Errata and addenda for Volume 3, 2012 printing

I collect here known errors and omissions, with their discoverers, in Volume 3 of my book *Measure Theory* (see my web page, `http://www1.essex.ac.uk/maths/people/fremlin/mt.htm`).

Part I

p 28 l 18 (312X) Add new exercises:

(l) Let $\mathfrak{A}$ be a Boolean algebra, and $A \subseteq \mathfrak{A}$ a set, closed under $\cup$ and $\cap$, such that 0, $1 \in A$. Let B be the set of elements of $\mathfrak{A}$ expressible as $a \setminus a'$ where a, $a' \in A$, and C the set of elements of $\mathfrak{A}$ expressible as $b_0 \cup \ldots \cup b_n$ where $b_0, \ldots , b_n \in B$ are disjoint. Show that C is a subalgebra of $\mathfrak{A}$.

(m) Let $\mathfrak{A}$, $\mathfrak{B}$ be Boolean algebras, and $A \subseteq \mathfrak{A}$ a set, closed under $\cup$ and $\cap$, such that $0_{\mathfrak{A}}$, $1_{\mathfrak{A}} \in A$; let $\mathfrak{C}$ be the subalgebra of $\mathfrak{A}$ generated by A. Let $\pi : A \to \mathfrak{B}$ be such that $\pi 0_{\mathfrak{A}} = 0_{\mathfrak{B}}$ and $\pi 1_{\mathfrak{A}} = 1_{\mathfrak{B}}$, and $\pi(a \cup a') = \pi a \cup \pi a'$, $\pi(a \cap a') = \pi a \cap \pi a'$ for all a, $a' \in \mathfrak{A}$. Show that π has a unique extension to a Boolean homomorphism from $\mathfrak{C}$ to $\mathfrak{B}$.

p 38 l 11 Add new exercises:

(s) Let $\mathfrak{A}$ and $\mathfrak{B}$ be Boolean algebras, $\pi : \mathfrak{A} \to \mathfrak{B}$ a Boolean homomorphism and D an order-dense subset of $\mathfrak{A}$ containing 0. Show that π is injective iff $\pi{\restriction}D$ is injective.

(t) Let $\mathfrak{A}$ be a Boolean algebra and $A_0, \ldots , A_n$ subsets of $\mathfrak{A}$ such that $\sup A_i$ is defined for each $i \le n$. Set $B = \{a_0 \cap \ldots \cap a_n : a_i \in A_i \text{ for each } i\}$. Show that $\sup B$ is defined and equal to $\inf_{i<n} \sup A_i$.

p 48 l 31 (314X) Add new exercise:

(l) Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra, $\mathfrak{B}$ an order-closed subalgebra of $\mathfrak{A}$, c a member of $\mathfrak{A}$ and $\mathfrak{C}$ the subalgebra of $\mathfrak{A}$ generated by $\mathfrak{B} \cup \{c\}$. Show that if $a \in \mathfrak{C}$, then $c \cap a = c \cap \operatorname{upr}(c \cap a, \mathfrak{B})$.

p 64 l 35 Add new result:

***316R Proposition** Let $\mathfrak{A}$ be a Boolean algebra, and $\mathfrak{B}$ a subalgebra of $\mathfrak{A}$ which is regularly embedded in $\mathfrak{A}$.

(a) Every atom of $\mathfrak{A}$ is included in an atom of $\mathfrak{B}$.

(b) If $\mathfrak{B}$ is atomless, so is $\mathfrak{A}$.

p 67 l 5 (316Y) Add new exercise:

(t) Suppose that $\mathfrak{A}$ is a weakly (σ, ∞)-distributive Boolean algebra, and that $\langle A_n\rangle_{n\in\mathbb{N}}$ is a sequence of upwards-directed subsets of $\mathfrak{A}$. Set

$$B = \bigcap_{n\in\mathbb{N}}\{b : b \in \mathfrak{A}, \text{ there is an } a \in A_n \text{ such that } b \cap a' \subseteq b \cap a \text{ for every } a' \in A_n\}.$$

Show that B is upwards-directed and $\sup B = 1$.

p 75 l 40 (322L) Add new part:

(d) Let $(\mathfrak{A}, \bar\mu)$ be a measure algebra, and $\langle e_i\rangle_{i\in I}$ a countable partition of unity in $\mathfrak{A}$. Then $(\mathfrak{A}, \bar\mu)$ is isomorphic to the product $\prod_{i\in I}(\mathfrak{A}_{e_i}, \bar\mu{\restriction}\mathfrak{A}_{e_i})$ of the corresponding principal ideals.

