Errata and addenda for Volume 3, 2004 printing

I collect here known errors and omissions, with their discoverers, in my book *Measure Theory* (`http://www1.essex.ac.uk/maths/people/fremlin/mt3.2004`).

p 17 l 21 (311G) Add new part:

(d) Note that a set $C \subseteq \mathfrak{A}$ is a partition of unity iff $C \cup \{0\}$ is a maximal disjoint set. If $A \subseteq \mathfrak{A}$ is any disjoint set, there is a partition of unity including A.

311Gd is now 311Ge.

p 17 l 22 (311Gd, now 311Ge): for '$cd = d$' read '$cd = c$'.

p 21 l 15 (Exercise 311Xf): for

$$`a \cup (b \cap c) = (a \cap b) \cup (a \cap c), \quad a \cup (b \cap c) = (a \cap b) \cup (a \cap c)'$$

read

$$`a \cup (b \cap c) = (a \cup b) \cap (a \cup c), \quad a \cap (b \cup c) = (a \cap b) \cup (a \cap c)'.$$

(S.Bianchini, K.Yates.)

p 20 l 34 (311Y) Add new exercises:

(c) Let $(\mathfrak{A}, \vee, ')$ be such that (i) $(\mathfrak{A}, \vee)$ is a non-empty commutative semigroup (ii) $' : \mathfrak{A} \to \mathfrak{A}$ is a function (iii) $((a \vee b)' \vee (a \vee b')')' = a$ for all a, $b \in \mathfrak{A}$. Show that there is a Boolean algebra structure on $\mathfrak{A}$ for which $\vee = \cup$ and $'$ is complementation.

(d) Let P be a distributive lattice, and Z the set of surjective lattice homomorphisms from P to $\{0, 1\}$. Show that there is a sublattice of $\mathcal{P}Z$ isomorphic to P.

p 23 l 25 (proof of 312E): for '$b \cap a = a$' read '$b \cap a = b$'. (J.Miller.)

p 27 l 8 (part (b) of the proof of 312N, now 312O): for '$\pi_1((a' \cap v) \cup (b' \setminus v))$' read '$\pi_1((a' \cap c) \cup (b' \setminus c))$'.

p 29 l 23 312T has been moved to 312K; consequently 312K-312S are now 312L-312T.

p 30 l 18 (312X) Add new exercises:

(l) Let $\mathfrak{A}$ be a Boolean algebra, and $A \subseteq \mathfrak{A}$ a set, closed under $\cup$ and $\cap$, such that $0, 1 \in A$. Let B be the set of elements of $\mathfrak{A}$ expressible as $a \setminus a'$ where a, $a' \in A$, and C the set of elements of $\mathfrak{A}$ expressible as $b_0 \cup \ldots \cup b_n$ where $b_0, \ldots, b_n \in B$ are disjoint. Show that C is a subalgebra of $\mathfrak{A}$.

(m) Let $\mathfrak{A}$, $\mathfrak{B}$ be Boolean algebras, and $A \subseteq \mathfrak{A}$ a set, closed under $\cup$ and $\cap$, such that $0_{\mathfrak{A}}$, $1_{\mathfrak{A}} \in A$; let $\mathfrak{C}$ be the subalgebra of $\mathfrak{A}$ generated by A. Let $\pi : A \to \mathfrak{B}$ be such that $\pi 0_{\mathfrak{A}} = 0_{\mathfrak{B}}$ and $\pi 1_{\mathfrak{A}} = 1_{\mathfrak{B}}$, and $\pi(a \cup a') = \pi a \cup \pi a'$, $\pi(a \cap a') = \pi a \cap \pi a'$ for all a, $a' \in \mathfrak{A}$. Show that π has a unique extension to a Boolean homomorphism from $\mathfrak{C}$ to $\mathfrak{B}$.

p 36 l 41 Add new part to Proposition 313L:

(d) If π is bijective, it is order-continuous.

p 39 l 6 (proof of 313R): for '$\bigcup_{a \in \mathfrak{A}} \widehat{a}$' read '$\bigcup_{a \in A} \widehat{a}$'. (J.M.)

p 40 l 25 (313X) Add new exercise:

(r) Let $\mathfrak{A}$ and $\mathfrak{B}$ be Boolean algebras, $\pi : \mathfrak{A} \to \mathfrak{B}$ an injective Boolean homomorphism and $\mathfrak{C}$ a Boolean subalgebra of $\mathfrak{A}$. Suppose that $a \in \mathfrak{A}$ is such that $c = \mathrm{upr}(a, \mathfrak{C})$ is defined. Show that $\pi c = \mathrm{upr}(\pi a, \pi[\mathfrak{C}])$.

(s) Let $\mathfrak{A}$ and $\mathfrak{B}$ be Boolean algebras, $\pi : \mathfrak{A} \to \mathfrak{B}$ a Boolean homomorphism and D an order-dense subset of $\mathfrak{A}$ containing 0. Show that π is injective iff $\pi\restriction D$ is injective.

(t) Let $\mathfrak{A}$ be a Boolean algebra and $A_0, \dots , A_n$ subsets of $\mathfrak{A}$ such that $\sup A_i$ is defined for each $i \le n$. Set $B = \{a_0 \cap \dots \cap a_n : a_i \in A_i \text{ for each } i\}$. Show that $\sup B$ is defined and equal to $\inf_{i<n} \sup A_i$.

313Xo-313Xp are now 313Xp-313Xq, 313Xq is now 313Xo.

p 41 l 4 (313Y) Add new exercises:

(d) Let P and Q be lattices, and $f : P \to Q$ a bijective lattice homomorphism. Show that f is order-continuous.

(h) Let $\mathfrak{A}$ be a Boolean algebra and $\mathfrak{C}$ a Boolean subalgebra. Let $\mathcal{I}$ be the set of those $a \in \mathfrak{A}$ such that the upper envelope $\operatorname{upr}(a, \mathfrak{C})$ is zero. (i) Show that $\mathcal{I}$ is an ideal in $\mathfrak{A}$. (ii) Show that $\mathfrak{C}$ is regularly embedded in $\mathfrak{A}$ iff $\mathcal{I} = \{0\}$. (iii) Let $\pi : \mathfrak{A} \to \mathfrak{A}/\mathcal{I}$ be the canonical map. Show that $\pi\restriction\mathfrak{C}$ is injective and order-continuous.

313Yb is now 313Ye, 313Yc-313Yd are now 313Yb-313Yc, 313Ye-313Yf are now 313Yf-313Yg.

p 43 l 41 (part (a-ii) of the proof of 314F): for '$\pi(a \cap a_0) = \pi a \cap \pi a_0 = \pi a$' read '$\pi(a \cap a_0) = \pi a \cap \pi a_0 = \pi a_0$'.

p 44 l 1 314Gb is now 314H, 314H is now 314G, and 314Ga is absorbed into 314Ha.

p 44 l 17 314Ia is now 314Ib and 314Ib is now 314Ia.

p 45 l 25 (part (b) of the proof of 314K): for '$a' \in \mathfrak{A}$' read '$a' \in \mathfrak{A}_1$'.

p 47 l 33 (part (e) of the proof of 314P): for '$G = \operatorname{int}\overline{\bigcap\mathcal{H}}$' read '$G = \operatorname{int}\bigcap\mathcal{H}$'.

p 48 l 24 (Lemma 314R) The former exercise 314Xi has been brought into the main text in the following form:

(b) Let X be a topological space.

(i) If $U \subseteq X$ is open, then $G \mapsto G \cap U$ is a surjective order-continuous Boolean homomorphism from $\mathrm{RO}(X)$ onto $\mathrm{RO}(U)$.

(ii) If $U \in \mathrm{RO}(X)$ then $\mathrm{RO}(U)$ is the principal ideal of $\mathrm{RO}(X)$ generated by U.

p 50 l 22 Upper envelopes (314V) have been moved to 313S.

p 50 l 21 (Dedekind completions.) The former exercise 314Xf has been brought into the main text as follows.

314Ub If $\mathfrak{C}$ is a Dedekind complete Boolean algebra and $\mathfrak{A}$ is an order-dense subalgebra of $\mathfrak{C}$, then the embedding $\mathfrak{A} \subseteq_{\rightarrow} \mathfrak{C}$ induces an isomorphism from $\widehat{\mathfrak{A}}$ to $\mathfrak{C}$.

p 51 l 10 Following the transfers of 314Xf and 314Xi, the exercises 314X have been rearranged: 314Xc is now 314Xg, 314Xd is now 314Xc, 314Xe is now 314Xi, 314Xg is now 314Xe, 314Xh is now 314Xf, 314Xj is now 314Xh.

p 51 l 22 (314X) Add new exercises:

(d) Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra, $\mathfrak{B}$ a Boolean algebra and $\pi : \mathfrak{A} \to \mathfrak{B}$ an surjective order-continuous Boolean homomorphism. (i) Show that the kernel of π is a principal ideal in $\mathfrak{A}$. (ii) Show that $\mathfrak{B}$ is isomorphic to the complementary principal ideal in $\mathfrak{A}$, and in particular is Dedekind complete.

(j) Let $\mathfrak{B}$ be a Dedekind complete Boolean algebra, and $\mathfrak{A}$ a Boolean algebra which can be regularly embedded in $\mathfrak{B}$. Show that the Dedekind completion of $\mathfrak{A}$ can be regularly embedded in $\mathfrak{B}$.

(k) Let X be a topological space and Y a dense subset of X. Show that $G \mapsto G \cap Y$ is a Boolean isomorphism from $\mathrm{RO}(X)$ to $\mathrm{RO}(Y)$.

(l) Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra, $\mathfrak{B}$ an order-closed subalgebra of $\mathfrak{A}$, c a member of $\mathfrak{A}$ and $\mathfrak{C}$ the subalgebra of $\mathfrak{A}$ generated by $\mathfrak{B} \cup \{c\}$. Show that if $a \in \mathfrak{C}$, then $c \cap a = c \cap \operatorname{upr}(c \cap a, \mathfrak{B})$.

p 54 l 25 Add new result:

315H Proposition Let X be a topological space, and $\mathcal{U}$ a disjoint family of open subsets of X with union dense in X. Then the regular open algebra $\mathrm{RO}(X)$ is isomorphic to the simple product $\prod_{U\in\mathcal{U}}\mathrm{RO}(U)$.

315H-315P are now 315I-315Q.

p 56 l 17 (proof of 315K, now 315L): for '$\varepsilon_i' : \mathfrak{A} \to \mathfrak{B}_k$' read '$\varepsilon_i' : \mathfrak{A}_i \to \mathfrak{B}_k$'. (J.M.)

p 58 l 19 I have added two paragraphs on projective and inductive limits of Boolean algebras, and another on simple products of regular open algebras, as follows:

315R Proposition Let $\langle\mathfrak{A}_i\rangle_{i\in I}$ be a family of Boolean algebras, and R a subset of $I\times I$; suppose that $\pi_{ji} : \mathfrak{A}_i \to \mathfrak{A}_j$ is a Boolean homomorphism for each $(i,j)\in R$.

(a) There are a Boolean algebra $\mathfrak{C}$ and a family $\langle\pi_i\rangle_{i\in I}$ such that

$\pi_i : \mathfrak{C} \to \mathfrak{A}_i$ is a Boolean homomorphism for each $i \in I$,

$\pi_j = \pi_{ji}\pi_i$ whenever $(i,j)\in R$,

and whenever $\mathfrak{B}$, $\langle\phi_i\rangle_{i\in I}$ are such that

$\mathfrak{B}$ is a Boolean algebra,

$\phi_i : \mathfrak{B} \to \mathfrak{A}_i$ is a Boolean homomorphism for each $i \in I$,

$\phi_j = \pi_{ji}\phi_i$ whenever $(i,j)\in R$,

then there is a unique Boolean homomorphism $\phi : \mathfrak{B} \to \mathfrak{C}$ such that $\pi_i\phi = \phi_i$ for every $i \in I$.

