setminus (W^\bullet)^{\rightharpoonup}.$$

By the definition in 5A3Cb, there are an $r \in P$, stronger than q, and a $V_1 \in \Lambda$ such that $V_1^\bullet = V^\bullet$ and $r \Vdash_{\mathbb{P}} \dot{x} = \vec{V}_1$. Let $E \in \Sigma \setminus \mathcal{I}$ be such that $E^\bullet = r$, and set

$$W_1 = (V_1 \cap (E \times \{0,1\}^I)) \cup (W \cap ((\Omega \setminus E) \times \{0,1\}^I)) \in \Lambda.$$

Then $E^\bullet \subseteq [\![\vec{W}_1 = \vec{V}_1]\!]$ (551Gb again). At the same time,

$$E^\bullet \subseteq [\![\vec{V}^\bullet = \vec{W}^\bullet]\!] = [\![\vec{h}_{V\triangle W} = 0]\!]$$

as in (i) just above. Now

$$\begin{aligned}
&\{\omega : \omega \in \Omega,\ \nu_I(W_1[\{\omega\}]\triangle W[\{\omega\}]) > 0\}\\
&\qquad = \{\omega : \omega \in E,\ \nu_I(V_1[\{\omega\}]\triangle W[\{\omega\}]) > 0\}\\
&\qquad \subseteq \{\omega : \omega \in \Omega,\ \nu_I(V_1[\{\omega\}]\triangle V[\{\omega\}]) > 0\} \cup \{\omega : \omega \in E,\ h_{V\triangle W})(\omega) > 0\}\\
&\qquad \in \mathcal{I}.
\end{aligned}$$

But this means that $W_1^\bullet = W^\bullet$ and $(\vec{W}_1, \mathbb{1}) \in (W^\bullet)^\rightharpoondown$, so that

$$\Vdash_{\mathbb{P}} \vec{W}_1 \in (W^\bullet)^\rightharpoondown, \quad r \Vdash_{\mathbb{P}} \dot{x} = \vec{V}_1 = \vec{W}_1 \in (W^\bullet)^\rightharpoondown;$$

but r is stronger than q and

$$q \Vdash_{\mathbb{P}} \dot{x} \notin (W^\bullet)^\rightharpoondown,$$

so we have a contradiction. **X**

(β) Thus if V, $W \in \Lambda$ and $p \in P$ are such that $p \Vdash_{\mathbb{P}} \vec{V}^\bullet = \vec{W}^\bullet$, we must have $p \Vdash_{\mathbb{P}} (V^\bullet)^\rightharpoondown \subseteq (W^\bullet)^\rightharpoondown$. Similarly, $p \Vdash_{\mathbb{P}} (W^\bullet)^\rightharpoondown \subseteq (V^\bullet)^\rightharpoondown$ and $p \Vdash_{\mathbb{P}} (W^\bullet)^\rightharpoondown = (V^\bullet)^\rightharpoondown$. Accordingly the conditions of 5A3Eb are satisfied and $\Vdash_{\mathbb{P}} \dot{\pi}$ is injective. **Q**

p 78 l 10 (part (c-iii) of the proof of 551Q): for '$p \in \mathfrak{A}^+$' read '$p \in \mathfrak{C}^+$'.

p 82 l 29 (part (a) of the proof of 552C): for '$[\![\check{h}(\check{m})^\smallsmile < \dot{f}(\check{m})]\!]$' read '$[\![\check{h}(\check{m}) < \dot{f}(\check{m})]\!]$'.

p 87 l 29 (part (b-v) of the proof of 552G): for '$\nu_\omega C'_m = 2^{-\#(\sigma_{\alpha(m)})-\#(\tau_{\alpha(m)})+j}$' read '$\nu_\omega C'_m \le 2^{-\#(\sigma_{\alpha(m)})-\#(\tau_{\alpha(m)})+j}$'.

p 97 l 18 (part (i) of the proof of 552N): for 'because $a_{\xi,z_{ni}} \in \mathfrak{C}_{\{\xi\}\times\kappa} \subseteq \mathfrak{C}_L$' read 'because $a_{\xi,z_{ni}} \in \mathfrak{C}_{J_{\xi,z_{ni}}\restriction I\cup(\{\xi\}\times\kappa)} \subseteq \mathfrak{C}_L$'.

p 98 l 17 (proof of 552P): for '$\mathcal{J} = \{W : W \in \Lambda,\ \nu_\lambda W[\{x\}] = 0$ for ν_λ-almost every $x \in \{0,1\}^\kappa\}$' read '$\mathcal{J} = \{W : W \in \Lambda,\ \nu_\lambda W[\{x\}] = 0$ for ν_κ-almost every $x \in \{0,1\}^\kappa\}$'.

p 98 l 24 (552X) Add new exercises:

(c) Suppose that the continuum hypothesis is true. Show that

$$\Vdash_{\mathbb{P}_{\omega_2}} \mathfrak{c} \text{ is a precaliber of every measurable algebra.}$$

(d) Describe Cichoń's diagram in the forcing universe $V^{\mathbb{P}_{\omega_2}}$ (i) if we start from $\mathfrak{c} = \omega_1$ (ii) if we start from $\mathfrak{m} = \mathfrak{c} = \omega_2$.

(e) Suppose that the continuum hypothesis is true. Show that

$\Vdash_{\mathbb{P}_{\omega_2}}$ there is a family $\langle\nu_\xi\rangle_{\xi<\mathfrak{c}}$ of additive functionals on $\mathcal{P}([0,1])$ such that $\sup_{\xi<\mathfrak{c}} \nu_\xi A$ is the Lebesgue outer measure of A for every $A \subseteq [0,1]$.

552Xc is now 552Xf.

p 113 l 28 Add new result:

553O Theorem Let κ be an infinite cardinal.

(a)

$\Vdash_{\mathbb{P}_\kappa}$ every universally measurable subset of $\{0,1\}^{\mathbb{N}}$ is expressible as the union of at most $\check{\mathfrak{c}}$ Borel sets.