322Ld is now 322Le.

p 91 l 37 (part (b-ii) of the proof of 324P): for 'even if C is' read 'even if A is'.

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p 92 l 11 (Exercise 324Xa) for '$\phi(a^\bullet) = \phi a$' read '$\pi(a^\bullet) = \phi a$'.

p 92 l 13 (Exercise 324Xb) for '$\mathfrak{B}$ a σ-subalgebra of $\mathfrak{A}$' read '$\mathfrak{B}$ an order-closed subalgebra of $\mathfrak{A}$'.

p 92 l 24 (Exercise 324Xa) for '$\phi(a^\bullet) = \phi a$' read '$\pi(a^\bullet) = \phi a$'.

p 92 l 41 Exercise 324Yg has been deleted; 324Yh is now 324Yg.

p 126 l 7 (328X) Add new exercise:

(c) Let $(\mathfrak{A}, \bar\mu)$ be a purely atomic probability algebra, I a non-empty set and $\mathcal{F}$ an ultrafilter on I. Show that $(\mathfrak{A}, \bar\mu)^I|\mathcal{F}$ is isomorphic to $(\mathfrak{A}, \bar\mu)$.

p 133 l 31 Add new result:

331P Proposition Let $(\mathfrak{A}, \bar\mu)$ be an atomless probability algebra of countable Maharam type. Then it is isomorphic to the measure algebras of the usual measure on $\{0,1\}^{\mathbb{N}}$ and of Lebesgue measure on $[0,1]$.

p 134 l 43 (331Y) Add new exercise:

(k) Let $\langle(\mathfrak{A}_i, \bar\mu_i)\rangle_{i\in I}$ be a non-empty family of homogeneous probability algebras, and $\mathcal{F}$ an ultrafilter on I. Show that the probability algebra reduced product $\prod_{i\in I}(\mathfrak{A}_i, \bar\mu_i)|\mathcal{F}$ is homogeneous.

p 173 l 24 (341Y) Add new exercise:

(f) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra and $\langle a_i\rangle_{i\in I}$ a family in $\mathfrak{A}$. Let $\mathcal{B}\mathfrak{a}_I$ be the Baire σ-algebra of $Y = \{0,1\}^I$, that is, the σ-algebra of subsets of Y generated by the family $\{E_i : i \in I\}$ where $E_i = \{y : y \in Y,\ y(i) = 1\}$ for $i \in I$. Show that there is a unique sequentially order-continuous Boolean homomorphism $\phi : \mathcal{B}\mathfrak{a}_I \to \mathfrak{A}$ such that $\phi E_i = a_i$ for every $i \in I$, and that $\phi[\mathcal{B}\mathfrak{a}_I]$ is the σ-subalgebra of $\mathfrak{A}$ generated by $\{a_i : i \in I\}$.

Part II

p 15 l 20 (352D) For

$$|u+v| = (u+v) \wedge ((-u)+(-v)) \le (|u|+|v|) \wedge (|u|+|v|) = |u|+|v|$$

read

$$|u+v| = (u+v) \vee ((-u)+(-v)) \le (|u|+|v|) \vee (|u|+|v|) = |u|+|v|.$$

p 15 l 21 (352D) For

$$||u|-|v|| = (|u|-|v|) \vee (|v|-|u|) \le |u-v| + |v-u| = |u-v|$$

read

$$||u|-|v|| = (|u|-|v|) \vee (|v|-|u|) \le |u-v| \vee |v-u| = |u-v|.$$

p 16 l 4 Add new fragment to 352Fa:

(ii) If $v_0, \dots, v_m, w_0, \dots, w_n \in U^+$ then

$$\textstyle\sum_{i=0}^m v_i \wedge \sum_{j=0}^n w_j \le \sum_{i=0}^m \sum_{j=0}^n v_i \wedge w_j.$$

p 24 l 24 (352Y) Add new exercises:

(a) Find an f-algebra with a non-commutative multiplication.