(b) There are a Boolean algebra $\mathfrak{C}$ and a family $\langle\pi_i\rangle_{i\in I}$ such that

$\pi_i : \mathfrak{A}_i \to \mathfrak{C}$ is a Boolean homomorphism for each $i \in I$,

$\pi_i = \pi_j\pi_{ji}$ whenever $(i,j)\in R$,

and whenever $\mathfrak{B}$, $\langle\phi_i\rangle_{i\in I}$ are such that

$\mathfrak{B}$ is a Boolean algebra,

$\phi_i : \mathfrak{A}_i \to \mathfrak{B}$ is a Boolean homomorphism for each $i \in I$,

$\phi_i = \phi_j\pi_{ji}$ whenever $(i,j)\in R$,

***315S Definitions** In 315Ra, we call $\mathfrak{A}$, together with $\langle\pi_i\rangle_{i\in I}$, 'the' **projective limit** of $(\langle\mathfrak{A}_i\rangle_{i\in I}, \langle\pi_{ji}\rangle_{(i,j)\in R})$; in 315Rb, we call $\mathfrak{A}$, together with $\langle\pi_i\rangle_{i\in I}$, 'the' **inductive limit** of $(\langle\mathfrak{A}_i\rangle_{i\in I}, \langle\pi_{ji}\rangle_{(i,j)\in R})$.

p 58 l 25 (315X) Add new exercises:

(b) Let $\langle\mathfrak{A}_i\rangle_{i\in I}$ be a family of Boolean algebras with simple product $\mathfrak{A} = \prod_{i\in I}\mathfrak{A}_i$. (i) Show that $\mathfrak{A}$ is Dedekind complete iff every $\mathfrak{A}_i$ is Dedekind complete. (ii) Show that $\mathfrak{A}$ is Dedekind σ-complete iff every $\mathfrak{A}_i$ is Dedekind σ-complete.

(c) Let $\langle\mathfrak{A}_i\rangle_{i\in I}$ be a family of Boolean algebras with simple product $\mathfrak{A} = \prod_{i\in I}\mathfrak{A}_i$. Suppose that for every $i \in I$ we are given a subalgebra $\mathfrak{B}_i$ of $\mathfrak{A}_i$. (i) Show that the simple product $\mathfrak{B} = \prod_{i\in I}\mathfrak{B}_i$ is a subalgebra of $\mathfrak{A}$. (ii) Show that $\mathfrak{B}$ is order-closed in $\mathfrak{A}$ iff $\mathfrak{B}_i$ is order-closed in $\mathfrak{A}_i$ for every $i \in I$.

315Xb-315Xq are now 315Xd-315Xs.

p 58 l 38 Part (ii) of Exercise 315Xe (now 315Xg) is now covered by 315H.

p 60 l 2 (315Y) Add new exercise:

(h) Let $\langle\mathfrak{A}_i\rangle_{i\in I}$ be a family of Boolean algebras, and R a subset of $I\times I$; suppose that $\pi_{ji} : \mathfrak{A}_i \to \mathfrak{A}_j$ is a Boolean homomorphism for each $(i,j)\in R$. For each $i \in I$, let Z_i be the Stone space of $\mathfrak{A}_i$; for $(i,j)\in R$, let $f_{ji} : Z_j \to Z_i$ be the continuous function corresponding to π_{ji} (312Q). Show that the Stone space of the inductive limit of the system $(\langle\mathfrak{A}_i\rangle_{i\in I}, \langle\pi_{ji}\rangle_{(i,j)\in R})$ can be identified with $\{z : z \in \prod_{i\in I} Z_i,\ f_{ji}(z(j)) = z(i) \text{ whenever } (i,j)\in R\}$.

p 62 l 32 Definition 316G has been rewritten, as follows:

Definition Let $\mathfrak{A}$ be a Boolean algebra. I will say that $\mathfrak{A}$ is **weakly** (σ,∞)**-distributive** if whenever $\langle A_n\rangle_{n\in\mathbb{N}}$ is a sequence of downwards-directed subsets of $\mathfrak{A}$ and $\inf A_n = 0$ for every n, then $\inf B = 0$, where

$$B = \{b : b \in \mathfrak{A},\text{ for every } n \in \mathbb{N} \text{ there is an } a \in A_n \text{ such that } b \supseteq a\}.$$

316H has been correspondingly changed to

Proposition Let $\mathfrak{A}$ be a Boolean algebra. Then the following are equiveridical:

(i) $\mathfrak{A}$ is weakly (σ,∞)-distributive;

(ii) whenever $\langle A_n\rangle_{n\in\mathbb{N}}$ is a sequence of partitions of unity in $\mathfrak{A}$, there is a partition of unity B in $\mathfrak{A}$ such that $\{a : a\in A_n,\ a\cap b\ne 0\}$ is finite for every $n\in\mathbb{N}$ and $b\in B$;

(iii) whenever $\langle A_n\rangle_{n\in\mathbb{N}}$ is a sequence of upwards-directed subsets of $\mathfrak{A}$, each with a supremum $c_n=\sup A_n$, and

$$B=\{b : b\in\mathfrak{A},\text{ for every }n\in\mathbb{N}\text{ there is an }a\in A_n\text{ such that }b\subseteq a\},$$

then $\inf\{c_n\setminus b : n\in\mathbb{N},\ b\in B\}=0$;

(iv) whenever $\langle A_n\rangle_{n\in\mathbb{N}}$ is a sequence of upwards-directed subsets of $\mathfrak{A}$, each with a supremum $c_n=\sup A_n$, and $\inf_{n\in\mathbb{N}}c_n=c$ is defined, then $c=\sup B$, where

$$B=\{b : b\in\mathfrak{A},\text{ for every }n\in\mathbb{N}\text{ there is an }a\in A_n\text{ such that }b\subseteq a\}.$$

Consequently 316K, 316L and 316M are now 316J-316L.

p 64 l 39 I have added four paragraphs on homogeneous Boolean algebras, with a note onas follows:

316N Definition (formerly 331M) A Boolean algebra $\mathfrak{A}$ is **homogeneous** if every non-trivial principal ideal of $\mathfrak{A}$ is isomorphic to $\mathfrak{A}$.

316O Lemma Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra such that

$$D=\{d : d\in\mathfrak{A},\ \mathfrak{A}\text{ is isomorphic to the principal ideal }\mathfrak{A}_d\}$$

is order-dense in $\mathfrak{A}$. Then $\mathfrak{A}$ is homogeneous.

***316P Proposition** Let $\mathfrak{A}$ be a homogeneous Boolean algebra. Then its Dedekind completion $\widehat{\mathfrak{A}}$ is homogeneous.

***316Q Proposition** The free product of any family of homogeneous Boolean algebras is homogeneous.

***316R Proposition** Let $\mathfrak{A}$ be a Boolean algebra, and $\mathfrak{B}$ a subalgebra of $\mathfrak{A}$ which is regularly embedded in $\mathfrak{A}$.

(a) Every atom of $\mathfrak{A}$ is included in an atom of $\mathfrak{B}$.

(b) If $\mathfrak{B}$ is atomless, so is $\mathfrak{A}$.

p 64 l 40 The exercises to §316 have been thoroughly re-arranged, with additions, as follows:

316Xg Show that a homogeneous Boolean algebra is either atomless or $\{0,1\}$.

(l) Let $\mathfrak{A}$ and $\mathfrak{B}$ be Boolean algebras, neither $\{0\}$, and $\mathfrak{A}\otimes\mathfrak{B}$ their free product. (i) Show that if $\mathfrak{A}\otimes\mathfrak{B}$ is ccc, then $\mathfrak{A}$ and $\mathfrak{B}$ are both ccc. (ii) Show that if $\mathfrak{A}\otimes\mathfrak{B}$ is weakly (σ,∞)-distributive, then $\mathfrak{A}$ and $\mathfrak{B}$ are both weakly (σ,∞)-distributive. (iii) Show that $\mathfrak{A}\otimes\mathfrak{B}$ is atomless iff either $\mathfrak{A}$ or $\mathfrak{B}$ is atomless. (iv) Show that $\mathfrak{A}\otimes\mathfrak{B}$ is purely atomic iff $\mathfrak{A}$ and $\mathfrak{B}$ are both purely atomic.

(m) Let $\mathfrak{A}$ be a Boolean algebra and $\mathfrak{A}_a$ a principal ideal of $\mathfrak{A}$. Show that if $\mathfrak{A}$ is homogeneous, then $\mathfrak{A}_a$ is homogeneous.

(r) Show that the algebra of open-and-closed subsets of $\{0,1\}^{\mathbb{N}}$, with its usual topology, is homogeneous.

(s) Show that the regular open algebra of $\mathbb{R}$ is homogeneous.

316Yf Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra with the Egorov property and I a σ-ideal of $\mathfrak{A}$. Show that $\mathfrak{A}/I$ has the Egorov property.

(p) Let $\langle\mathfrak{A}_i\rangle_{i\in I}$ be a family of Boolean algebras, none of them $\{0\}$, and $\mathfrak{A}$ their simple product. Show that $\mathfrak{A}$ is homogeneous iff (i) $\mathfrak{A}_i$ is isomorphic to $\mathfrak{A}_j$ for all i, $j\in I$ (ii) for every $i\in I$ there is a partition of unity $A\subseteq\mathfrak{A}_i\setminus\{0\}$ with $\#(A)=\#(I)$.

(q) Let $\mathfrak{A}$ be a Boolean algebra such that $\{d : d\in\mathfrak{A},\ \mathfrak{A}_d\cong\mathfrak{A}\}$ is order-dense in $\mathfrak{A}$. Show that the Dedekind completion $\widehat{\mathfrak{A}}$ is homogeneous.

(r) Write $[\mathbb{N}]^{<\omega}$ for the ideal of $\mathcal{P}\mathbb{N}$ consisting of the finite subsets of $\mathbb{N}$. Show that the Dedekind completion of $\mathcal{P}\mathbb{N}/[\mathbb{N}]^{<\omega}$ is homogeneous.

(s) Show that the regular open algebra of $\{0,1\}^I$ is homogeneous for any infinite set I.

(t) Suppose that $\mathfrak{A}$ is a weakly (σ,∞)-distributive Boolean algebra, and that $\langle A_n\rangle_{n\in\mathbb{N}}$ is a sequence of upwards-directed subsets of $\mathfrak{A}$. Set

$$B=\bigcap_{n\in\mathbb{N}}\{b:b\in\mathfrak{A},\text{ there is an }a\in A_n\text{ such that }b\cap a'\subseteq b\cap a\text{ for every }a'\in A_n\}.$$

Show that B is upwards-directed and $\sup B=1$.

316Xa is now 316Xh, 316Xb is now part of 316Xm, 316Xc is now part of 316Xn, 316Xd is now 316Xo, 316Xe is now split between 316Xp and 316Xs, 316Xf is now part of 316Xj, 316Xg is now 316Xa, 316Xh is now 316Xc, 316Xi is now 316Xb, 316Xj is now 316Xd, 316Xk is now part of 316Xm, 316Xl is now part of 316Xn, 316Xm is now part of 316Xi, 316Xn is now part of 316Xk, 316Xo is now 316Xj, 316Xp is now part of 316Xr, 316Xq is now part of 316Xo, 316Xr is now part of 316Xi, 3[]16Xs is now 316Xf, 316Xt is now part of 316Xk, 316Xu is now part of 316Xm, 316Xv is now part of 316Xn, 316Xw is now 316Xe, 316Xx has been dropped.

316Yb is now 316Yj, 316Yc-316Ye are now 316Yb-316Yd, 316Yf is now 316Yg, 316Yg is now 316Ye, 316Yj-316Yl are now 316Yk-316Ym, 316Ym is now 316M, 316Yn has been dropped, 316Yo is now 316Yr, 316Yp is now 316Yn.

p 76 l 39 Add new paragraph:

322K Indefinite-integral measures: Proposition Let (X,Σ,μ) be a measure space and ν an indefinite-integral measure over μ. Then the measure algebra of ν can be identified, as Boolean algebra, with a principal ideal of the measure algebra of μ.

322K-322Q have been moved to 322L-322R.

p 77 l 10 (322K, now 322L) Add new part:

(d) Let $(\mathfrak{A},\bar\mu)$ be a measure algebra, and $\langle e_i\rangle_{i\in I}$ a countable partition of unity in $\mathfrak{A}$. Then $(\mathfrak{A},\bar\mu)$ is isomorphic to the product $\prod_{i\in I}(\mathfrak{A}_{e_i},\bar\mu\restriction\mathfrak{A}_{e_i})$ of the corresponding principal ideals.

322Kd is now 322Le.

p 80 l 1 (proof of 322O, now 322P): for '$\{a:a\in\mathfrak{A},\,a\subseteq c\}$' read '$A=\{a:a\in\mathfrak{A},\,a\subseteq c\}$'.

p 80 l 44 (Exercise 322Xa) The definition of I_∞ must be amended to include the zero element; e.g., 'let I_∞ be the set of those $a\in\mathfrak{A}$ which are either 0 or purely infinite, that is, $\bar\mu b=\infty$ for every non-zero $b\subseteq a$'.

p 81 l 14 Exercise 322Xf has been moved to 322K. Consequently 322Xg-322Xi are now 322Xf-322Xh.

p 81 l 22 (Exercise 322Xh): for 'let $\mathfrak{B}$ be an order-closed subalgebra of $\mathfrak{A}$' read 'let $\mathfrak{B}$ be a regularly embedded σ-subalgebra of $\mathfrak{A}$'.

p 83 l 11 Parts (d) and (e) of 323A are now

(d) On the ideal $\mathfrak{A}^f$ we have an actual metric ρ defined by saying that $\rho(a,b)=\bar\mu(a\,\triangle\,b)$ for a, $b\in\mathfrak{A}^f$; the is the **measure metric** or **Fréchet-Nikodým metric**. I will call the topology it generates the **strong measure-algebra topology** on $\mathfrak{A}^f$.