(b) If the cardinal power $\kappa^{\mathfrak{c}}$ is equal to κ, then

$$\Vdash_{\mathbb{P}_\kappa} \text{there are exactly } \mathfrak{c} \text{ universally measurable subsets of } \{0,1\}^{\mathbb{N}}.$$

p 113 l 31 (Exercise 553Xa) Add new fragment: (iii) Suppose that $\mathfrak{m} = \mathfrak{c} > \omega_1$. Show that

$$\Vdash_{\mathbb{P}_{\omega_1}} \text{there is a set of strong measure zero with cardinal greater than } \mathfrak{m}_{\text{countable}}.$$

p 116 l 20 (part (b) of the proof of 554D): the clause 'In particular, each point of B belongs to countably many members of $\mathcal{A}$' is looking at the relation from the wrong side, and ought to be 'Each member of $\mathcal{A}$ is meager, so meets B in a countable set'.

p 118 l 26 (part (c) of the proof of 554G): for '$p \Vdash_{\mathbb{Q}_\kappa} \dot{x} \in \dot{\theta}$' read '$p \Vdash_{\mathbb{Q}_\kappa} \dot{x}$ is a value of $\dot{\theta}$'.

p 119 l 41 (554X) Add new exercises:

(b) Show that if I is any set, every regular uncountable cardinal is a precaliber of $\mathfrak{G}_I$.

(c) Let I be any set. (i) Show that $(\mathcal{C}_I, \supseteq)$ is isomorphic to $(\mathrm{Fn}_{<\omega}(I;\{0,1\}), \subseteq)$ (definition: 552A). (ii) Show that $\mathfrak{G}_I$ can be identified with the regular open algebra $\mathrm{RO}^\uparrow(\mathrm{Fn}_{<\omega}(I;\{0,1\}))$.

(d) Let κ be an infinite cardinal such that $\mathbb{R}$ has a Lusin set of size κ. Show that there is a first-countable compact Hausdorff space X such that $\kappa \in \mathrm{Mah}_{\mathrm{R}}(X)$.

(f) Describe Cichoń's diagram in the forcing universe $V^{\mathbb{Q}_{\omega_2}}$ (i) if we start from $\mathfrak{c} = \omega_1$ (ii) if we start from $\mathfrak{m} = \mathfrak{c} = \omega_2$.

554Xb is now 554Xe.

p 120 l 5 (part (ii) of exercise 554Yb) T should be the set of *injective* functions $t : \xi + 1 \to \mathbb{N}$ almost agreeing with $e_{\xi+1}$.

p 120 l 10 (Exercise 554Yc): the definition of $\mathbb{Q}_\kappa$ mutated while I was writing this section. The given formula for $\dot{\preccurlyeq}$ works better if you use $\mathrm{Fn}_{<\omega}(\kappa; \{0,1\})$ instead. In terms of the definition of $\mathbb{Q}_\kappa$ in 554A we need

$$\{((\check{\eta}, \check{\xi}), p) : \eta \le \xi < \omega_1,\, p \in \mathbb{Q}_\kappa,\, p \subseteq \{x : x(e_\eta(\zeta)) = x(e_\xi(\zeta)) \text{ for every } \zeta < \eta\}^\bullet\}.$$

p 120 l 40 (statement of Theorem 555B): add a new part

(a)(i) $\dot{J}$ is a $\mathbb{P}$-name and $p \in P$ is such that $p \Vdash_{\mathbb{P}} \dot{J} \in \dot{\mathcal{I}}$, there is an $I \in \mathcal{I}$ such that $p \Vdash_{\mathbb{P}} \dot{J} \subseteq \check{I}$.

p 124 l 11 (statement of Theorem 555E): for 'κ-additive' read 'λ-additive'.

p 124 l 39 (part (b) of the proof of 555E): for '$\int \bar{\nu}_\kappa(a \cap \sigma_\alpha(\xi) \cap \sigma_\beta(\xi))\mu(d\xi) \ge 3t\bar{\nu}_\kappa a$' read

'$\int \bar{\nu}_\kappa(a \cap (\sigma_\alpha(\xi) \triangle \sigma_\beta(\xi)))\mu(d\xi) \ge 3t\bar{\nu}_\kappa a$'.

p 125 l 27 (part (b-ii) of the proof of 555F): for '$u_{ij} \in \mathfrak{C}_I$' read '$u_{ij} \in L^\infty(\mathfrak{C}_I)$'.

p 129 l 41 (part (a) of the proof of 555J): for '$b_n = \{x : x \in X,\, n = \max_{m \le g(x)} f_m(x)\}$' read '$b_n = \{x : x \in X,\, n = \max_{m \le g(x)} f_x(m)\}$'.

p 130 l 7 Lemma 555K has been moved to 517Rc, and Lemma 555L to 539I. 555M-555Q are now 555K-555O.

p 131 l 23 (proof of 555O, now 555M): for '$f(s) \ge \xi$ for every $s \in A$' read '$f(s) \ge \xi$ for every $s \notin A$'.

p 132 l 13 (part (d) of the proof of 555P, now 555N): for '$I = \{s : s \in S \setminus \{\emptyset\},\, J_s \not\subseteq K\}$' read '$I = \{s : s \in S \setminus \{\emptyset\},\, J_s \not\subseteq K\} \cup \{\emptyset\}$'.

p 133 l 6 (555X) Add new exercise:

(e) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra. Show that $\mathfrak{A}$ has the Egorov property iff for every sequence $\langle u_n \rangle_{n\in\mathbb{N}}$ in $L^0 = L^0(\mathfrak{A})$ there is a sequence $\langle \alpha_n \rangle_{n\in\mathbb{N}}$ in $]0,\infty[$ such that $\{\alpha_n u_n : n \in \mathbb{N}\}$ is order-bounded in L^0.

p 133 l 22 (555Y) Add new exercises:

(e) Let κ be a cardinal. Suppose that for every set X there is a κ-additive maximal proper ideal $\mathcal{I}$ of subsets of $S = [X]^{<\kappa}$ such that

(α) $\{s : s \in S,\, x \notin s\} \in \mathcal{I}$ for every $x \in X$,

(β) if $A \subseteq S$, $A \notin \mathcal{I}$ and $f : A \to X$ is such that $f(s) \in s$ for every $s \in A$, then there is an $x \in X$ such that $\{s : s \in A,\, f(s) = x\} \notin \mathcal{I}$.