(b) Let U be an f-algebra. Show that the multiplication of U is commutative iff $u \times v = (u \wedge v) \times (u \vee v)$ for all u, $v \in U$.

(c) Let U be an f-algebra. Show that $u \times \mathrm{med}(v_1, v_2, v_3) = \mathrm{med}(u\times v_1, u\times v_2, u\times v_3)$ whenever u, v_1, v_2, $v_3 \in U$.

p 25 l 45 (proof of 353E) for '$nv_0 \le u \Longrightarrow nv_0 \le v \Longrightarrow (n+1)v_0 \le v + w = u$' read '$n(w \wedge v_0) \le u \Longrightarrow n(w \wedge v_0) \le v \Longrightarrow (n+1)(w \wedge v_0) \le v + w = u$', and similarly in the next line.

p 26 l 6 353Q has been brought forward to 353G. 353G-353P are now 353H-353Q.

p 27 l 4 (statement of 353J(a-iii), now 353K) for 'the canonical map from U to V' read 'the canonical map from U to U/V'.

p 31 l 24 (proof of 353Q, now 353G) for '$nw \wedge n(\tilde{u}-v)^+ = n((v-\tilde{u})^+ \wedge (\tilde{u}-v)^+)$' read '$nw \wedge n(\tilde{u}-v)^+ \le n((v-\tilde{u})^+ \wedge (\tilde{u}-v)^+)$'.

p 31 l 40 Exercises 353Yd-353Ye have been exchanged.

p 69 l 33 (361X) Add new exercise:

(l) Let $\mathfrak{A}$ be a Boolean algebra, U a partially ordered linear space and $\nu : \mathfrak{A} \to U$ a non-negative additive function. (i) Show that ν is order-continuous iff $\nu 1 = \sup_{J \subseteq I \text{ is finite}} \sum_{i \in J} \nu a_i$ whenever $\langle a_i \rangle_{i \in I}$ is a partition of unity in U. (i) Show that ν is order-continuous iff $\nu 1 = \sup_{n \in \mathbb{N}} \sum_{i=0}^{n} \nu a_i$ whenever $\langle a_i \rangle_{i \in \mathbb{N}}$ is a partition of unity in U.

p 119 l 37 Corollary 365J has been dropped. 365K-365U are now 365J-365T.

p 128 l 13 (statement of 365U, now 365T): for '$\int (u - M\chi a)^+ \le \epsilon$' read '$\int (|u| - M\chi a)^+ \le \epsilon$'.

p 128 l 37 (365X) Add new exercise:

(f) Let $(\mathfrak{A}, \bar{\mu})$ be a measure algebra and $u, v \in L^0(\mathfrak{A})^+$. Show that $\int u \times v \, d\bar{\mu} = \int_0^\infty (\int_{[\![u>\alpha]\!]} v \, d\bar{\mu}) d\alpha$.

Other exercises have been rearranged; 365Xf-365Xl are now 365Xg-365Xm, 365Xm-365Xp are now 365Xo-365Xr, 365Xq is now 365Xn.

p 128 l 39 (statement (iii) of Exercise 365Xf, now 365Xg): for '$\nu a \le \epsilon$ whenever $a \subseteq c$ and $\bar{\mu} a \le \delta$' read '$|\nu a| \le \epsilon$ whenever $a \subseteq c$ and $\bar{\mu} a \le \delta$'.

p 150 l 7 (367R) Add new part:

(d) A non-empty set $A \subseteq L^0$ is bounded in the linear topological space sense iff $\inf_{k \in \mathbb{N}} \sup_{u \in A} \bar{\mu}(a \cap [\![|u| > k]\!]) = 0$ for every $a \in \mathfrak{A}^f$.

p 178 l 32 (369Xb) for '$\phi(\alpha x + (1-\alpha) y) \ge \alpha \phi(x) + (1-\alpha)\phi(y)$' read '$\phi(\alpha x + (1-\alpha) y) \le \alpha \phi(x) + (1-\alpha)\phi(y)$'.

p 179 l 18 The exercises for §369 have been rearranged; 369Xh-369Xk are now 369Xj-369Xm, 369Xl-369Xm are now 369Xh-369Xi, 369Yc-369Yf are now 369Yd-369Yg, 369Yg is now 369Yc.

p 179 l 34 (369Xj, now 369Xl) To show that $\| \, \|_{1,\infty}$ cannot be represented as an Orlicz norm, we need a further hypothesis on the measure algebra $(\mathfrak{A}, \bar{\mu})$; e.g., that $\mathfrak{A}$ is atomless and $\bar{\mu}$ is not totally finite.

p 180 l 24 Exercise 369Ye, now 369Yf, is wrong as stated; it now reads

(f) Let $(\mathfrak{A}, \bar{\mu})$ be a semi-finite measure algebra and $\phi : [0, \infty[\, \to [0, \infty[$ be a strictly increasing Young's function such that $\sup_{t>0} \phi(2t)/\phi(t)$ is finite. Show that if $\mathcal{F}$ is a filter on L^{τ_ϕ}, then $\mathcal{F} \to u \in L^{\tau_\phi}$ for the norm τ_ϕ iff (i) $\mathcal{F} \to u$ for the topology of convergence in measure (ii) $\limsup_{v \to \mathcal{F}} \tau_\phi(v) \le \tau_\phi(u)$.

p 180 l 27 Exercise 369Yf, now 369Yg, has been elaborated, and now reads

(g) Give examples of extended Fatou norms τ on measure spaces $L^0(\mathfrak{A})$, where $(\mathfrak{A}, \bar{\mu})$ is a semi-finite measure algebra, such that (α) $\tau \restriction L^\tau$ is order-continuous (β) there is a sequence $\langle u_n \rangle_{n \in \mathbb{N}}$ in L^τ, converging in measure to $u \in L^\tau$, such that $\lim_{n \to \infty} \tau(u_n) = \tau(u)$ but $\langle u_n \rangle_{n \in \mathbb{N}}$ does not converge to u for the norm on L^τ. Do this (i) with τ an Orlicz norm (ii) with $(\mathfrak{A}, \bar{\mu})$ the measure algebra of Lebesgue measure on $\mathbb{R}$.

p 200 l 33 Add new parts to Exercise 372Yi(ii): (β) Show that if ψ is weakly mixing then θ is weakly mixing. (γ) Show that if ϕ and ψ are mixing then θ is mixing.

p 276 l 11 (381Y) Add new exercise:

(e)(i) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra, and a, b two elements of $\mathfrak{A}$. Suppose that $\pi : \mathfrak{A}_a \to \mathfrak{A}_b$ is a Boolean isomorphism such that there is no disjoint sequence $\langle c_n \rangle_{n \in \mathbb{N}}$ of non-zero elements of $\mathfrak{A}_{a \cap b}$ such that $\pi c_n = c_{n+1}$ for every $n \in \mathbb{N}$. Show that there is a Boolean automorphism of $\mathfrak{A}$ extending π. (ii) Let $(\mathfrak{A}, \bar{\mu})$ be a measure algebra, and a, $b \in \mathfrak{A}$ two elements of $\mathfrak{A}$ such that $\bar{\mu}(a \cap b) < \infty$. Show that any measure-preserving isomorphism from $\mathfrak{A}_a$ to $\mathfrak{A}_b$ extends to a measure-preserving automorphism of $\mathfrak{A}$. (Compare 332L.)