When $\bar\mu$ is totally finite, that is, $\mathfrak{A}^f=\mathfrak{A}$, $\rho=\rho_1$ defines the measure-algebra topology and uniformity of $\mathfrak{A}$.

p 83 l 17 (323B) Add new part:

(b) $\mathfrak{A}^f$ is dense in $\mathfrak{A}$.

p 83 l 35 (part (c) of the proof of 323C): for '$|\bar\mu(a\,\triangle\,b)-\bar\mu(a\,\triangle\,c)|\le\rho_a(b,c)$' read '$|\bar\mu(a\cap b)-\bar\mu(a\cap c)|\le\rho_a(b,c)$'. (S.B.)

p 88 l 14 (proof of 323L): for '$\tau_j(b(j),c(j))$' read '$\tau_j(b,c)$' (twice).

p 88 l 17 Add new result:

323M Proposition Let $(\mathfrak{A},\bar\mu)$ be a measure algebra, and give $\mathfrak{A}^f$ its measure metric.

(a) The Boolean operations $\triangle$, $\cap$, $\cup$ and $\setminus$ on $\mathfrak{A}^f$ are uniformly continuous.

(b) $\bar\mu\restriction\mathfrak{A}^f:\mathfrak{A}^f\to[0,\infty[$ is 1-Lipschitz, therefore uniformly continuous.

(c) $\mathfrak{A}^f$ is complete.

p 88 l 18 The exercises 323X have been rearranged; 323Xa-323Xe are now 323Xc-323Xg, 323Xf is now 323Xa, 323Xg is now 323Xb.

A new exercise has been added:

(h) Let $(\mathfrak{A},\bar{\mu})$ be a measure algebra. Show that its localization (322P) can be identified with its completion under its measure-algebra uniformity.

p 94 l 38 (Exercise 324Xb) for '$\mathfrak{B}$ a σ-subalgebra of $\mathfrak{A}$' read '$\mathfrak{B}$ an order-closed subalgebra of $\mathfrak{A}$'.

p 97 l 35 (part (a) of the proof of 325C): for '$c \in \mathfrak{A} \otimes \mathfrak{B}$' read '$c \in \mathfrak{A}_1 \otimes \mathfrak{A}_2$'. (S.B.)

p 100 l 1 (part (b) of the proof of 325F): for '$a \cap a_n = 0$' read '$a \cap (a_n \otimes a_n) = 0$'. (S.B.)

p 101 l 34 (part (b) of the proof of 325I) for '$c \in \mathfrak{C}$' read '$c \in \bigotimes_{i \in I} \mathfrak{A}_i$'.

p 102 l 37 Part (b) of Theorem 325M has been elaborated, as follows:

(b)(i) For any $c \in \mathfrak{C}$, there is a unique smallest $J_c \subseteq I$ such that $c \in \mathfrak{C}_{J_c}$, and this J_c is countable.

(ii) If c, $d \in \mathfrak{C}$ and $c \subseteq d$, then there is an $e \in \mathfrak{C}_{J_c \cap J_d}$ such that $c \subseteq e \subseteq d$.

p 102 l 44 (part (a) of the proof of 325M) for '$\langle \psi\phi_i \rangle_{i \in I}$' read '$\langle \psi\phi_i \rangle_{i \in J}$'.

p 103 l 23 (325X) Add new exercise:

(c) Let $(\mathfrak{A},\bar{\mu})$ and $(\mathfrak{B},\bar{\nu})$ be semi-finite measure algebras with localizations $(\widehat{\mathfrak{A}},\tilde{\mu})$ and $(\widehat{\mathfrak{B}},\tilde{\nu})$. Show that the localizable measure algebra free products $(\mathfrak{A},\bar{\mu})\widehat{\otimes}_{\mathrm{loc}}(\mathfrak{B},\bar{\nu})$ and $(\widehat{\mathfrak{A}},\tilde{\mu})\widehat{\otimes}_{\mathrm{loc}}(\widehat{\mathfrak{B}},\tilde{\nu})$ are isomorphic.

Exercises 325Xc and 325Xe-325Xg are now 325Xd and 325Xf-325Xh. Exercises 325Xd and 325Xh have been re-written, and are now

(e) Let $\langle(\mathfrak{A}_i,\bar{\mu}_i)\rangle_{i \in I}$ and $\langle(\mathfrak{A}'_i,\bar{\mu}'_i)\rangle_{i \in I}$ be two families of probability algebras, and $(\mathfrak{C},\bar{\lambda},\langle\psi_i\rangle_{i \in I})$, $(\mathfrak{C}',\bar{\lambda}',\langle\psi'_i\rangle_{i \in I})$ their probability algebra free products. Suppose that for each $i \in I$ we are given a measure-preserving Boolean homomorphism $\pi_i : \mathfrak{A}_i \to \mathfrak{A}'_i$. Show that there is a unique measure-preserving Boolean homomorphism $\pi : \mathfrak{C} \to \mathfrak{C}'$ such that $\pi\psi_i = \psi'_i\pi_i$ for every $i \in I$.

(i) Let $\langle(\mathfrak{A}_i,\bar{\mu}_i)\rangle_{i \in I}$ be a family of probability algebras with probability algebra free product $(\mathfrak{C},\bar{\lambda},\langle\psi_i\rangle_{i \in I})$. For $J \subseteq I$ let $\mathfrak{C}_J$ be the closed subalgebra of $\mathfrak{C}$ generated by $\bigcup_{i \in J}\psi_i[\mathfrak{A}_i]$. Show that for any J, $K \subseteq I$ and $c \in \mathfrak{C}$, the upper envelope $\mathrm{upr}(c,\mathfrak{C}_{J\cap K})$ is equal to $\mathrm{upr}(\mathrm{upr}(c,\mathfrak{C}_J),\mathfrak{C}_K)$.

p 103 l 31 (325Xe, now 325Xf): for '$\inf_{i \in j} a_j$' read '$\inf_{i \in J} a_i$'.

p 104 l 1 (325Y) Add new exercise:

(b) Let $(\mathfrak{A}_1,\bar{\mu}_1)$, $(\mathfrak{A}_2,\bar{\mu}_2)$, $(\mathfrak{A}'_1,\bar{\mu}'_1)$ and $(\mathfrak{A}'_2,\bar{\mu}'_2)$ be semi-finite measure algebras with localizable measure algebra free products $(\mathfrak{C},\bar{\lambda},\psi_1,\psi_2)$ and $(\mathfrak{C}',\bar{\lambda}',\psi'_1,\psi'_2)$. Suppose that $\pi_1 : \mathfrak{A}_1 \to \mathfrak{A}'_1$ and $\pi_2 : \mathfrak{A}_2 \to \mathfrak{A}'_2$ are measure-preserving Boolean homomorphisms. Show that there is a measure-preserving Boolean homomorphism $\pi : \mathfrak{C} \to \mathfrak{C}'$ such that $\pi\psi_i = \psi'_i\pi_i$ for both i, but that π is not necessarily unique.

325Yb-325Yf are now 325Yc-325Yg.

p 104 l 2 (Exercise 325Yb, now 325Yc): we must add a hypothesis '$\mu a > 0$ for every $a \neq 0$'.

p 106 l 39 New results have been added:

326F Definition Let $\mathfrak{A}$ be a Boolean algebra, and ν a finitely additive functional on $\mathfrak{A}$. I will say that ν is **properly atomless** if for every $\epsilon > 0$ there is a finite partition $\langle a_i\rangle_{i \in I}$ of unity in $\mathfrak{A}$ such that $|\nu a| \le \epsilon$ whenever $i \in I$ and $a \subseteq a_i$.

326G Lemma Let $\mathfrak{A}$ be a Boolean algebra.

(a)(i) If ν, $\nu' : \mathfrak{A} \to \mathbb{R}$ are properly atomless finitely additive functionals and $\alpha \in \mathbb{R}$, then $\alpha\nu$ and $\nu + \nu'$ are properly atomless additive functionals.

(ii) If $\nu : \mathfrak{A} \to \mathbb{R}$ is a properly atomless finitely additive functional, then ν is bounded and ν can be expressed as the difference of two non-negative properly atomless additive functionals.

(b) Suppose that $\mathfrak{A}$ is Dedekind σ-complete and that $\langle\nu_i\rangle_{i \in I}$ is a family of non-negative additive functionals on $\mathfrak{A}$ such that for every $a \in \mathfrak{A}$ there are an $\alpha \in [\frac{1}{3},\frac{2}{3}]$ and an $a' \subseteq a$ such that

$\nu_i a' = \alpha\nu_i a$ for every $i \in I$. Then for any $a \in \mathfrak{A}$ there is a non-decreasing family $\langle a_t\rangle_{t\in[0,1]}$ in $\mathfrak{A}$ such that $a_0 = 0$, $a_1 = a$ and $\nu_i a_t = t\nu_i a$ for every $t \in [0, 1]$ and $i \in I$.

(c) Suppose that $\mathfrak{A}$ is Dedekind σ-complete and that $\nu_0, \dots , \nu_n : \mathfrak{A} \to [0, \infty[$ are properly atomless additive functionals such that $\nu_i a \le \nu_0 a$ for every $i \le n$ and $a \in \mathfrak{A}$. Then for any $a \in \mathfrak{A}$ there is a non-decreasing family $\langle a_t\rangle_{t\in[0,1]}$ in $\mathfrak{A}$ such that $a_0 = 0$, $a_1 = a$ and $\nu_i a_t = t\nu_i a$ for every $t \in [0, 1]$ and $i \le n$.

326H Liapounoff's convexity theorem Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra, and $r \ge 1$ an integer. Suppose that $\nu : \mathfrak{A} \to \mathbb{R}^r$ is additive in the sense that $\nu(a \cup b) = \nu a + \nu b$ whenever $a \cap b = 0$, and properly atomless in the sense that for every $\epsilon > 0$ there is a finite partition $\langle a_j\rangle_{j\in J}$ of unity in $\mathfrak{A}$ such that $\|\nu a\| \le \epsilon$ whenever $j \in J$ and $a \subseteq a_j$. Then $\{\nu a : a \in \mathfrak{A}\}$ is a convex set in $\mathbb{R}^r$.

326E-326P are now 326I-326T, 326Q is now 326E.

p 112 l 38 (326X) Add new exercises:

(b) Let $\mathfrak{A}$ be a Boolean algebra. (i) Show that a finitely additive functional ν is properly atomless iff there is a properly atomless additive functional ν' such that $|\nu a| \le \nu' a$ for every $a \in \mathfrak{A}$. (ii) Show that a non-negative finitely additive functional ν on $\mathfrak{A}$ is properly atomless iff whenever ν' is a non-zero finitely additive functional such that $0 \le \nu' a \le \nu a$ for every $a \in \mathfrak{A}$ there is an $a \in \mathfrak{A}$ such that $\nu' a$ and $\nu'(1 \setminus a)$ are both non-zero.

(c) (i) Suppose that $\mathfrak{A}$ is a Dedekind σ-complete Boolean algebra and $\nu : \mathfrak{A} \to \mathbb{R}$ is countably additive. Show that $\mathcal{I} = \{a : \nu b = 0 \text{ for every } b \subseteq a\}$ is an ideal of $\mathfrak{A}$. Show that the following are equiveridical: (α) ν is properly atomless; (β) whenever $\nu a \ne 0$ there is a $b \subseteq a$ such that $\nu b \notin \{0, \nu a\}$; ($\gamma$) the quotient algebra $\mathfrak{A}/\mathcal{I}$ is atomless. (ii) Find an atomless Dedekind complete Boolean algebra $\mathfrak{A}$ and a finitely additive $\nu : \mathfrak{A} \to [0, 1]$ such that $\nu a > 0$ for every non-zero $a \in \mathfrak{A}$ but ν is not properly atomless.

(i) Let $\mathfrak{A}$ be an atomless Boolean algebra. Show that every completely additive functional on $\mathfrak{A}$ is properly atomless.