Show that κ is supercompact.

(g) Suppose that λ is a two-valued-measurable cardinal, and $\kappa > \lambda$ a cardinal. Show that

$\Vdash_{\mathbb{P}_\kappa}$ there is a probability measure on ω_1 with Maharam type greater than the least atomlessly-measurable cardinal.

(h) In 555C, suppose that $X = \kappa$ and that μ is a $\{0,1\}$-valued measure on κ witnessing that κ is two-valued-measurable. For $J \subseteq \kappa$ let $P_J : L^\infty(\mathfrak{B}_\kappa) \to L^\infty(\mathfrak{B}_\kappa)$ be the corresponding conditional expectation as in part (b) of the proof of 555F. Show that for every $\sigma \in \mathfrak{B}_\kappa^\kappa$ there is a countable set $J \subseteq \kappa$ such that $\mu\{\xi : \xi < \kappa,\, u_\sigma = P_J(\chi\sigma(\xi))\} = 1$.

555Ye is now 555Yf.

p 137 l 30 Theorem 556C has been rewritten, and now reads

556C Theorem Let $\mathfrak{A}$ be a Boolean algebra, not $\{0\}$, and $\mathfrak{C}$ a subalgebra of $\mathfrak{A}$. Let $\mathbb{P}$ be the forcing notion $\mathfrak{C}^+$, active downwards, $\dot{\mathfrak{A}}$ the forcing name for $\mathfrak{A}$ over $\mathfrak{C}$, and $\pi : \mathfrak{A} \to \mathfrak{A}$ a ring homomorphism such that $\pi c \subseteq c$ for every $c \in \mathfrak{C}$; write $\dot{\pi}$ for the forcing name for π over $\mathfrak{C}$.

(a)(i)

$$\Vdash_{\mathbb{P}} \dot{\pi} \text{ is a ring homomorphism from } \dot{\mathfrak{A}} \text{ to itself}$$

and

$$\Vdash_{\mathbb{P}} \dot{\pi}(\dot{a}) = (\pi a)^{\bullet}$$

for every $a \in \mathfrak{A}$.

(ii) If π is injective, $\Vdash_{\mathbb{P}} \dot{\pi}$ is injective.

(iii) If $\phi : \mathfrak{A} \to \mathfrak{A}$ is another ring homomorphism such that $\phi c \subseteq c$ for every $c \in \mathfrak{C}$, with corresponding forcing name $\dot{\phi}$, then

$$\Vdash_{\mathbb{P}} \dot{\pi}\dot{\phi} = (\pi\phi)^{\bullet}.$$

(b) Now suppose that π is a Boolean homomorphism.

(i) $\pi c = c$ for every $c \in \mathfrak{C}$.

(ii) $\Vdash_{\mathbb{P}} \dot{\pi}$ is a Boolean homomorphism.

(iii) If π is surjective, $\Vdash_{\mathbb{P}} \dot{\pi}$ is surjective.

(iv) If $\pi \in \operatorname{Aut}\mathfrak{A}$ then

$$\Vdash_{\mathbb{P}} \dot{\pi} \text{ is a Boolean automorphism and } (\dot{\pi})^{-1} = (\pi^{-1})^{\bullet}.$$

(v) If the fixed-point subalgebra of π is $\mathfrak{C}$ exactly, then

$$\Vdash_{\mathbb{P}} \text{the fixed-point subalgebra of } \dot{\pi} \text{ is } \{0, 1\}.$$

p 142 l 16 (part (a-ii-β) of the proof of 556F): for '$p \Vdash_{\mathbb{P}} \dot{x} \in \dot{\mathfrak{A}}$ and $\dot{\psi}(\dot{x}) = 0$' read '$p \Vdash_{\mathbb{P}} \dot{x} \in \check{\mathfrak{A}}$ and $\dot{\psi}(\dot{x}) = 0$'.

p 161 l 26 (part (d-ix) of the proof of 556S): for '$\Vdash_{\mathbb{P}} \#(\dot{\mathfrak{A}}) \le \mathfrak{c}$' read '$\Vdash_{\mathbb{P}} \#(\dot{\mathfrak{Z}}) \le \mathfrak{c}$'.

p 162 l 9 (556Y) Add new exercise:

(d) Show that the argument of 556Q is sufficient to take us from (†) there to Theorem 395N, as well as to 395P.

556Yd is now 556Ye.

p 164 l 32 Lemma 561C has been elaborated on, and now reads

Let $\mathcal{E}$ be the set of non-empty closed subsets of $\mathbb{N}^{\mathbb{N}}$. Then there is a family $\langle f_F \rangle_{F \in \mathcal{E}}$ such that, for each $F \in \mathcal{E}$, f_F is a continuous function from $\mathbb{N}^{\mathbb{N}}$ to F and $f_F(\alpha) = \alpha$ for every $\alpha \in F$.

p 171 l 8 (part (a) of the proof of 561I): for 'Set $\gamma = \min\{\|u - w\| : w \in C\}$' read 'Set $\gamma = \inf\{\|u - w\| : w \in C\}$'.

p 171 l 37 Exercises have been moved: 561Xd-561Xi are now 561Xe-561Xj, 561Xj-561Xo are now 561Xm-561Xr, 561Xp is now 561Xl, 561Xq is now 561Xk, 561Xr is now 561Xd.