p 290 l 37 (part (b) of the statement of Lemma 383G): for 'every element of $\mathfrak{A}$' read 'every non-zero element of $\mathfrak{A}$'.

p 293 l 4 The exercises 383X have been rearranged: 383Xb-383Xj are now 383Xd-383Xl, 383Xk-383Xl are now 383Xb-383Xc.

p 293 l 44 (Exercise 383Xk, now 383Xb): for '$\pi_{fg} = \pi_f \pi_g$' read '$\pi_{fg} = \pi_g \pi_f$'.

p 294 l 3 (Exercise 383Xl, now 383Xc): part (i) should read 'show that $f \mapsto \pi_f^{-1}$ is a group homomorphism'. In part (iii), we need to suppose that F is a countable subgroup of Φ.

p 294 l 11 (Exercise 383Yb): for '$5 \le \#(X) \le \omega$' read '$5 \le \#(X) < \omega$'.

p 302 l 23 (384X) Add new exercise:

(f) Show that if X is any set such that $\#(X) \ne 6$, the group G of all permutations of X has no outer automorphisms.

384Xf is now 384Xg

p 313 l 45 The exercises to §385 have been rearranged: 385Xf-385Xh are now 385Xg-385Xi, 385Xi-385Xm are now 385Xk-385Xo, 385Xn-385Xp are now 385Xq-385Xs, 385Xq, in revised form, is now 385Xf, 385Xr is now 385Xj, 385Xs is now 385Xp.

385Yb-385Yg are now 385Yc-385Yh, 385Yh is now 385Yb.

p 314 l 18 I have not been able to decide whether every automorphism which is conjgate to its inverse is expressible as a product of at most two involutions. Exercise 385Xl (now 385Xn) therefore now reads

(n) Let $(\mathfrak{A}, \bar{\mu})$ be a probability algebra and $\pi : \mathfrak{A} \to \mathfrak{A}$ a two-sided Bernoulli shift. (i) Show that π^{-1} is a two-sided Bernoulli shift. (ii) Show that π and π^{-1} are conjugate in $\mathrm{Aut}_{\bar{\mu}}\mathfrak{A}$. (iii) Show that π is expressible as the product of at most two involutions.

p 314 l 36 There is an error in Exercise 385Xq: the lattice operation $\wedge$ is not in general continuous for the entropy metric. This exercise now reads

(f) Let $(\mathfrak{A}, \bar{\mu})$ be a probability algebra, and write $\mathcal{A}$ for the set of partitions of unity in $\mathfrak{A}$ not containing 0, ordered by saying that $A \le B$ if B refines A. (i) Show that $\mathcal{A}$ is a Dedekind complete lattice, and can be identified with the lattice of purely atomic closed subalgebras of $\mathfrak{A}$. Show that for A, $B \in \mathcal{A}$, $A \vee B$, as defined in 385F, is $\sup\{A, B\}$ in $\mathcal{A}$. (ii) Show that $H(A \vee B) + H(A \wedge B) \le H(A) + H(B)$ for all A, $B \in \mathcal{A}$, where $\vee$, $\wedge$ are the lattice operations on $\mathcal{A}$. (iii) Set $\mathcal{A}_1 = \{A : A \in \mathcal{A}, H(A) < \infty\}$. For A, $B \in \mathcal{A}_1$ set $\rho(A, B) = 2H(A \vee B) - H(A) - H(B)$. Show that ρ is a metric on $\mathcal{A}_1$ (the **entropy metric**). (iv) Show that $H : \mathcal{A}_1 \to [0, \infty[$ is 1-Lipschitz for ρ. (v) Show that the lattice operation $\vee$ is uniformly ρ-continuous on $\mathcal{A}_1$. (vi) Show that $H : \mathcal{A}_1 \to [0, \infty[$ is order-continuous. (vii) Show that if $\mathfrak{B}$ is any closed subalgebra of $\mathfrak{A}$, then $A \mapsto H(A|\mathfrak{B})$ is order-continuous and 1-Lipschitz on $\mathcal{A}_1$. (viii) Show that if $\pi : \mathfrak{A} \to \mathfrak{A}$ is a measure-preserving Boolean homomorphism, $A \mapsto h(\pi, A) : \mathcal{A}_1 \to [0, \infty[$ is 1-Lipschitz for ρ.

p 320 l 23 Definition 386G has been dropped, in favour of expressing the results here in terms of the function q of 385A.