Other exercises have been renamed: 326Xb-326Xf are now 326Xd-326Xh, 326Xg-326Xi are now 326Xj-326Xl.

p 113 l 26 Exercises 326Ya-326Ye (Liapounoff's convexity theorem) have been moved into the main text as 326F-326H. Note that I have decided to use the phrase 'properly atomless' for the property of 326Ya, rather than 'atomless'. New exercises are

(i) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra and $\nu : \mathfrak{A} \to \mathbb{R}$ a countably additive functional. Show that $\nu[\mathfrak{A}]$ is a compact subset of $\mathbb{R}$.

(j) Let $\mathfrak{G}$ be the regular open algebra of $\mathbb{R}$ (314P). Find a properly atomless finitely additive $\nu : \mathfrak{G} \to \mathbb{R}$ such that $\nu[\mathfrak{G}]$ is not closed.

(k) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra and $r \ge 1$ an integer. (i) Let $C \subseteq \mathbb{R}^r$ be a non-empty bounded convex set, and for $z \in \mathbb{R}^r$ set $H_z = \{x : x\,\textbf{.}\,z = \sup_{y\in C} y\,\textbf{.}\,z\}$. Suppose that $H_z \cap \overline{C} \subseteq C$ for every $z \in \mathbb{R}^r \setminus \{0\}$. Show that C is closed. (ii) Suppose that $\nu : \mathfrak{A} \to \mathbb{R}^r$ is countably additive in the sense that all its coordinates are countably additive functionals. Show that $\nu[\mathfrak{A}]$ is compact.

Other exercises have been rearranged: 326Yf is now 326Yh, 326Yg-326Yk are now 326Ya-326Ye, 326Yt-326Yu are now 326Yf-326Yg.

p 117 l 26 (proof of (c)(i)⇒(ii) in 327B): we are supposed only to be assuming that ν is continuous at 0. However, if you look at the proof of (a), you will see that only 'continuous at 0' is needed.

p 119 l 10 (proof of 327D): for '$\nu E = \nu E^\bullet$' read '$\nu E = \bar{\nu} E^\bullet$'. (J.M.)

p 121 l 16 (327X) Some exercises have been rearranged: 327Xa is now 327Xc, 327Xb-327Xc are now 327Xa-327Xb.

p 122 l 35 There is a new section §328, 'Reduced products and other constructions'.

p 127 l 26 (331I): for '$\{b_\eta : \eta \le \xi\}$' read '$\{b_\eta : \eta < \xi\}$'.

p 129 l 43 Add new results:

331O Proposition Let $(\mathfrak{A}, \bar{\mu})$ be a measure algebra with countable Maharam type. Then $\mathfrak{A}$ is separable in its measure-algebra topology.

331P Proposition Let $(\mathfrak{A}, \bar{\mu})$ be an atomless probability algebra of countable Maharam type. Then it is isomorphic to the measure algebras of the usual measure on $\{0,1\}^{\mathbb{N}}$ and of Lebesgue measure on $[0,1]$.

p 130 l 23 (331X) Add new exercises:

(l) Let (X, Σ, μ) be a measure space, and A a subset of X which has a measurable envelope. Show that the Maharam type of the subspace measure on A is less than or equal to the Maharam type of μ.

(m) Let $\mathfrak{A}$ be a Boolean algebra, and $\mathfrak{B}$ an order-dense subalgebra of $\mathfrak{A}$. Show that $\tau(\mathfrak{A}) \leq \tau(\mathfrak{B})$.

(n) Let (X, Σ, μ) be a semi-finite measure space, and $\tilde{\mu}$ the c.l.d. version of μ. Show that the Maharam type of $\tilde{\mu}$ is at most the Maharam type of μ.

(o) Let X be a set and $\langle \mu_i \rangle_{i \in I}$ a non-empty countable family of σ-finite measures on X all with the same domain; let μ be the sum measure $\sum_{i \in I} \mu_i$. Writing $\tau(\mu)$, $\tau(\mu_i)$ for the Maharam types of the measures, show that $\sup_{i \in I} \tau(\mu_i) \leq \tau(\mu) \leq \max(\omega, \sup_{i \in I} \tau(\mu_i))$.

p 130 l 40 (331Ye) For 'localizable' read 'semi-finite'.

p 131 l 6 (331Y) Exercises 331Yg-331Yi have been deleted; 331Yj is now 331Yg. Add new exercises:

(h) Let κ be an infinite cardinal, ν_κ the usual measure on $\{0,1\}^\kappa$ and $(\mathfrak{B}_\kappa, \bar{\nu}_\kappa)$ its measure algebra. Suppose that $(\mathfrak{A}, \bar{\mu})$ is a totally finite measure algebra and such that $\tau(\mathfrak{A}) < \kappa$, and $\pi : \mathfrak{B}_\kappa \to \mathfrak{A}$ a Boolean homomorphism. Show that (i) for every $\epsilon > 0$ there is a $b \in \mathfrak{B}_\kappa$ such that $\bar{\nu}_\kappa b \geq 1 - \epsilon$ and $\bar{\mu}(\pi b) \leq \epsilon$ (ii) π is not injective.

(i) Give an example of a semi-finite measure space (X, Σ, μ) such that the Maharam type of μ is greater than the Maharam type of its c.l.d. version.

(j) Let $(\mathfrak{A}, \bar{\mu})$ be a semi-finite measure algebra which is separable when given its measure-algebra topology. Show that it has countable Maharam type.

(k) Let $\langle (\mathfrak{A}_i, \bar{\mu}_i) \rangle_{i \in I}$ be a non-empty family of homogeneous probability algebras, and $\mathcal{F}$ an ultrafilter on I. Show that the probability algebra reduced product $\prod_{i \in I} (\mathfrak{A}_i, \bar{\mu}_i) | \mathcal{F}$ is homogeneous.

p 136 l 47 (proof of 332O): for '$\sup_{\kappa' \geq \kappa} f_\kappa = f_v^*$' read '$\sup_{\kappa' \geq \kappa} f_\kappa \subseteq f_v^*$'.

p 138 l 1 (part (b-γ) of the proof of 332P): for '$\bar{\mu}(e_{\kappa_0} \setminus c_\pi)$' read '$\bar{\mu}(e_{\kappa_0} \setminus c_{\tilde{\pi}})$'.

p 144 l 10 (part (b-i-α) of the proof of 333C): for '$\mathfrak{D}'_\xi = \phi_\xi[\mathfrak{C}_\xi]$' read '$\mathfrak{D}'_\xi = \phi_\xi[\mathfrak{C}'_\xi]$'.

p 144 l 16 (part (b-i-β) of the proof of 333C): for 'the closed subalgebra of $\mathfrak{B}$ generated by $\mathfrak{D} \cup \{b_\eta : \eta \leq \xi\} \cup \{b'_\eta : \eta < \xi\}$' read 'the closed subalgebra of $\mathfrak{B}$ generated by $\mathfrak{D} \cup \{b_\eta : \eta \leq \xi\} \cup \{b'_\eta : \eta \leq \xi\}$'.

p 146 l 26 (statement of part (a) of Corollary 333G): we must assume that $\mathfrak{C} \neq \{0\}$.

p l (part (c) of the proof of 333K): in the formula $\{\kappa : \kappa \geq \omega, \exists\, a \in A, \tau_{\mathfrak{C}_a}(\mathfrak{A}_a) = \kappa\}$, I should have said that A was the set of those $a \in \mathfrak{A}$ which are relatively Maharam-type-homogeneous over $\mathfrak{C}$.

p 154 l 19 (proof of 333Q): the proof assumes that $(\mathfrak{C}, \bar{\mu}_0)$ is a probability algebra. Of course the general result announced follows immediately from the special case.

p 154 l 28 (proof of 333Q): for 'Take any $\epsilon > 0$' read 'Take any $\epsilon \in \left]0, \frac{1}{4}\right]$'.

p 156 l 37 (333X) Add new exercise:

(b) In Lemma 333J, show that every relative atom in $\mathfrak{A}$ over $\mathfrak{C}$ belongs to the closed subalgebra of $\mathfrak{A}$ generated by $\mathfrak{C} \cup \{a_n : n \in \mathbb{N}\}$.

Exercises 333Xb-333Xc are now 333Xc-333Xd.

p 156 l 38 (Exercise 333Xb, now 333Xc): to make this work, we need to assume that I is countable.

p 160 l 29 (Exercises 334X): 334Xb and 334Xd have been moved to 331Xo. 334Xc is now 334Xb, 334Xe-334Xg are now 334Xc-334Xe.

p 161 l 5 Exercise 334Ya is incorrect as written; the final clause should be '$\#(\{i : \kappa_i \neq 0\}) \leq \kappa'$', not 'either $\kappa' = 0$ or $\#(I) < \kappa$'.

p 167 l 14 (part (A-c) of the proof of 341I): for '$\mathfrak{A}_{\zeta(n)} \subseteq \mathfrak{A}_\xi$ for every ξ' read '$\mathfrak{A}_{\zeta(n)} \subseteq \mathfrak{A}_\xi$ for every n'. (J.M.)

p 172 l 36 (Exercise 341Xc, now 341Xd): for 'if $\underline{\theta} \in P$ and $a \in \mathfrak{A}$' read 'if $\underline{\theta} \in P$ and $a \in \mathfrak{A} \setminus \{0\}$'. Other exercises have been rearranged: 341Xc-341Xe are now 341Xd-341Xf, 341Xf is now 341Xc.

p 173 l 27 The former exercise 341Ye is now 341Xg. Add new exercises:

(e) Give an example of a complete probability space (X, Σ, μ), a subalgebra T of Σ, and a partial lower density $\underline{\phi} : \mathrm{T} \to \Sigma$ which has no extension to a lower density for μ.

(f) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra and $\langle a_i \rangle_{i \in I}$ a family in $\mathfrak{A}$. Let $\mathcal{B}\mathfrak{a}_I$ be the Baire σ-algebra of $Y = \{0, 1\}^I$, that is, the σ-algebra of subsets of Y generated by the family $\{E_i : i \in I\}$ where $E_i = \{y : y \in Y,\, y(i) = 1\}$ for $i \in I$. Show that there is a unique sequentially order-continuous Boolean homomorphism $\phi : \mathcal{B}\mathfrak{a}_I \to \mathfrak{A}$ such that $\phi E_i = a_i$ for every $i \in I$, and that $\phi[\mathcal{B}\mathfrak{a}_I]$ is the σ-subalgebra of $\mathfrak{A}$ generated by $\{a_i : i \in I\}$.

p 179 l 25 (part (a) of the proof of 342M): for '$A = \{\alpha : \alpha \in \mathbb{R},\, \{x : x \in X,\, f(x) \leq r\}$' read '$A = \{\alpha : \alpha \in \mathbb{R},\, \{x : x \in X,\, f(x) \leq \alpha\}$'. (J.M.)

p 179 l 30 (part (a) of the proof of 342M): for '$\{x : f(x) > \gamma_2^{-k}\}$' read '$\{x : f(x) > \gamma + 2^{-k}\}$'. (J.M.)

p 181 l 27 (342X) Add new part to Exercise 342Xn:

(x) a sum of perfect measures is perfect.

p 181 l 33 (342Y) Add new exercise:

(b) Give an example of a compact class $\mathcal{K}$ of subsets of $\mathbb{N}$ such that there is no compact Hausdorff topology on $\mathbb{N}$ for which every member of $\mathcal{K}$ is closed.

Exercises 342Ya-342Yb are now 342Yc-342Yd, and 342Yc is now 342Ya.

p 183 l 8 (343Ac): for '$f^{-1}[E]$ is negligible' read '$f^{-1}[G]$ is negligible'. (J.M.)

p 184 l 47 (part (g-α) of the proof of 343B): for 'the family of sets $K \subseteq E$' read 'the family of sets $K \subseteq X$'.

p 185 l 34 (343C) Add new part:

(d) In the other direction, if (X, Σ, μ) is a compact probability space with Maharam type at most $\kappa \geq \omega$, then there is an inverse-measure-preserving function from $\{0, 1\}^\kappa$ to X.

Re-name the former 343Cd as 343Ce.

p 189 l 23 (343X) Add new exercise:

(j) Let (X, Σ, μ) be a complete compact measure space, Y a set and $f : Y \to X$ a surjection; set

$$\mathrm{T} = \{F : F \subseteq Y,\, f[F] \in \Sigma,\, \mu(f[F] \cap f[Y \setminus F]) = 0\}, \quad \nu F = \mu f[F] \text{ for } F \in \mathrm{T}.$$

so that ν is a measure on Y and f is inverse-measure-preserving (234Ye). Show that ν is a compact measure.

p 189 l 6 (343X) Some exercises have been re-arranged: 343Xf-343Xh are now 343Xg-343Xi, 343Xi is now 343Xf.

p 189 l 26 Part (i) of Exercise 343Yb is wrong, and should read

(i) Show that a countably separated semi-finite measure space has magnitude and Maharam type at most $2^{\mathfrak{c}}$.