p 172 l 28 (561X) Add new exercise:

(s) In 561C, show that $(F, \alpha) \mapsto f_F(\alpha) : \mathcal{E} \times \mathbb{N}^{\mathbb{N}} \to \mathbb{N}^{\mathbb{N}}$ is continuous if $\mathcal{E}$ is given its Vietoris topology and $\mathbb{N}^{\mathbb{N}}$ its usual topology.

p 172 l 38 Exercise 561Yd (now 561Ye) has been expanded to include the former 561Yj, and now reads

(e) (i) Let (X, ρ) be a complete metric space. Show that X has a well-orderable dense subset iff it has a well-orderable base iff it has a well-orderable π-base iff it has a well-orderable network iff there is a choice function for the family of its non-empty closed subsets. (ii) Let X be a locally compact Hausdorff space. (α) Show that if it has a well-orderable π-base then it has a well-orderable dense subset. (β) Show that if it has a well-orderable base then it is completely regular and there is a choice function for the family of its non-empty closed subsets.

561Ye is now 561Yd. There is a new exercise:

(j) Let (X,ρ) be a complete metric space with a well-orderable base. Show that a closed totally bounded subset of X is compact.

p 174 l 27 The initial paragraph 562A and the introduction to resolvable sets in 562F have been split, and are now 562A-562B and 562G-562H. Consequently 562B-562E are now 562C-562F and 562G-562T are now 562I-562V.

p 171 l 36 (562Ae, now 562Ba): for '$X\setminus((X\setminus(X\setminus E_0))\cup(X\setminus E_0))$' read '$X\setminus((X\setminus(X\setminus U_0))\cup(X\setminus U_0))$'.

p 176 l 4 (562Bb, now 562Cc): for '$\phi(\tilde{T})=T$' read '$\phi(\tilde{T})=\phi(T)$'.

p 176 l 16 (562Bb, now 562Cc): for

$$\Theta'(T,T')=\{<n>:i_n\in A_{\tilde{T}},\,j_n\in A_{\tilde{T}'}\}\cup\{<n>^\frown\sigma:n\in\mathbb{N},\,\sigma\in\Theta(\tilde{T}_{<i_n>},\tilde{T}'_{<j_n>})\}.$$

read

$$\Theta'(T,T')=\{<n>:n\in A\}\cup\{<n>^\frown\sigma:n\in A,\,\sigma\in\Theta(\tilde{T}_{<i_n>},\tilde{T}'_{<j_n>})\}$$

where $A=\{n:i_n\in A_{\tilde{T}},\,j_n\in A_{\tilde{T}'}\}$.

p 177 l 7 Exercise 562Ya has been incorporated into Theorem 562E (now 562F), which accordingly reads

Theorem (a) If X is a Hausdorff second-countable space and A, B are disjoint analytic subsets of X, there is a codable Borel set $E\subseteq X$ such that $A\subseteq E$ and $B\cap E=\emptyset$.

(b) Let X be a Polish space. Then a subset E of X is a codable Borel set iff E and $X\setminus E$ are analytic.

p 177 l 37 (part (c) of the proof of 562E, now part (b-iii) of the proof of 562F): the check that f'_T is surjective is not quite trivial, and depends on the fact that there is a choice function for the non-empty closed subsets of $\mathbb{N}^{\mathbb{N}}$.

p 178 l 5 Resolvable sets The concluding remark in 562F (now 562G) has been given a formal statement and proof in a new paragraph 562H.

p 185 l 15 (562R(c-i), now 562T(c-ii)): for '$g^{-1}[\phi(T_{nq})]=g_n^{-1}[\phi(T'_{nq})]=\{x:h'_n(x)>q\}$' read '$g^{-1}[\phi(T_{nq})]=g_n^{-1}[\phi(T'_{nq})]=\{x:f_n(x)>q\}$'.

p 185 l 39 (562R, now 562T) Add new parts:

(c)(vi) If $E\subseteq X$, then $E\in\mathcal{B}\mathfrak{a}_c(X)$ iff $\chi E:X\to\mathbb{R}$ is a codable Baire function.

(d) If $\langle f_n\rangle_{n\in\mathbb{N}}$ is a codable sequence of codable Baire functions from X to $\mathbb{R}$, then $\langle f_n^{-1}[H]\rangle_{n\in\mathbb{N},H\subseteq\mathbb{R}\text{ is open}}$ is codable.

p 188 l 8 Exercise 562Ya has been incorporated into 562F. Add new exercises:

(a) Let X be a second-countable space, Y a T_0 second-countable space and $f:X\to Y$ a function with graph $\Gamma\subseteq X\times Y$. (i) Show that if f is a codable Borel function, then Γ is a codable Borel subset of $X\times Y$. (ii) Show that if X and Y are Polish and Γ is a codable Borel subset of $X\times Y$, then f is a codable Borel function.

(b) Show that there is an analytic subset of $\mathbb{N}^{\mathbb{N}}$ which is not a codable Borel set.

562Yb-562Yd are now 562Yc-562Ye.

p 189 l 17 Parts (b) and (c) of 563B have been exchanged. The former has been rewritten, and is now

Suppose that $\mathfrak{T}$ is T_1. If $\mathcal{E}$ is the algebra of resolvable subsets of X, then $\mu\restriction\mathcal{E}$ is countably additive in the sense that $\mu E=\sum_{n=0}^{\infty}\mu E_n$ for any disjoint family $\langle E_n\rangle_{n\in\mathbb{N}}$ in $\mathcal{E}$ such that $E=\sup_{n\in\mathbb{N}}E_n$ is defined in $\mathcal{E}$.

p 191 l 42 (part (c-ii-β) of the proof of 563F: for 'μ and ν have the same sets of finite measure' read 'μ and ν have the same sets of zero measure'.

p 192 l 18 (part (d-(iii)$\Rightarrow$(i)-α) of the proof of 563F): for 'difference of open sets' read 'difference of G_δ sets'. On the next line, for '$F'=\mathcal{B}_c(F)$' read '$F'\in\mathcal{B}_c(F)$'.

p 192 l 30 (part (f) of the proof of 563F): for '$F_{n+1}=F\cap\bigcup_{i\in J_n\cap k_n}\overline{U}_i$' read '$F_{n+1}=F_n\cap\bigcup_{i\in J_n\cap k_n}\overline{U}_i$'. On the next line, I omitted to justify the claim that $K=\bigcap_{n\in\mathbb{N}}F_n$ is compact. The mere fact that it is a closed totally bounded set in a Polish space won't quite do (561Y(c-iii)). The point is that the F_n come equipped with witnesses that each can be covered by finitely many sets of small diameter, and we can argue as follows.