386H-386O are now 386G-386N.

p 322 l 12 (proof of 386K, now 386J) for '$\rho(c_i, B_k)$' read '$\rho(c_j, B_k)$'.

p 324 l 29 (part (c) of the proof of 386N, now 386M) for '$\rho(c_b, \mathfrak{C})$' read '$\rho(b, \mathfrak{C})$'.

p 326 l 25 (387A) Add new part:

(d) Let $\mathfrak{B}$, $\mathfrak{C}$ be closed subalgebras of $\mathfrak{A}$ such that $\pi[\mathfrak{B}] \subseteq \mathfrak{B}$ and $\pi[\mathfrak{C}] \subseteq \mathfrak{C}$. I will write $\mathrm{Hom}_{\bar{\mu},\pi}(\mathfrak{B}; \mathfrak{C})$ for the set of Boolean homomorphisms $\phi : \mathfrak{B} \to \mathfrak{C}$ such that

$$\bar{\mu}\phi b = \bar{\mu} b, \quad \pi\phi b = \phi\pi b$$

for every $b \in \mathfrak{B}$. On $\operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$ the **weak uniformity** will be the uniformity generated by the pseudometrics

$$(\phi,\psi) \mapsto \bar{\mu}(\phi b \mathbin{\triangle} \psi b)$$

for $b \in \mathfrak{B}$; the **weak topology** on $\operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$ will be the associated topology.

p 326 l 26 387B has been rewritten, and is now

387B Elementary facts Suppose that $(\mathfrak{A},\bar{\mu})$ is a probability algebra, $\pi \in \operatorname{Aut}_{\bar{\mu}}\mathfrak{A}$ and that $\langle b_i\rangle_{i\in I}$ is a Bernoulli partition for π. Write $\mathfrak{B}_0$ for the closed subalgebra of $\mathfrak{A}$ generated by $\{b_i : i \in I\}$, $\mathfrak{B}$ for the closed subalgebra generated by $\{\pi^j b_i : i \in I,\ j \in \mathbb{Z}\}$, and B for $\{b_i : i \in I\} \setminus \{0\}$.

(a) $\pi{\restriction}\mathfrak{B}$ is a two-sided Bernoulli shift with root algebra $\mathfrak{B}_0$ and entropy $H(B) = h(\pi,B) \le h(\pi)$.

(b) If $H(B) > 0$ then $\mathfrak{A}$ is atomless.

(c) Suppose now that $\langle c_i\rangle_{i\in I}$ is another Bernoulli partition for π with $\bar{\mu}c_i = \bar{\mu}b_i$ for every i; let $\mathfrak{C}$ be the closed subalgebra of $\mathfrak{A}$ generated by $\{\pi^j c_i : i \in I,\ j \in \mathbb{Z}\}$. Then we have a unique $\phi \in \operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$ such that $\phi b_i = c_i$ for every $i \in I$, and ϕ is an isomorphism between $(\mathfrak{B},\bar{\mu}{\restriction}\mathfrak{B},\pi{\restriction}\mathfrak{B})$ and $(\mathfrak{C},\bar{\mu}{\restriction}\mathfrak{C},\pi{\restriction}\mathfrak{C})$.

p 330 l 24 (part (e) of the proof of 387C): for '$\bar{\mu}(\sup B) \ge 1-2\delta$' read '$\bar{\mu}(\sup B') \ge 1-2\delta$'.