I have added another part to this exercise:

(iii) Show that a countably separated perfect measure space has countable Maharam type.

p 192 l 41 (part (a) of the proof of 344E, when choosing the $F_{\phi i}$) Of course we take $F_{\iota i} = E_i$ for every i.

p 194 l 15 (part (b-iv) of the proof of 344E): for '$x \notin H_{\theta\psi,\phi} \cup H_{\phi\theta,\psi} \cup H_{\theta,\phi} \cup H_{\psi,\phi} \cup h_{\phi\psi}^{-1}[V_z]$' read '$x \notin H_{\theta\psi,\phi} \cup H_{\psi,\phi\theta} \cup H_{\theta,\phi} \cup H_{\psi,\phi} \cup h_{\phi\psi}^{-1}[V_z]$'.

p 195 l 7 (part (b-vii-(γ) of the proof of 344E): for '$\tilde{g}_\phi\tilde{g}_\psi(x) = h_\phi h_\psi(x) = h_{\psi\phi}(x) = g_{\psi\phi}(x)$' read '$\tilde{g}_\phi\tilde{g}_\psi(x) = h_\phi h_\psi(x) = h_{\psi\phi}(x) = \tilde{g}_{\psi\phi}(x)$'.

p 195 l 14 (part (b-viii) of the proof of 344E): for '$q[E_i \cap H] = E_i$' read '$q[E_i] = E_i \cap H$'.

p 196 l 11 (part (d) of the proof of 344E): for '$f_{\phi,\xi}f_{\psi,\xi} = f_{\phi\psi,\xi}$' read '$f_{\phi,\xi}f_{\psi,\xi} = f_{\psi\phi,\xi}$'.

p 196 l 28 344H has been strengthened, and now reads

344H Lemma Let (X, Σ, μ) be a perfect semi-finite measure space. If $H \in \Sigma$ is a non-negligible set which includes no atom, there is a negligible subset of H with cardinal $\mathfrak{c}$.

p 198 l 2 Theorem 344L has been rewritten, as follows:

Theorem Let I be an infinite set, and ν_I the usual measure on $\{0,1\}^I$. If $E \subseteq \{0,1\}^I$ is a measurable set of non-zero measure, the subspace measure on E is isomorphic to $(\nu_I E)\nu_I$.

p 198 l 36 (exercise 344Xf): for 'every automorphism of $\mathfrak{A}$' read 'every measure-preserving automorphism of $\mathfrak{A}$'.

p 198 l 38 Exercise 344Xg has been deleted (now being covered by 344L).

p 199 l 15 (exercise 344Yd): for 'let $\mathcal{I}$ be an ω_1-saturated ideal of $\mathcal{B}\mathfrak{a}$' read 'let $\mathcal{I}$ be an ω_1-saturated σ-ideal of $\mathcal{B}\mathfrak{a}_I$'.
Similarly, in 344Ye, we need σ-ideals again.

p 203 l 19 (part (c-vi) of the proof of 345C): for '$V_E \triangle E$' read '$V \triangle E$'.

p 203 l 38 (part (c-viii) of the proof of 345C): for '$\{y : g'_n(y) \le \alpha\}$' read '$\{y : g'_n(y) \ge \alpha\}$'.

p 205 l 8 The statement of lemma 345E has been slightly strengthened, and now reads

Give $X = \{0,1\}^{\mathbb{N}}$ its usual measure $\nu_{\mathbb{N}}$, and let $E \subseteq X$ be any non-negligible measurable set. Then there is an $n \in \mathbb{N}$ such that for every $k \ge n$ there are x, $x' \in E$ which differ at k and nowhere else.

p 207 l 36 (Exercise 345Xf): for '$\underline{\phi}G \subseteq G$' read '$\underline{\phi}G \supseteq G$'.

p 209 l 8 Proposition 346B has been revised, and now reads

Lemma (a) Let (X, Σ, μ) be a measure space with a lifting $\phi : \Sigma \to \Sigma$. Suppose that Y is a set and $f : X \to Y$ a surjective function such that whenever $E \in \Sigma$ is such that $f^{-1}[f[E]] = E$, then $f^{-1}[f[\phi E]] = \phi E$. Then we have a lifting ψ for the image measure μf^{-1} defined by the formula

$$f^{-1}[\psi F] = \phi(f^{-1}[F]) \text{ whenever } F \subseteq Y \text{ and } f^{-1}[F] \in \Sigma.$$

(b) Let $\langle(X_i, \Sigma_i, \mu_i)\rangle_{i\in I}$ be a family of probability spaces, with product (Z, Λ, λ). For $J \subseteq I$ let $(Z_J, \Lambda_J, \lambda_J)$ be the product of $\langle(X_i, \Sigma_i, \mu_i)\rangle_{i\in J}$, and $\pi_J : Z \to Z_J$ the canonical map. Let $\phi : \Lambda \to \Lambda$ be a lifting. If $J \subseteq I$ is such that ϕW is determined by coordinates in J whenever $W \in \Lambda$ is determined by coordinates in J, then ϕ induces a lifting $\phi_J : \Lambda_J \to \Lambda_J$ defined by the formula

$$\pi_J^{-1}[\phi_J E] = \phi(\pi_J^{-1}[E]) \text{ for every } E \in \Lambda_J.$$

p 209 l 20 (proof of 346C): for '$E + y = y$' read '$E + y = E$'. [(J.M.)]

p 217 l 4 (Exercise 346Xe, now 346Xg): I do not know whether the result declared in this exercise is true, but it looks rather optimistic, and the hint is certainly inadequate. So I have changed the exercise to

Let $\underline{\phi}$ be lower Lebesgue density on $\mathbb{R}$, and ϕ a translation-invariant lifting for Lebesgue measure on $\mathbb{R}$ such that $\phi E \supseteq \underline{\phi}E$ for every measurable set E. Show that ϕ is consistent.

I have added a new exercise:

(e) Describe the connections between 346B, 346D and 346F.
Other exercises have been re-arranged: 346Xa is now 346Xd, 346Xb is now 346Xc, 346Xc-346Xd are now 346Xa-346Xb, 346Xe-346Xf are now 346Xg-346Xh, 346Xg is now 346Xf.

p 217 l 32 Exercise 346Ye has been deleted.
Other exercises have been re-arranged: 346Ya-346Yb are now 346Yc-346Yd, 346Yc is now 346Ya, 346Yd is now 346Ye, 346Yf is now 346Yb.

p 225 l 34 Exercises 351Yc-351Yd have been exchanged, and are now 351Yd-351Yc.

p 225 l 39 (351Y) Add new exercise:
(e) Show that a reduced power $\mathbb{R}^X|\mathcal{F}$, as described in 351M, is Archimedean iff $\bigcap_{n\in\mathbb{N}} F_n \in \mathcal{F}$ whenever $\langle F_n\rangle_{n\in\mathbb{N}}$ is a sequence in $\mathcal{F}$.

p 227 l 15 (352D) For

$$|u+v| = (u+v)\wedge((-u)+(-v)) \le (|u|+|v|)\wedge(|u|+|v|) = |u|+|v|$$

read

$$|u+v| = (u+v)\vee((-u)+(-v)) \le (|u|+|v|)\vee(|u|+|v|) = |u|+|v|.$$

p 227 l 16 (352D) For

$$||u|-|v|| = (|u|-|v|)\vee(|v|-|u|) \le |u-v|+|v-u| = |u-v|$$

read

$$||u|-|v|| = (|u|-|v|)\vee(|v|-|u|) \le |u-v|\vee|v-u| = |u-v|.$$

p 228 l 2 Add new fragment to 352Fa:
(ii) If $v_0,\dots,v_m,w_0,\dots,w_n \in U^+$ then

$$\sum_{i=0}^m v_i \wedge \sum_{j=0}^n w_j \le \sum_{i=0}^m \sum_{j=0}^n v_i \wedge w_j.$$

p 237 l 14 (352Y) Add new exercises:
(a) Find an f-algebra with a non-commutative multiplication.
(b) Let U be an f-algebra. Show that the multiplication of U is commutative iff $u\times v = (u\wedge v)\times(u\vee v)$ for all u, $v\in U$.
(c) Let U be an f-algebra. Show that $u\times\mathrm{med}(v_1,v_2,v_3) = \mathrm{med}(u\times v_1, u\times v_2, u\times v_3)$ whenever u, v_1, v_2, $v_3\in U$.

p 238 l 39 (proof of 353E) for '$nv_0 \le u \Longrightarrow nv_0 \le v \Longrightarrow (n+1)v_0 \le v+w = u$' read '$n(w\wedge v_0) \le u \Longrightarrow n(w\wedge v_0) \le v \Longrightarrow (n+1)(w\wedge v_0) \le v+w = u$', and similarly in the next line.

p 239 l 5 Add new result:
353G Proposition Let U be a Riesz space and V an order-dense Riesz subspace of U. If V is Archimedean, so is U.
353G-353P are now 353H-353Q.

p 240 l 4 (statement of 353J(a-iii), now 353K) for 'the canonical map from U to V' read 'the canonical map from U to U/V'.

p 245 l 5 Exercises 353Yd-353Ye have been exchanged.

p 258 l 32 Theorem 355F can be strengthened, as follows:
Theorem Let U and V be Riesz spaces, $U_0 \subseteq U$ a Riesz subspace and $T_0 : U_0 \to V$ a positive linear operator such that $Su = \sup\{T_0 w : w\in U_0,\ 0\le w\le u\}$ is defined in V for every $u\in U^+$. Suppose *either* that U_0 is order-dense and that T_0 is order-continuous *or* that U_0 is solid.
(a) There is a unique positive linear operator $T : U\to V$, extending T_0, which agrees with S on U^+.
(b) If T_0 is a Riesz homomorphism so is T.

(c) If T_0 is order-continuous so is T.

(d) If U_0 is order-dense and T_0 is an injective Riesz homomorphism, then T is injective.

(e) If U_0 is order-dense and T_0 is order-continuous then T is the only order-continuous positive linear operator from U to V extending T_0.

p 261 l 45 (Exercise 355Xe) For 'show that a matrix represents a Riesz homomorphism' read 'show that a positive matrix represents a Riesz homomorphism'.

p 265 l 6 (part (a-ii) of the proof of 356D): for 'if f, $g \in U^*$' read 'if $f \in U^\sim$, $g \in U^*$'.

p 266 l 10 (part (c) of the proof of 356F): for '356D' read '356E'.

p 266 l 16 (proof of 356G): for '$f(v) \le |f|(v) \le \frac{1}{n}|f|(u)$' read '$|f(v)| \le |f|(v) \le \frac{1}{n}|f|(u)$'.

p 273 l 5 Exercise 356Yd is wrong, and should be deleted. Consequently 356Ye-356Yi should be renamed 356Yd-356Yh.

p 273 l 23 (Exercise 356Yi, now 356Yh): for 'let U be a Banach lattice with an order-continuous norm with the Levi property' read 'let U be a perfect Banach lattice'.

p 278 l 30 (part (b) of the proof of 361E): for '$\sum_{j=0}^{k}\gamma_{ij}\chi b_j$' read '$\sum_{j=0}^{m}\gamma_{ij}\chi b_j$'.

p 278 l 33 (part (b) of the proof of 361E): for '$j \le k$' read '$j \le m$'.

p 279 l 5 (part (f) of the proof of 361E): for '$\sum_{j=0}^{m}\beta_j b_j$' read '$\sum_{j=0}^{m}\beta_j\chi b_j$'.

p 280 l 1 (part (a) of the proof of 361F): for '$\sum_{j=0}^{m}\sum_{i=0}^{n}\alpha_i\gamma_{ij}\nu b_i$' read '$\sum_{j=0}^{m}\sum_{i=0}^{n}\alpha_i\gamma_{ij}\nu b_j$'.

p 282 l 10 (part (b) of the proof of 361H): for

$$|Tv| = |\sum_{i=0}^{n}\alpha_i\chi a_i| \le \sum_{i=0}^{n}|\alpha_i||\nu a_i| \le \alpha\sum_{i=0}^{n}|\nu a_i| = \alpha\theta a$$

read

$$|Tv| = |\sum_{i=0}^{n}\alpha_i\nu a_i| \le \sum_{i=0}^{n}|\alpha_i||\nu a_i| \le \alpha\sum_{i=0}^{n}|\nu a_i| \le \alpha\theta a.$$

p 282 l 32 (part (b-ii) of the proof of 361I): for '$\gamma_i = \sum_{j=0}^{m}\delta_{ij}$, $\gamma'_j = \sum_{i=0}^{n}\delta_{ij}$' read '$\gamma_i = \sum_{j=0}^{n}\delta_{ij}$, $\gamma'_j = \sum_{i=0}^{m}\delta_{ij}$'.