Set $L = \prod_{n\in\mathbb{N}} J_n \cap k_n \subseteq \mathbb{N}^{\mathbb{N}}$. Then L is compact (561D). Set $L' = \{\alpha : \alpha \in L,\ F \cap \bigcap_{i\le n} \overline{U}_{\alpha(i)} \ne \emptyset$ for every $n\}$; then L' is a closed subset of L so is compact. For $\alpha \in L'$, $\{F \cap \overline{U}_{\alpha(i)} : i \in \mathbb{N}\}$ generates a filter $\mathcal{F}_\alpha$ on X which is a Cauchy filter because $\operatorname{diam} \overline{U}_{\alpha(i)} = \operatorname{diam} U_{\alpha(i)} \le 2^{-i}$ for every i; because X is ρ-complete, $f(\alpha) = \lim \mathcal{F}_\alpha$ is defined, and belongs to $F \cap \bigcap_{i\in\mathbb{N}} \overline{U}_{\alpha(i)} \subseteq K$. If α, $\beta \in L'$ and $\alpha(i) = \beta(i)$, then $f(\alpha)$, $f(\beta)$ both belong to $\overline{U}_{\alpha(i)}$ so $\rho(f(\alpha), f(\beta)) \le 2^{-i}$; thus f is continuous and $f[L']$ is a compact subset of K. On the other hand, given $x \in K$, we can set $\alpha(n) = \min\{i : i \in J_n \cap k_n,\ x \in \overline{U}_i\}$ for each n, and now $\alpha \in L'$ and $f(\alpha) = x$. So $K = f[L']$ is compact.

p 195 l 23 (part (f) of the proof of 563I): for '$\mu(H_n \cap E \setminus K_n)$' read '$\mu(E \setminus K_n)$'.

p 196 l 29 (part (a-iii) of the proof of 563L): for 'If $\langle E_n\rangle_{n\in\mathbb{N}}$ is a codable sequence in $\mathcal{B}\mathfrak{a}_c(X)$' read 'If $\langle E_n\rangle_{n\in\mathbb{N}}$ is a disjoint codable sequence in $\mathcal{B}\mathfrak{a}_c(X)$'.

p 197 l 11 (part (b) of the proof of 563N): for '$E \subseteq G$ and $\mu(G \setminus E) \le \epsilon$' read '$E \cap F \subseteq G$ and $\mu(G \setminus (E \cap F)) \le \epsilon$'.

p 198 l 2 (563X) Add new exercise:

(e) Let X be a zero-dimensional compact Hausdorff space, $\mathcal{E}$ the algebra of open-and-closed subsets of X and $\mu_0 : \mathcal{E} \to [0, \infty[$ an additive functional. Show that there is a unique Baire-coded measure on X extending μ_0.

p 202 l 7 (part (b-ii) of the proof of 564G): for '$2^m \int h_{n_k} \times \chi E_{mk} = 2^m \int (h_{n_k} - f + g) \times \chi E_{mk}$' read '$2^m \int h_{n_k} \times \chi E_{mk} \le 2^m \int (h_{n_k} - f + g) \times \chi E_{mk}$'.

p 202 l 17 (part (a) of the proof of 564H): for '$\sup_{n\in\mathbb{N}} f(nu \wedge \chi X)$' read '$\sup_{n\in\mathbb{N}} f(u_n)$'.

p 203 l 17 (part (a) of the proof of 564I): for '$v = v_1 \vee v_2$' read '$v = v_1 \wedge v_2$'.

p 210 l 36 (part (b-i) of the proof of 564O): for '$\{k : F_k \ne X_k\}$ is finite' read '$\{k : G_k \ne X_k\}$ is finite'.

p 211 l 31 (part (c-ii-β) of the proof of 564O): for 'λW_n' and 'λW' read '$\lambda_0 W_n$' and '$\lambda_0 W$'.

p 212 l 9 (564X) Add new exercise:

(b) Let X be a topological space, μ a Baire-coded measure on X, and f a non-negative integrable real-valued function defined almost everywhere in X. Set $\nu E = \int f \times \chi E$ for $E \in \mathcal{B}\mathfrak{a}_c(X)$. Show that ν is a Baire-coded measure, and that $\int g\,d\nu = \int g \times f\,d\mu$ for every ν-integrable g, if we interpret $(g \times f)(x)$ as 0 when $f(x) = 0$ and $g(x)$ is undefined.