p 332 l 11 (part (i) of the proof of 387C): for

This means that there must be some $b \in B$ and $d' \in D_n(C,\pi)$ such that $d \cap f(d) \cap b \cap f(d') \ne 0$ and $\#(I_{bd'}) \ge n(1-\beta-4\delta)$; of course $b = f(d)$ and $d' = d$ (because f is injective), so that $\#(I_{f(d),d})$ must be at least $n(1-\beta-4\delta)$

read

This means that there must be some $b \in B$ and $d' \in D_n(C,\pi)$ such that $d \cap f(d) \cap b \cap d' \ne 0$ and $\#(I_{bd'}) \ge n(1-\beta-4\delta)$; of course $d' = d$ and $b = f(d)$, so that $\#(I_{f(d),d})$ must be at least $n(1-\beta-4\delta)$.

p 334 l 29 Add new result:

387F Lemma Let $(\mathfrak{A},\bar{\mu})$ be a probability algebra, π a measure-preserving automorphism of $\mathfrak{A}$, and $\mathfrak{B}$, $\mathfrak{C}$ closed subalgebras of $\mathfrak{A}$ such that $\pi[\mathfrak{B}] = \mathfrak{B}$ and $\pi[\mathfrak{C}] = \mathfrak{C}$.

(a) Suppose that $\phi \in \operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$.

(i) $\pi^j\phi = \phi\pi^j$ for every $j \in \mathbb{Z}$.

(ii) $\phi[\mathfrak{B}]$ is a closed subalgebra of $\mathfrak{C}$ and $\pi[\phi[\mathfrak{B}]] = \phi[\mathfrak{B}]$; ϕ is an isomorphism between $(\mathfrak{B},\bar{\mu}{\restriction}\mathfrak{B},\pi{\restriction}\mathfrak{B})$ and $(\phi[\mathfrak{B}],\bar{\mu}{\restriction}\phi[\mathfrak{B}],\pi{\restriction}\phi[\mathfrak{B}])$.

(iii) If $\psi \in \operatorname{Hom}_{\bar{\mu},\pi}(\phi[\mathfrak{B}];\mathfrak{C})$ then $\psi\phi \in \operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$.

(iv) If $\langle b_i\rangle_{i\in I}$ is a Bernoulli partition for $\pi{\restriction}\mathfrak{B}$, then $\langle \phi b_i\rangle_{i\in I}$ is a Bernoulli partition for $\pi{\restriction}\mathfrak{C}$.

(b) $\operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$ is complete under its weak uniformity.

(c) Let $B \subseteq \mathfrak{B}$ be such that $\mathfrak{B}$ is the closed subalgebra of itself generated by $\bigcup_{i\in\mathbb{Z}}\pi^i[B]$. Then the weak uniformity of $\operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$ is the uniformity defined by the pseudometrics $(\phi,\psi) \mapsto \bar{\mu}(\phi b \mathbin{\triangle} \psi b)$ as b runs over B.

387F-387L are now 387G-387M.

p 347 l 12 (387X) Add new exercises:

(c) Let $(\mathfrak{A},\bar{\mu})$ be a measure algebra and $\pi \in \operatorname{Aut}_{\bar{\mu}}\mathfrak{A}$. Show that $(\phi,\psi) \mapsto \psi\phi : \operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{A};\mathfrak{A}) \times \operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{A};\mathfrak{A}) \to \operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{A};\mathfrak{A})$ is continuous for the weak topology on $\operatorname{Hom}_{\bar{\mu},\pi}(\mathfrak{A};\mathfrak{A})$.

(d) Let $(\mathfrak{A},\bar{\mu})$ be a probability algebra, and write ι for the identity map on $\mathfrak{A}$; regard $\operatorname{Aut}_{\bar{\mu}}\mathfrak{A}$ as a subset of $\operatorname{Hom}_{\bar{\mu},\iota}(\mathfrak{A};\mathfrak{A})$ with its weak topology. Show that $\pi \mapsto \pi^{-1} : \operatorname{Aut}_{\bar{\mu}}\mathfrak{A} \to \operatorname{Aut}_{\bar{\mu}}\mathfrak{A}$ is continuous.