p 284 l 19 (part (g) of the proof of 361J): for '$T_\phi T(\chi a)$' read '$T_\phi T_\pi(\chi a)$'.

p 284 l 30 (part (b) of the proof of 361K): for '$V_a + V_{1\setminus a} = U$' read '$V_a + V_{1\setminus a} = S$'.

p 286 l 15 The exercises for §361 have been rearranged: 361Xh-361Xj are now 361Xi-361Xk, 361Xk is now 361Xh, 361Yb-361Ye are now 361Yc-361Yf, 361Yf is now 361Yb.

p 286 l 20 (Exercise 361Xj, now 361Xk): for '$\mathbb{C}^X$' read '$\mathbb{C}^Z$'.

p 286 l 25 (361X) Add new exercise:

(l) Let $\mathfrak{A}$ be a Boolean algebra, U a partially ordered linear space and $\nu : \mathfrak{A} \to U$ a non-negative additive function. (i) Show that ν is order-continuous iff $\nu 1 = \sup_{J\subseteq I \text{ is finite}}\sum_{i\in J}\nu a_i$ whenever $\langle a_i\rangle_{i\in I}$ is a partition of unity in U. (i) Show that ν is order-continuous iff $\nu 1 = \sup_{n\in\mathbb{N}}\sum_{i=0}^{n}\nu a_i$ whenever $\langle a_i\rangle_{i\in\mathbb{N}}$ is a partition of unity in U.

p 293 l 2 (part (f-i-(γ) $\Rightarrow$ (α)) of the proof of 362B): for '$\lim_{k\to\infty}\mu c_k = 0$' read '$\lim_{k\to\infty}|\mu|c_k = 0$'; two lines later, read '$\lim_{k\to\infty}|\mu|(c_k \setminus b) = 0$' for '$\lim_{k\to\infty}\mu(c_k \setminus b) = 0$'.

p 293 l 21 (part (f-i-(γ) $\Rightarrow$ (α)) of the proof of 362B): for '$\langle c_k\rangle_{k\in\mathbb{N}}$ is a non-decreasing sequence' read '$\langle c_k\rangle_{k\in\mathbb{N}}$ is a non-increasing sequence'.

p 293 l 40 (362B) Add

Remark The L-space norm $\| \|$ on M, described in (a) above, is the **total variation norm**.

p 296 l 8 Exercise 362Xe has been rewritten, and is now

Let $\mathfrak{A}$ be a Boolean algebra, and M the space of bounded additive functionals on $\mathfrak{A}$. Let us say that a non-zero finitely additive functional $\nu : \mathfrak{A} \to \mathbb{R}$ is **atomic** if whenever a, $b \in \mathfrak{A}$ and $a \cap b = 0$ then at least one of νa, νb is zero. (i) Show that for a non-zero finitely additive functional ν on $\mathfrak{A}$ the following are equiveridical: (α) ν is atomic; (β) $\nu \in M$ and $|\nu|$ is atomic; (γ) $\nu \in M$ and the corresponding linear functional $f_{|\nu|} = |f_\nu| \in S(\mathfrak{A})^\sim$ is a Riesz homomorphism; (δ) there are a multiplicative linear functional $f : S(\mathfrak{A}) \to \mathbb{R}$ and an $\alpha \in \mathbb{R}$ such that $\nu a = \alpha f(\chi a)$ for every $a \in \mathfrak{A}$; (ϵ) $\nu \in M$ the band in M generated by ν is the set of multiples of ν. (ii) Show that a completely additive functional $\nu : \mathfrak{A} \to \mathbb{R}$ is atomic iff there are $a \in \mathfrak{A}$ and $\alpha \in \mathbb{R} \setminus \{0\}$ such that a is an atom in $\mathfrak{A}$ and $\nu b = \alpha$ when $a \subseteq b$, 0 when $a \cap b = 0$.

p 297 l 6 (Exercise 362Yd): for 'whenever $\langle u_n \rangle_{n\in\mathbb{N}}$ is a non-increasing sequence in U^+ and $\lim_{n\to\infty} f(u_n) = 0$' read 'whenever $\langle u_n \rangle_{n\in\mathbb{N}}$ is a non-increasing sequence in U^+ and $\lim_{n\to\infty} |f|(u_n) = 0$'.

p 297 l 30 (362Y) Add new exercise:

(k) Let $\mathfrak{A}$ be a Boolean algebra and M the L-space of bounded additive real-valued functionals on $\mathfrak{A}$. Suppose that M_0 is a norm-closed linear subspace of M and that $a \mapsto \nu(a \cap c) : \mathfrak{A} \to \mathbb{R}$ belongs to M_0 whenever $\nu \in M_0$ and $c \in \mathfrak{A}$. Show that M_0 is a band in M. (*Hint*: 436L.)

p 303 l 13 (part (b-ii) of the proof of 363H): for '$h = -\alpha\mathbf{1} \vee (f \wedge \mathbf{1})$' read '$h = \mathrm{med}(-\alpha\chi X, f, \alpha\chi X)$'.

p 306 l 10 (part (a-i) of the proof of 363M): for '$\mathcal{I}$' read '$\mathcal{M}$'.

p 312 l 24 Exercise 363Xj has been moved to 366Xl.

p 312 l 35 The exercises 363Y have been rearranged: 363Yd is now 363Yi, 363Yj is now 366Ym, 363Ye-363Yi are now 363Yd-363Yh, 363Yk is now part of 372Yq.

p 315 l 8 The remarks in 364B has been incorporated into 364A as parts (b)-(g), with a new subparagraph:

***(f)** Indeed, we have the option of declaring $L^0(\mathfrak{A})$ to be the set of functions $\alpha \mapsto [\![u > \alpha]\!] : \mathbb{Q} \to \mathfrak{A}$ such that

(α'') $[\![u > q]\!] = \sup_{q'\in\mathbb{Q},q'>q} [\![u > q']\!]$ for every $q \in \mathbb{Q}$,

(β') $\inf_{n\in\mathbb{N}} [\![u > n]\!] = 0$,

(γ') $\sup_{n\in\mathbb{N}} [\![u > -n]\!] = 1$.

364C-364L are now 364C-364K.

p 321 l 16 364M-364N have been merged into 364L. 364O-364R are now 364M-364P.

p 321 l 36 (part (b-i) of the proof of 364M, now part (a-ii-α) of the proof of 364L): for '$\sup_{u\in A} [\![u > q]\!] = c_\alpha$' read '$\sup_{u\in A} [\![u > \alpha]\!] = c_\alpha$'.

p 323 l 14 (proof of 364P, now 364N): for '$\pi\{z : |f(z)| > 0\} = [\![Sf > 0]\!] = 1$' read '$\pi\{z : |f(z)| > 0\} = [\![S|f| > 0]\!] = 1$'.

p 323 l 28 (proof of 364Q, now 364O): for '$V_1 + V_{1\setminus a}$' read '$V_a + V_{1\setminus a}$'.

p 325 l 26 I have brought the former 364Xr and 372H together in the following form:

364Q Proposition Let X and Y be sets, Σ, T σ-algebras of subsets of X, Y respectively, and $\mathcal{I}$, $\mathcal{J}$ σ-ideals of Σ, T. Set $\mathfrak{A} = \Sigma/\mathcal{I}$ and $\mathfrak{B} = \mathrm{T}/\mathcal{J}$. Suppose that $\phi : X \to Y$ is a function such that $\phi^{-1}[F] \in \Sigma$ for every $F \in \mathrm{T}$ and $\phi^{-1}[F] \in \mathcal{I}$ for every $F \in \mathcal{J}$.

(a) There is a sequentially order-continuous Boolean homomorphism $\pi : \mathfrak{B} \to \mathfrak{A}$ defined by saying that $\pi F^\bullet = \phi^{-1}[F]^\bullet$ for every $F \in \mathrm{T}$.

(b) Let $T_\pi : L^0(\mathfrak{B}) \to L^0(\mathfrak{A})$ be the Riesz homomorphism corresponding to π, as defined in 364P. If we identify $L^0(\mathfrak{B})$ with $\mathcal{L}^0_\mathrm{T}/\mathcal{W}_\mathcal{J}$ and $L^0(\mathfrak{A})$ with $\mathcal{L}^0_\Sigma/\mathcal{W}_\mathcal{I}$ in the manner of 364B-364C, then $T_\pi(g^\bullet) = (g\phi)^\bullet$ for every $g \in \mathcal{L}^0_\mathrm{T}$.

(c) Let Z be a third set, Υ a σ-algebra of subsets of Z, $\mathcal{K}$ a σ-ideal of Υ, and $\psi : Y \to Z$ a function such that $\psi^{-1}[G] \in \mathrm{T}$ for every $G \in \Upsilon$ and $\psi^{-1}[G] \in \mathcal{J}$ for every $F \in \mathcal{K}$. Let $\theta : \mathfrak{C} \to \mathfrak{B}$ and $T_\theta : L^0(\mathfrak{C}) \to L^0(\mathfrak{B})$ be the homomorphisms corresponding to ψ as in (a)-(b). Then $\pi\theta : \mathfrak{C} \to \mathfrak{A}$ and $T_\pi T_\theta : L^0(\mathfrak{C}) \to L^0(\mathfrak{A})$ correspond to $\psi\phi : X \to Y$ in the same way.

(d) Now suppose that μ and ν are measures with domains Σ, T and null ideals $\mathcal{N}(\mu)$, $\mathcal{N}(\nu)$ respectively, and that $\mathcal{I} = \Sigma \cap \mathcal{N}(\mu)$ and $\mathcal{J} = \mathrm{T} \cap \mathcal{N}(\nu)$. In this case, identifying $L^0(\mathfrak{A})$, $L^0(\mathfrak{B})$ with $L^0(\mu)$ and $L^0(\nu)$ as in 364Ic, we have $g\phi \in \mathcal{L}^0(\mu)$ and $T_\pi(g^\bullet) = (g\phi)^\bullet$ for every $g \in \mathcal{L}^0(\nu)$.

364S-364W are now 364R-364V.

p 325 l 34 (proof of 364S, now 364R): for '$[\![Tu > \alpha]\!]$' and '$[\![Tv > \alpha]\!]$' read '$[\![u > \alpha]\!]$' and '$[\![v > \alpha]\!]$', and again in the next line.

p 327 l 16 (part (b) of the proof of 364U, now 364T): for '$\sup_{\beta>\alpha} \phi_f(\alpha)$' read '$\sup_{\beta>\alpha} \phi_f(\beta)$'.

p 327 l 19 (part (b) of the proof of 364U, now 364T): for '$G_1 \cap \operatorname{int} \overline{\{x : f(x) > \gamma\}} = \emptyset$' read '$G_1 \cap \operatorname{int} \overline{\{x : f(x) > \beta\}} = \emptyset$'.

p 329 l 17 (part (i) of the proof of 364U, now 364T): for 'counting $\inf \emptyset$ as $-\infty$' read 'counting $\sup \emptyset$ as $-\infty$'.

p 332 l 15 (364X) Add new exercise:

(j) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra and $\nu : \mathfrak{A} \to \mathbb{R}$ a non-negative finitely additive functional. Let $f : L^\infty(\mathfrak{A}) \to \mathbb{R}$ be the corresponding linear functional, as in 363L. Write U for the set of those $u \in L^0(\mathfrak{A})$ such that $\sup\{f v : v \in L^\infty(\mathfrak{A}),\ v \le |u|\}$ is finite. Show that f has an extension to a non-negative linear functional on U.

364Xr is now part of 364Q. Other exercises have been re-arranged: 364Xj-364Xq are now 364Xl-364Xr, 364Xs is now 364Xv, 364Xt-364Xv are now 364Xs-364Xu.

p 332 l 18 Exercise 364Xk (now 364Xl) is wrong; in both parts, the result is true for open sets E, but false for Borel sets in general.

p 333 l 15 Exercise 364Ya has been extended, as follows:

(a) (i) Show directly, without using the Loomis-Sikorski theorem or the Stone representation, that if $\mathfrak{A}$ is any Dedekind σ-complete Boolean algebra then the formulae of 364D define a group operation $+$ on $L^0(\mathfrak{A})$, and generally an f-algebra structure. (ii) Defining $\chi : \mathfrak{A} \to L^0(\mathfrak{A})$ by the formula in 364Jc, show that $S(\mathfrak{A})$ and $L^\infty(\mathfrak{A})$ can be identified with the linear span of $\{\chi a : a \in \mathfrak{A}\}$ and the solid linear subspace of $L^0(\mathfrak{A})$ generated by $e = \chi 1$. (iii) Still without using the Loomis-Sikorski theorem, explain how to define $\bar{h} : L^0(\mathfrak{A}) \to L^0(\mathfrak{A})$ for continuous functions $h : \mathbb{R} \to \mathbb{R}$. (iv) Check that these ideas are sufficient to yield 364L-364R, except that in 364Pd we may not be able to handle all Borel functions h.