564Xb-564Xd are now 564Xc-564Xe.

p 212 l 13 Part (iii) of Exercise 564Xb (now 564Xc) is now

(iii) Show that a norm-bounded sequence $\langle u_n\rangle_{n\in\mathbb{N}}$ in the normed space $C(X)$ is weakly convergent to 0 iff it is pointwise convergent to 0.

p 212 l 21 (564Y) Add new exercise:

(a) Let X be a topological space and $\langle f_n\rangle_{n\in\mathbb{N}}$ a codable sequence of bounded Baire-codable real-valued functions on X such that $\{\int f_n d\mu : n \in \mathbb{N}\}$ is bounded for every totally finite Baire-coded measure μ on X. (i) Show that if $\langle E_n\rangle_{n\in\mathbb{N}}$ is a disjoint codable sequence in $\mathcal{B}\mathfrak{a}_c(X)$ and μ is a Baire-coded measure on X, then $\lim_{n\to\infty} \int f_n \times \chi E_n d\mu = 0$. (ii) Now suppose in addition that $\lim_{n\to\infty} f_n(x) = 0$ for every $x \in X$. Show that if μ is a Baire-coded measure on X, then $\lim_{n\to\infty} \int f_n d\mu = 0$. (iii) Use this result to strengthen (iii) of 564Xb (now 564Xc) to 'a sequence $\langle u_n\rangle_{n\in\mathbb{N}}$ in $C(X)$ is weakly convergent to 0 iff it is bounded for the weak topology and pointwise convergent to 0'.

564Ya-564Yc are now 564Yb-564Yd.

p 216 l 17 (proof of 565F): for '$\sum_{n=0}^{\infty}\sum_{i=m_n}^{\infty}\hat{\mu}C'_{\mathcal{I}ni}$' read '$\sum_{n=0}^{\infty}\sum_{i=m_{kn}}^{\infty}\hat{\mu}C'_{\mathcal{I}ni}$'.

p 217 l 13 (part (b) of the proof of 565J): for '$|\int g^+ - \int_E g| \le \int |g \times \chi(E\triangle F)|$' read '$\int g^+ - \int_E g \le \int |g \times \chi(E\triangle F)|$'.

p 217 l 41 (part (a) of the proof of 565K): for '$\int C$' read '$\operatorname{int} C$'.

p 220 l 26 (part (a) of the proof of 565O): in the formulae $\sum_{i=0}^{n}\operatorname{diam} U_i$, $\sum_{n=0}^{\infty}\operatorname{diam} U_n$, $\sum_{i=0}^{m_n}\operatorname{diam} U_{ni}$ we need $\sum_{i=0}^{n}(\operatorname{diam} U_i)^s$, $\sum_{n=0}^{\infty}(\operatorname{diam} U_n)^s$ and $\sum_{i=0}^{m_n}(\operatorname{diam} U_{ni})^s$. More substantially, in the formula for G_n, we need to write

Let $\langle\eta_n\rangle_{n\in\mathbb{N}}$ be a sequence in $\left]0,\frac{1}{4}\delta\right[$ such that $\sum_{n=0}^{\infty}(2\eta_n + \operatorname{diam} A_n)^s \le \theta_s K + \epsilon$. For each $n \in \mathbb{N}$, set $G_n = \{x : \rho(x, A_n) < \eta_n\}$.

p 225 l 12 Lemma 566F has been refined; the function f can be made non-decreasing in the second variable.

p 226 l 20 (proof of 566H): for 'note that $d_n \in \mathfrak{B}_n$' read 'note that d_n is an atom of $\mathfrak{B}_{n+1}$'.

p 228 l 48 Parts (a) and (c) of Proposition 566M are now in 566Xc; 566Mb and 566Md are now 566Ma-566Mb.

p 229 l 13 (part (b-i) of the proof of 566M, now part (a-i)): for 'take $d \in A$' read 'take $a \in A$'.

p 232 l 13 (part (b) of the proof of 566O): the sentence 'set $\phi E = \tilde{\pi}(f^{-1}[E])$ for $E \in \mathcal{B}(\mathbb{R})$' should be moved to the end of the next paragraph, following the definition of $\tilde{\pi}$.

p 233 l 22 (part (d) of the proof of 566P): for '$\liminf_{u\to\mathcal{H}}\|u\|^2$' read '$\limsup_{u\to\mathcal{H}}\|u\|^2$'.

p 237 l 46 (566X) Add new exercise:

(c)(i) Let $\mathfrak{A}$ be a Boolean algebra, and $D \subseteq \mathfrak{A}$ an order-dense set. Show that $a = \sup\{d : d \in D,\ d \subseteq a\}$ for every $a \in \mathfrak{A}$. (ii) Let $(\mathfrak{A}, \bar{\mu})$ be a semi-finite measure algebra. Show that $a = \sup\{b : b \subseteq a,\ \bar{\mu}b < \infty\}$ for every $a \in \mathfrak{A}$.

Other exercises have been renamed: 566Xc-566Xk are now 566Xd-566Xl.

p 240 l 17 (part (c) of the proof of 567B): for '$w = (v(0), \tau(v{\restriction}1), v(2), \tau(v{\restriction}2), \dots, v(n-1))$' read '$w = (v(0), \tau(v{\restriction}1), v(1), \tau(v{\restriction}2), \dots, v(n-1))$'.

p 240 l 40 (proof of 567D): for 'the resulting play $(n, g(n)(0), 0, g(n)(1), 0, \dots)$ must belong to A' read 'the resulting play $(n, g(n)(0), 0, g(n)(1), 0, \dots)$ must not belong to A'.

p 248 l 40 The exercises 567X have been re-arranged: 567Xe is now 567Xo, 567Xf-567Xj are now 567Xe-567Xi, 567Xo-567Xq are now 567Xp-567Xr. 567Xe (now 567Xo) has been revised, and now reads

(o)(i) Show that there is a set $A \subseteq \omega_1^{\mathbb{N}}$ such that $\partial(\omega_1, A)$ is not determined. (ii) Show that there is a set $A \subseteq (\mathcal{P}\mathbb{R})^{\mathbb{N}}$ such that $\partial(\mathcal{P}\mathbb{R}, A)$ is not determined.

Add new exercise:

(j) [DC] Let I be a set, and $\widehat{\mathcal{B}}$ the Baire-property algebra of $\mathcal{P}I$ with its usual topology. Show that every $\widehat{\mathcal{B}}$-measurable real-valued finitely additive functional on $\mathcal{P}I$ is completely additive.

p 249 l 25 (567Y) Add new exercises:

(c) [AD] Suppose that $r \ge 3$, and that $c : \mathcal{P}\mathbb{R}^r \to [0,\infty]$ is Choquet-Newton capacity. Show that $c(A) = \sup\{c(K) : K \subseteq A \text{ is compact}\}$ for every $A \subseteq \mathbb{R}^r$.