387Xc-387Xe are now 387Xc-387Xg.

p 357 l 31 (388X) Add new exercises:

(c) Let $\mathfrak{A}$ be a Boolean algebra and $\pi : \mathfrak{A} \to \mathfrak{A}$ a relatively von Neumann automorphism with fixed-point subalgebra $\mathfrak{C}$ and a dyadic cycle system $\langle d_{mi}\rangle_{m\in\mathbb{N},i<2^m}$ such that $\{d_{mi} : m \in \mathbb{N},$

$i < 2^m\} \cup \mathfrak{C}$ τ-generates $\mathfrak{A}$. Show that for any $n \in \mathbb{N}$ the fixed-point subalgebra of π^{2^n} is the subalgebra of $\mathfrak{A}$ generated by $\{d_{ni} : i < 2^n\} \cup \mathfrak{C}$.

(d) Let $(\mathfrak{A}, \bar\mu)$ be a probability algebra, and $\pi : \mathfrak{A} \to \mathfrak{A}$ a measure-preserving automorphism. (i) Show that π is weakly von Neumann iff it has a factor (definition: 387Ac) which is a von Neumann automorphism. (ii) Show that if π is a relatively von Neumann automorphism then no non-trivial factor of π can be weakly mixing.

Other exercises have been moved: 388Xc-388Xe are now 388Xe-388Xg.

388Yd is now 388Yh, 388Yf is now 388Yd, 388Yg is now 388Yf.

p 363 l 37 Add new result:

391L Proposition (a) If $\mathfrak{A}$ is a measurable algebra, all its principal ideals and σ-subalgebras are, in themselves, measurable algebras.

(b) The simple product of countably many measurable algebras is a measurable algebra.

(c) If $\mathfrak{A}$ is a measurable algebra, $\mathfrak{B}$ is a Boolean algebra and $\pi : \mathfrak{A} \to \mathfrak{B}$ is a surjective order-continuous Boolean homomorphism, then $\mathfrak{B}$ is a measurable algebra.

p 382 l 27 §394 has been rewritten to incorporate the generalization of Talagrand's example by Ž.Perović and B.Veličković. 394A now gives the definition of 'PV norm' on $[\mathbb{N}]^{<\omega}$; 394C-394D have moved to 394B-394C; the former 394A-394B are now 394D.

p 386 l 1 There is a blunder in the proof of Lemma 394G. In part (c) of the proof, the device suggested for ensuring that $r \notin W$ doesn't work. This has to be done by modifying Lemma 394B to

Suppose that $r \in \mathbb{N}$ and $\langle K_i\rangle_{i<s}$ is a family of finite subsets of $\mathbb{N}$, all of size at least $m \ge 2$, such that $\max K_i < \min K_{i+1}$ for $i \le s-2$. Let $\mathcal{J}$ be a finite subset of $([\mathbb{N}]^{<\omega} \setminus \{\emptyset\}) \times [0,\infty[$, and set $\gamma = \sum_{(I,w)\in\mathcal{J}} w$. Then we can find $\langle u_i\rangle_{i<s}$ and $\langle v_i\rangle_{i<s}$ such that u_i, v_i are successive members of K_i for each $i < s$ and, setting $W = \bigcup_{i<s} v_i \setminus u_i$,

$$\sum_{\substack{(J,w)\in\mathcal{J}\\ \#(J\cap W)\ge\frac{1}{2}\#(J)}} w \le \frac{2\gamma}{m-1}$$

and $r \notin W$.

A small refinement of the proof can achieve this, and the result can now be applied directly in 394G with a trifling simplification there.

p 415 l 5 (3A3B) Add new part:

(h) Any subspace of a Hausdorff space is Hausdorff.

p 419 l 22 (3A4C, uniform continuity) Add new parts:

(e) Two metrics ρ, σ on a set X are **uniformly equivalent** if they give rise to the same uniformity

(f) If U and V are linear topological spaces, and $T : U \to V$ is a continuous linear operator, then T is uniformly continuous for the uniformities associated with the topologies of U and V.