364Yb is now 364Ye, 364Yc-364Ye are now 364Yb-364Yd.

p 333 l 34 (364Y) Add new exercise:

(g) Let $\mathfrak{A}$ and $\mathfrak{B}$ be Dedekind σ-complete Boolean algebras and $\pi : \mathfrak{A} \to \mathfrak{B}$ a sequentially order-continuous ring homomorphism. (i) Show that we have a multiplicative sequentially order-continuous Riesz homomorphism $T_\pi : L^0(\mathfrak{A}) \to L^0(\mathfrak{B})$ defined by the formula

$$[\![T_\pi u > \alpha]\!] = \pi[\![u > \alpha]\!]$$

whenever $u \in L^0(\mathfrak{A})$ and $\alpha > 0$. (ii) Show that T_π is order-continuous iff π is order-continuous, injective iff π is injective, and surjective iff π is surjective. (iii) Show that if $\mathfrak{C}$ is another Dedekind σ-complete Boolean algebra and $\theta : \mathfrak{B} \to \mathfrak{C}$ another sequentially order-continuous ring homomorphism then $T_{\theta\pi} = T_\theta T_\pi : L^0(\mathfrak{A}) \to L^0(\mathfrak{C})$.

364Yg-364Ym are now 364Yh-364Yn, 364Yn has been moved to 366M and 366Yj-366Yl, 364Yo is now 364Yg.

p 348 l 26 Add new result:

365T Theorem Let $(\mathfrak{A}, \bar{\mu})$ be a measure algebra. Set $L^1 = L^1(\mathfrak{A}, \bar{\mu})$.

(a) For a non-empty subset A of L^1, the following are equiveridical:

(i) A is uniformly integrable in the sense of 354P;

(ii) for every $\epsilon > 0$ there are an $a \in \mathfrak{A}^f$ and an $M \ge 0$ such that $|\int (u - M\chi a)^+ \le \epsilon$ for every $u \in \mathfrak{A}$;

(iii)(α) $\sup_{u \in A} |\int_a u|$ is finite for every atom $a \in \mathfrak{A}$,

(β) for every $\epsilon > 0$ there are $c \in \mathfrak{A}^f$ and $\delta > 0$ such that $|\int_a u| \le \epsilon$ whenever $u \in A$, $a \in \mathfrak{A}$ and $\bar{\mu}(a \cap c) \le \delta$;

(iv)(α) $\sup_{u \in A} |\int_a u|$ is finite for every atom $a \in \mathfrak{A}$,

(β) $\lim_{n\to\infty} \sup_{u \in A} |\int_{a_n} u| = 0$ for every disjoint sequence $\langle a_n \rangle_{n\in\mathbb{N}}$ in $\mathfrak{A}$;

(iv) A is relatively weakly compact in L^1.

(b) If $(\mathfrak{A}, \bar{\mu})$ is a probability algebra and $A \subseteq L^1$ is uniformly integrable, then there is a solid convex norm-closed uniformly integrable set $C \supseteq A$ such that $P[C] \subseteq C$ whenever $P : L^1 \to L^1$ is the conditional expectation operator associated with a closed subalgebra of $\mathfrak{A}$.

p 340 l 4 Corollary 365J has been dropped. 365K-365T are now 365J-365S.

p 348 l 26 Add new result:

365T Theorem Let $(\mathfrak{A}, \bar{\mu})$ be a measure algebra. Set $L^1 = L^1(\mathfrak{A}, \bar{\mu})$.

(a) For a non-empty subset A of L^1, the following are equiveridical:

(i) A is uniformly integrable in the sense of 354P;

(ii) for every $\epsilon > 0$ there are an $a \in \mathfrak{A}^f$ and an $M \ge 0$ such that $\int (|u| - M\chi a)^+ \le \epsilon$ for every $u \in \mathfrak{A}$;

(iii)(α) $\sup_{u \in A} |\int_a u|$ is finite for every atom $a \in \mathfrak{A}$,

(β) for every $\epsilon > 0$ there are $c \in \mathfrak{A}^f$ and $\delta > 0$ such that $|\int_a u| \le \epsilon$ whenever $u \in A$, $a \in \mathfrak{A}$ and $\bar{\mu}(a \cap c) \le \delta$;

(iv)(α) $\sup_{u \in A} |\int_a u|$ is finite for every atom $a \in \mathfrak{A}$,

(β) $\lim_{n\to\infty} \sup_{u \in A} |\int_{a_n} u| = 0$ for every disjoint sequence $\langle a_n \rangle_{n\in\mathbb{N}}$ in $\mathfrak{A}$;

(v) A is relatively weakly compact in L^1.

(b) If $(\mathfrak{A}, \bar{\mu})$ is a probability algebra and $A \subseteq L^1$ is uniformly integrable, then there is a solid convex norm-closed uniformly integrable set $C \supseteq A$ such that $P[C] \subseteq C$ whenever $P : L^1 \to L^1$ is the conditional expectation operator associated with a closed subalgebra of $\mathfrak{A}$.

p 349 l 1 (365X) Add new exercises:

(f) Let $(\mathfrak{A}, \bar{\mu})$ be a measure algebra and $u, v \in L^0(\mathfrak{A})^+$. Show that $\int u \times v \, d\bar{\mu} = \int_0^\infty (\int_{[\![u>\alpha]\!]} v \, d\bar{\mu}) d\alpha$.

(n) Let $(\mathfrak{A}, \bar{\mu})$ and $(\mathfrak{B}, \bar{\nu})$ be probability algebras, $\pi : \mathfrak{A} \to \mathfrak{B}$ a measure-preserving Boolean homomorphism, and $T : L^0(\mathfrak{A}) \to L^0(\mathfrak{B})$ the corresponding Riesz homomorphism. Let $\mathfrak{C}$ be a closed subalgebra of $\mathfrak{A}$ and $P : L^1(\mathfrak{A}, \bar{\mu}) \to L^1(\mathfrak{C}, \bar{\mu} \restriction \mathfrak{C}) \subseteq L^1(\mathfrak{A}, \bar{\mu})$, $Q : L^1(\mathfrak{B}, \bar{\nu}) \to L^1(\mathfrak{B}, \bar{\nu})$ the conditional expectation operators defined from $\mathfrak{C} \subsetneqq \mathfrak{A}$ and $\pi[\mathfrak{C}] \subsetneqq \mathfrak{B}$. Show that $TP = QT$.

Other exercises have been rearranged; 365Xf-365Xl are now 365Xg-365Xm, 365Xm-365Xp are now 365Xo-365Xr.

p 349 l 4 (statement (iii) of Exercise 365Xf, now 325Xg): for '$\nu a \le \epsilon$ whenever $a \subseteq c$ and $\bar{\mu} a \le \delta$' read '$|\nu a| \le \epsilon$ whenever $a \subseteq c$ and $\bar{\mu} a \le \delta$'.

p 352 l 30-31 (part (c) of the proof of 366D: for '$(w^-)^p$' read '$(w^-)^q$' (four times).

p 354 l 2-4 (part (d) of the proof of 366G): for 'u'' read 'v' (nine times).

p 355 l 11 (part (a-iii) of the proof of 366H): for '$0 < \chi\pi a \le T|u|$' read '$0 < \alpha\chi\pi a \le T|u|$'.

p 355 l 19 (part (a-iv-β) of the proof of 366H): for '$\chi a \le |u|$ so $\chi(\pi a) \le |Tu|$' read '$\alpha\chi a \le |u|$ so $\alpha\chi(\pi a) \le |Tu|$'.

p 355 l 22 (part (a-iv-γ) of the proof of 366H): for '$Tu_n \le Tu$' read '$Tu_n \le T|u| = |Tu|$'.

p 356 l 9 (part (b-i-β) of the proof of 366H): for '$\inf_{n\ge1} c_n \le \inf_{n\ge1} \frac{2}{n} \int (v_0 - \frac{n}{2}\chi 1_{\mathfrak{B}})^+$' read '$\inf_{n\ge1} \bar{\mu} c_n \le \inf_{n\ge1} \frac{2}{n} \int (v_0 - \frac{n}{2}\chi 1_{\mathfrak{B}})^+$'.

p 359 l 29 (Exercise 366Ya): for 'is atomless' read 'has no atom of finite measure'.

p 365 l 4 To make room for the new 367Q, 367D and 367E have been brought together as parts (a) and (b) of 367D; 367F-367Q are now 367E-367P.

p 369 l 6 Add new result:

367Q Theorem Let $(\mathfrak{A}, \bar{\mu})$ be a probability algebra; for each closed subalgebra $\mathfrak{B}$ of $\mathfrak{A}$, let $P_{\mathfrak{B}} : L^1 = L^1(\mathfrak{A}, \bar{\mu}) \to L^1 \cap L^0(\mathfrak{B})$ be the conditional expectation operator.

(a) If $\mathbb{B}$ is a non-empty downwards-directed family of closed subalgebras of $\mathfrak{A}$ with intersection $\mathfrak{C}$, then for every $u \in L^1$, $P_{\mathfrak{C}}u$ is the $\| \|_1$-limit of $P_{\mathfrak{B}}$ as $\mathfrak{B}$ decreases through $\mathbb{B}$, in the sense that

for every $\epsilon > 0$ there is a $\mathfrak{B}_0 \in \mathbb{B}$ such that $\|P_{\mathfrak{B}}u - P_{\mathfrak{C}}u\|_1 \le \epsilon$ whenever $\mathfrak{B} \in \mathbb{B}$ and $\mathfrak{B} \subseteq \mathfrak{B}_0$.

(b) If $\mathbb{B}$ is a non-empty upwards-directed family of closed subalgebras of $\mathfrak{A}$ and $\mathfrak{C}$ is the closed subalgebra generated by $\bigcup \mathbb{B}$, then for every $u \in L^1$, $P_{\mathfrak{C}}u$ is the $\| \|_1$-limit of $P_{\mathfrak{B}}$ as $\mathfrak{B}$ increases through $\mathbb{B}$, in the sense that

for every $\epsilon > 0$ there is a $\mathfrak{B}_0 \in \mathbb{B}$ such that $\|P_{\mathfrak{B}}u - P_{\mathfrak{C}}u\|_1 \le \epsilon$ whenever $\mathfrak{B} \in \mathbb{B}$ and $\mathfrak{B} \supseteq \mathfrak{B}_0$.

(c) Suppose that $\mathbb{B}$ is a non-empty upwards-directed family of closed subalgebras of $\mathfrak{A}$, and $\langle u_{\mathfrak{B}} \rangle_{\mathfrak{B} \in \mathbb{B}}$ is a $\| \|_1$-bounded family in L^1 such that $u_{\mathfrak{B}} = P_{\mathfrak{B}} u_{\mathfrak{C}}$ whenever $\mathfrak{B}, \mathfrak{C} \in \mathbb{B}$ and $\mathfrak{B} \subseteq \mathfrak{C}$. Then there is a $u \in L^1$ which is the limit $\lim_{\mathfrak{B} \to \mathcal{F}(\mathbb{B}\uparrow)} u_{\mathfrak{B}}$ for the topology of convergence in measure, where $\mathcal{F}(\mathbb{B}\uparrow)$ is the filter on $\mathbb{B}$ generated by $\{\{\mathfrak{C} : \mathfrak{B} \subseteq \mathfrak{C} \in \mathbb{B}\} : \mathfrak{B} \in \mathbb{B}\}$.

p 369 l 10 (367R) Add new parts:

(b) If $\mathfrak{A}$ has countable Maharam type, then L^0 is separable.

(c) Suppose that $\mathfrak{B}$ is a subalgebra of $\mathfrak{A}$ which is closed for the measure-algebra topology. Then $L^0(\mathfrak{B})$ is closed in $L^0(\mathfrak{A})$.