(d) [AC($\mathbb{R}; \omega$)] Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra, and $\nu : \mathfrak{A} \to \mathbb{R}$ an additive functional which is Borel measurable for the order-sequential topology on $\mathfrak{A}$. Show that ν is countably additive.

567Yc-567Yd are now 567Ye-567Yf.

p 249 l 25 (Exercise 567Yc, now 567Ye): for 'Let Θ be the least ordinal such that there is no surjection from $\mathcal{P}\mathbb{N}$ onto Θ' read 'Let Θ be the least non-zero ordinal such that there is no surjection from $\mathcal{P}\mathbb{N}$ onto Θ.'.

p 252 l 16 New material has been added, as follows:

5A1C Concatenation Suppose that σ, τ are two functions with domains α, β respectively which are ordinals. Then we can form their **concatenation** $\sigma^\frown\tau$, setting

$$\mathrm{dom}(\sigma^\frown\tau) = \alpha + \beta$$

(the ordinal sum),

$$(\sigma^\frown\tau)(\xi) = \sigma(\xi) \text{ if } \xi < \alpha,$$
$$(\sigma^\frown\tau)(\alpha + \eta) = \tau(\eta) \text{ if } \eta < \beta.$$

The operator $^\frown$ is associative, so we can omit brackets and speak of $\sigma^\frown\tau^\frown\upsilon$. The empty function $\emptyset$ is an identity in the sense that

$$\emptyset^\frown\sigma = \sigma^\frown\emptyset = \sigma$$

whenever $\mathrm{dom}(\sigma)$ is an ordinal.

In this context, it will often be helpful to have a special notation for functions with domain the singleton set $\{0\} = 1$; I will write $<t>$ for the function with domain $\{0\}$ and value t.

We can also have infinite concatenations. If $\langle\sigma_n\rangle_{n\in\mathbb{N}}$ is a sequence of functions with ordinal domains, we can form the concatenations

$$\sigma_0^\frown\sigma_1, \quad \sigma_0^\frown\sigma_1^\frown\sigma_2, \quad \sigma_0^\frown\sigma_1^\frown\sigma_2^\frown\sigma_3, \quad \ldots$$

to get a sequence of functions each extending its predecessors. The union will be a function with domain the ordinal $\sup_{n\in\mathbb{N}} \mathrm{dom}(\sigma_0) + \ldots + \mathrm{dom}(\sigma_n)$. I will generally denote it $\sigma_0^\frown\sigma_1^\frown\sigma_2^\frown\ldots$ or in some similar form.

5A1C-5A1O are now 5A1D-5A1P.

p 252 l 23 (5A1C, now 5A1D) Add new part:

(d) If X is a Polish space and $\le$ is a well-founded relation on X such that $\{(x,y) : x < y\}$ is analytic, then the height of $\le$ is countable.

p 253 l 31 (part (e-iv) of 5A1E, now 5A1F): for $\mathrm{cf}[\kappa]^{\le\omega} = \max(\kappa, \mathrm{cf}[\lambda]^{\le\omega})$ read $\mathrm{cf}[\kappa]^{\le\omega} \le \max(\kappa, \mathrm{cf}[\lambda]^{\le\omega})$.

p 256 l 18 (5A1I, now 5A1J) Add new parts:

(d) If $R \subseteq X \times X$ is an equivalence relation on a set X I will say that a set $A \subseteq X$ is **R-free** if A meets each equivalence class for R in at most one point.

(e) Let X be a set and R an equivalence relation on X.

(i) For any cardinal κ, there is a partition $\langle X_\xi\rangle_{\xi<\kappa}$ of X into R-free sets iff every R-equivalence class has cardinal at most κ.

(ii) If $A \subseteq X$ is R-free then $R[B] \cap R[C] = \emptyset$ whenever B, $C \subseteq A$ are disjoint.

p 256 l 27 (proof of 5A1J, now 5A1K): delete 'and $S = S_M$'.

p 257 l 11 (Lemma 5A1M, now 5A1N) Add new part:

(d) If X and Y are sets and $\mathcal{I}$ is a maximal ideal of $\mathcal{P}X$, then there is an $F \subseteq Y^X$ such that $\#(F) = \mathrm{Tr}_{\mathcal{I}}(X;Y)$ and $\{x : f(x) = g(x)\} \in \mathcal{I}$ for all distinct f, $g \in F$.

p 258 l 26 The last part of §5A1 has been moved to a new §5A6, so that 5A1P-5A1W are now 5A6A-5A6H.

p 257 l 24 (part (b) of the proof of 5A1M) For '$\langle fg_\alpha\rangle_{\alpha\in S}$' read '$\langle fg_\alpha\rangle_{\alpha\in S_1}$'.

5A6I $\mathfrak{u}$, $\mathfrak{g}$ and the filter dichotomy: Definitions (a) The **ultrafilter number** $\mathfrak{u}$ is the least cardinal of any filter base generating a free ultrafilter on $\mathbb{N}$.

(b)(i) A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is **groupwise dense** if

(α) whenever $A \in \mathcal{A}$, $A' \in [\mathbb{N}]^\omega$ and $A' \setminus A$ is finite, then $A' \in \mathcal{A}$,

(β) whenever $\langle I_k\rangle_{k\in\mathbb{N}}$ is a disjoint sequence of finite subsets of $\mathbb{N}$, there is an infinite $C \subseteq \mathbb{N}$ such that $\bigcup_{k\in C} I_k \in \mathcal{A}$.

(ii) The **groupwise density number** $\mathfrak{g}$ is the least cardinal of any collection $\mathbb{A}$ of groupwise dense subsets of $[\mathbb{N}]^\omega$ such that $\bigcap \mathbb{A} = \emptyset$.

(c) The **filter dichotomy** is the statement

(FD) For every free filter $\mathcal{F}$ on $\mathbb{N}$ there is a finite-to-one function $f : \mathbb{N} \to \mathbb{N}$ such that $f[[\mathcal{F}]]$ is either the Fréchet filter or an ultrafilter.

5A6J Proposition If $\mathfrak{u} < \mathfrak{g}$ then the filter dichotomy is true.

p 261 l 6 Part (c) of 5A1U (now 5A6F) should read

(c) CL implies that CTP(κ, λ) is false except when $\lambda \leq \omega$.

p 261 l 36 (5A2Ab) for 'cf$(\prod_{i\in I} P_i) \leq \operatorname{cf} P$' read 'cf$(\prod_{i\in F} P_i) \leq \operatorname{cf} P$'.

p 263 l 13 (part (b-ii) of the proof of 5A2B): for 'because $\operatorname{add} P \geq \delta > \#(\xi)$' read 'because $\operatorname{add} P \geq \lambda^+ \#(\xi)$'.

p 268 l 28 (part (e) of the proof of 5A2G): for

$$`g^*(\eta) = \sup\{f(\xi) : \xi < \gamma_0,\ h^*(\xi) = \eta\}'$$

read

$$`g^*(\eta) = \sup\{f(\xi) : \xi < \gamma_0,\ f(\xi) < h^*(\xi) = \eta\}.'$$

p 274 l 27 Part (b) of 5A3H has been expanded, and now reads

(b) In this case,

$$\Vdash_{\mathbb{P}} \operatorname{dom} \dot{f} = \dot{A} \text{ and } \dot{f}[\dot{A}] = \dot{B},$$

and the following are equiveridical:

(i) $\Vdash_{\mathbb{P}} \dot{f}$ is injective;

(ii) whenever $(\dot{x}_0, \dot{y}_0, p_0)$, $(\dot{x}_1, \dot{y}_1, p_1)$ belong to R, $p \in \mathbb{P}$ is stronger than both p_0 and p_1 and $p \Vdash_{\mathbb{P}} \dot{y}_0 = \dot{y}_1$, then $p \Vdash_{\mathbb{P}} \dot{x}_0 = \dot{x}_1$.

p 280 l 17 (part (ii) of the argument for 5A3Nd) for '$K_\xi \not\subseteq K$' read '$K \not\subseteq K_\xi$'.

p l Add '$t(Y) \leq t(X)$' to the list in 5A4Bb.

p 284 l 11 Part (d-iii) of 5A4C has been revised, and now reads

So if there is a continuous surjection from a closed subset of X onto $\{0,1\}^\kappa$, there is a non-empty closed $K \subseteq X$ such that $\chi(x, K) \geq \kappa$ for every $x \in K$.

p 285 l 17 5A4Eb is now 4A3S(a-i), so has been deleted. 5A4Ec-5A4Ed are now 5A4Eb-5A4Ec.

p 311 l 30 (5A4E(c-iii), now 5A4E(b-iii)) for '$V \in \mathcal{V}_n$' read '$V \in \mathcal{V}'_n$'.

p 285 l 41 (5A4E) Add new fragment:

(c)(iii) Every subset of X with the Baire property is expressible as $G \triangle M$ where G is a cozero set and M is meager.

p 286 l 17 The note on Baire σ-algebras (5A4G) has been expanded, as follows.

(b)(i) If $\langle X_i \rangle_{i\in I}$ is a family of separable metrizable spaces with product X, then $\#(\mathcal{B}\mathfrak{a}(X)) \leq \max(\mathfrak{c}, \#(I)^\omega)$.

(ii) If $\kappa \geq 2$ is a cardinal, then the set of Baire measurable functions from $\{0,1\}^\kappa$ to $\{0,1\}^\omega$ has cardinal κ^ω.

p 286 l 18 Blumberg's theorem (5A4H) has been dropped. 5A4I and 5A4J are now 5A4H and 5A4L.

p 288 l 5 Compact-open topologies I have interpolated a couple of paragraphs on a class of topologies on spaces of functions, and a note on irreducible surjections.

5A4Ia Definition Let X and Y be topological spaces and F a set of functions from X to Y. The **compact-open** topology on F is the topology generated by sets of the form $\{f : f \in F, f[K] \subseteq H\}$ where $K \subseteq X$ is compact and $H \subseteq Y$ is open.

(b) Let X be a compact topological space and I a set; give $Y=\{0,1\}^I$ its usual product topology and $Z=C(X;Y)$ its compact-open topology. Let $\mathcal{E}$ be the algebra of open-and-closed subsets of X. (i) Z is homeomorphic to $\mathcal{E}^I$ with its product topology, where here we give $\mathcal{E}$ its discrete topology. (ii) Set $H_i=\{y:y\in Y,\ y(i)=1\}$ for $i\in I$. Then $\{\{f:f\in Z,\ f^{-1}[H_i]=E\}:i\in I,\ E\in\mathcal{E}\}$ is a subbase for the topology of Z.

5A4J Proposition Let X be a set and $\mathcal{A}$ a family of countable sets which is stationary over X. Give X its discrete topology and $X^{\mathbb{N}}$ the product topology; let $\mathcal{M}(X^{\mathbb{N}})$ be the associated ideal of meager sets. Then $\operatorname{non}\mathcal{M}(X^{\mathbb{N}})\le\max(\#(\mathcal{A}),\operatorname{non}\mathcal{M}(\mathbb{R}))$.

5A4K Lemma Let X be a topological space and K, L closed subsets of X such that $K\cup L=X$. Set $Z=\{(x,1):x\in K\}\cup\{(x,0):x\in L\}\subseteq X\times\{0,1\}$, and wwrite $\pi:Z\to X$ for the first-coordinate map. Then π is an irreducible continuous surjection.

p 288 l 29 The former §5A6 has been dropped.