(d) A non-empty set $A \subseteq L^0$ is bounded in the linear topological space sense iff $\inf_{k \in \mathbb{N}} \sup_{u \in A} \bar{\mu}(a \cap [\![|u| > k]\!]) = 0$ for every $a \in \mathfrak{A}^f$.

p 374 l 30 (367X) Add new exercise:

(t) Let $(\mathfrak{A}, \bar{\mu})$ be a measure algebra, and give $L^0 = L^0(\mathfrak{A})$ its topology of convergence in measure. Show that a set $A \subseteq L^0$ is bounded in the sense of 3A5N iff for every $a \in \mathfrak{A}^f$ and $\epsilon > 0$ there is an $n \in \mathbb{N}$ such that $\bar{\mu}(a \cap [\![|u| > n]\!]) \le \epsilon$ for every $u \in A$.

p 376 l 1 (367Y) Add new exercise:

(q) In 367Qc, show that $u = \lim_{\mathfrak{B} \to \mathcal{F}(\mathbb{B}\uparrow)} u_{\mathfrak{B}}$ for the norm topology of L^1 iff $\{u_{\mathfrak{B}} : \mathfrak{B} \in \mathbb{B}\}$ is uniformly integrable, and that in this case $u_{\mathfrak{B}} = P_{\mathfrak{B}}u$ for every $\mathfrak{B} \in \mathbb{B}$.

p 378 l 35 (part (c-ii) of the proof of 368E): for '$e \wedge (u - \inf_{\beta > \alpha} e)^+$' read '$e \wedge (u - \inf_{\beta > \alpha} \beta e)^+$'.

p 383 l 4 (part (c) of the proof of 368M): for '$Tv \ge T_0 = u$' read '$Tv \ge T_0 u = u$'.

p 384 l 4 (part (b) of the proof of 368P: the letter u is being used for two things at once. In the phrases 'there are a $u > 0$ in U' (line 4), '$\sup_{n \in \mathbb{N}} u_n = u$' (line 5), '$0 < v \le u$' (line 6) and '$v = v \wedge u$' (line 16) we need a different symbol, e.g., u^*.

p 385 l 32 (368X) Add new exercise:

(b) Let U be a linear space, $\mathfrak{A}$ a Dedekind complete Boolean algebra, and $p : U \to L^0 = L^0(\mathfrak{A})$ a function such that $p(u + v) \le p(u) + p(v)$ and $p(\alpha u) = \alpha p(u)$ whenever u, $v \in U$ and $\alpha \ge 0$. Suppose that $U_0 \subseteq U$ is a linear subspace and $T_0 : U_0 \to L^0$ is a linear operator such that $T_0 u \le p(u)$ for every $u \in U_0$. Show that there is a linear operator $T : U \to L^0$, extending T_0, such that $Tu \le p(u)$ for every $u \in U$.

Exercises 368Xb-368Xf are now 368Xc-368Xg.

p 399 l 30 (369Xb) for '$\phi(\alpha x + (1-\alpha)y) \ge \alpha\phi(x) + (1-\alpha)\phi(y)$' read '$\phi(\alpha x + (1-\alpha)y) \le \alpha\phi(x) + (1-\alpha)\phi(y)$'.

p 400 l 21 The exercises for §369 have been rearranged; 369Xh-369Xk are now 369Xj-369Xm, 369Xl-369Xm are now 369Xh-369Xi, 369Yc-369Yf are now 369Yd-369Yg, 369Yg is now 369Yc.

p 400 l 27 (369Xj, now 369Xl) To show that $\| \|_{1,\infty}$ cannot be represented as an Orlicz norm, we need a further hypothesis on the measure algebra $(\mathfrak{A}, \bar{\mu})$; e.g., that $\mathfrak{A}$ is atomless and $\bar{\mu}$ is not totally finite.

p 401 l 28 Exercise 369Ye, now 369Yf, is wrong as stated; it now reads

(f) Let $(\mathfrak{A},\bar{\mu})$ be a semi-finite measure algebra and $\phi : [0,\infty[\ \to [0,\infty[$ be a strictly increasing Young's function such that $\sup_{t>0}\phi(2t)/\phi(t)$ is finite. Show that if $\mathcal{F}$ is a filter on L^{τ_ϕ}, then $\mathcal{F} \to u \in L^{\tau_\phi}$ for the norm τ_ϕ iff (i) $\mathcal{F} \to u$ for the topology of convergence in measure (ii) $\limsup_{v\to\mathcal{F}} \tau_\phi(v) \le \tau_\phi(u)$.

p 401 l 31 Exercise 369Yf, now 369Yg, has been elaborated, and now reads

(g) Give examples of extended Fatou norms τ on measure spaces $L^0(\mathfrak{A})$, where $(\mathfrak{A},\bar{\mu})$ is a semi-finite measure algebra, such that (α) $\tau\restriction L^\tau$ is order-continuous (β) there is a sequence $\langle u_n\rangle_{n\in\mathbb{N}}$ in L^τ, converging in measure to $u \in L^\tau$, such that $\lim_{n\to\infty}\tau(u_n) = \tau(u)$ but $\langle u_n\rangle_{n\in\mathbb{N}}$ does not converge to u for the norm on L^τ. Do this (i) with τ an Orlicz norm (ii) with $(\mathfrak{A},\bar{\mu})$ the measure algebra of Lebesgue measure on $\mathbb{R}$.

p 407 l 5 (371X) Exercises 371Xb (on weakly compact linear operators) and 371Ya (on compact linear operators) have been exchanged.

p 408 l 26 (proof of 372A): for '$h(u_n) = u$' read '$h(u_n) = h(u)$'.

p 411 l 10 372E (the second form of the Ergodic Theorem) and Corollary 372F have been exchanged.

p 412 l 1 Lemma 372H is now 364Qd. 372I-372O are now 372H-372N.

p 415 l 34 (part (b) of the proof of 372N, now 372M): for 'ν is inverse-measure-preserving.' read 'ϕ is inverse-measure-preserving.'.

p 417 l 12 I have changed the definition of 'ergodic Boolean homomorphism' in 372P (now 372O), and now say

(a)(i) Let $\mathfrak{A}$ be a Boolean algebra. Then a Boolean homomorphism $\pi : \mathfrak{A} \to \mathfrak{A}$ is **ergodic** if whenever a, $b \in \mathfrak{A}\setminus\{0\}$ there are m, $n \in \mathbb{N}$ such that $\pi^m a \cap \pi^n b \ne 0$.

To relate this to the previous definition, I interpolate a new paragraph:

372P Proposition Let $\mathfrak{A}$ be a Boolean algebra and $\pi : \mathfrak{A} \to \mathfrak{A}$ a Boolean homomorphism, with fixed-point subalgebra $\mathfrak{C}$.

(a) If π is ergodic, then $\mathfrak{C} = \{0,1\}$.

(b) If π is an automorphism, then π is ergodic iff $\sup_{n\in\mathbb{Z}}\pi^n a = 1$ for every $a \in \mathfrak{A}\setminus\{0\}$.

(c) If π is an automorphism and $\mathfrak{A}$ is Dedekind σ-complete, then π is ergodic iff $\mathfrak{C} = \{0,1\}$.

The remarks in 372Pc are now in 372R; 372R is now 372S.

In addition there are new definitions in 372O:

(a)(iii) Let $(\mathfrak{A},\bar{\mu})$ be a probability algebra and $\pi : \mathfrak{A} \to \mathfrak{A}$ a measure-preserving Boolean homomorphism. Then π is **weakly mixing** if $\lim_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n-1}|\bar{\mu}(\pi^n a \cap b) - \bar{\mu}a\cdot\bar{\mu}b| = 0$ for all a, $b \in \mathfrak{A}$.

***(b)(iii)** ϕ is **weakly mixing** if $\lim_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n-1}|\mu(F\cap\phi^{-n}[E]) - \mu E\cdot\mu F| = 0$ for all E, $F \in \Sigma$.

p 417 l 30 372Q(a-i) is now

(i) If π is mixing, it is weakly mixing.

(ii) If π is weakly mixing, it is ergodic.

372Q(a-ii) is now 372Q(a-iii). Two more fragments have been added to 372Qa:

(iv) The following are equiveridical: (α) π is mixing; (β) $\lim_{n\to\infty}(T^n u|v) = \int u\int v$ for all u, $v \in L^2(\mathfrak{A},\bar{\mu})$.

(v) The following are equiveridical: (α) π is weakly mixing; (β) $\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n-1}|(T^k u|v) - \int u\int v| = 0$ for all u, $v \in L^2(\mathfrak{A},\bar{\mu})$.

Similarly, 372Q(b-ii) is now

(ii) ϕ is mixing iff π is, and in this case ϕ is weakly mixing.

(iii) ϕ is weakly mixing iff π is, and in this case ϕ is ergodic.

p 419 l 38 The exercises to §372 have been rearranged, as follows: 372Xh is now 372Xi, 372Xi is now 372Xj, 372Xj is now 372Xl, 372Xk-372Xo are now 372Xn-372Xr, 372Xp and 372Xq have been collected into 372Xs, 372Xr-372Xv are now 372Xt-372Xx, 372Xw is now 372Xm, 372Yd is now 372Ye, 372Ye is now 372Yg, 372Yf is now 372Yh, 372Yg is now 372Yo, 372Yh-372Yk are now 372Yj-372Ym, 372Yl is now 372Yi, 372Ym is now 372Yf, 372Yo is now 372Yd.

p 419 l 38 (Exercise 372Xh, now 372Xi): for '1, 2, 1, 2, ... ' read '1, 2, 2, 2, ... '.

p 420 l 4 (Exercise 372Xj, now 372Xl): for '$|x-r_n(x)| \le 1/q_n(x)^2k_n(x)$' read '$|x-r_n(x)| \le 1/q_n(x)^2k_{n+1}(x)$'.

p 420 l 10 Exercise 372Xl (now 372Xo) now reads

(o) Let $(\mathfrak{A}, \bar{\mu})$ be a probability algebra, and $\pi : \mathfrak{A} \to \mathfrak{A}$ a measure-preserving Boolean homomorphism. (i) Show that if $n \ge 1$ then π is mixing iff π^n is mixing. (ii) Show that if $n \ge 1$ then π is weakly mixing iff π^n is weakly mixing. (iii) Show that if $n \ge 1$ and π^n is ergodic then π is ergodic. (iv) Show that if π is an automorphism then it is ergodic, or mixing, or weakly mixing, iff π^{-1} is.

p 420 l 27 Part (v) of exercise 372Xo (now 372Xr) has been changed to 'Show that ϕ is not weakly mixing'.

p 421 l 10 (372X) Add new exercises:

(k) Set $x = (\sqrt{5} - 1)/2$. Show that, in the notation of 372L, $k_n(x) = 1$ and $q_n(x) = p_{n-1}(x)$ for every $n \ge 1$ and that $\langle p_n(x) \rangle_{n \in \mathbb{N}}$ is the Fibonacci sequence.

(y)(i) Let $\mathfrak{A}$ be a Boolean algebra, $\pi : \mathfrak{A} \to \mathfrak{A}$ a Boolean homomorphism, and $\phi : \mathfrak{A} \to \mathfrak{A}$ a Boolean automorphism. Show that if π is ergodic then $\phi\pi\phi^{-1}$ is ergodic. (ii) Let $(\mathfrak{A}, \bar{\mu})$ be a probability algebra, $\pi : \mathfrak{A} \to \mathfrak{A}$ a measure-preserving Boolean homomorphism, and $\phi : \mathfrak{A} \to \mathfrak{A}$ a measure-preserving Boolean automorphism. Show that if π is mixing, or weakly mixing, then so is $\phi\pi\phi^{-1}$.

p 422 l 1 Exercise 372Yh (now 372Yj) has a further part

(iii) Show that if every ϕ_i is weakly mixing so is ϕ.

p 422 l 12 Exercise 372Yl (now 372Yi) has been changed to

(i)(i) Let (X, Σ, μ) and (Y, T, ν) be probability spaces, with c.l.d. product $(X \times Y, \Lambda, \lambda)$. Suppose that $\phi : X \to X$ is a weakly mixing inverse-measure-preserving function and $\psi : Y \to Y$ is an ergodic inverse-measure-preserving function. Define $\theta : X \times Y \to X \times Y$ by setting $\theta(x, y) = (\phi(x), \psi(y))$ for all x, y. Show that θ is an ergodic inverse-measure-preserving function. (ii)(α) Let $(\mathfrak{A}, \bar{\mu})$ and $(\mathfrak{B}, \bar{\nu})$ be probability algebras, with probability algebra free product $(\mathfrak{C}, \bar{\lambda})$. Suppose that $\phi : \mathfrak{A} \to \mathfrak{A}$ is a weakly mixing measure-preserving Boolean homomorphism and $\psi : \mathfrak{B} \to \mathfrak{B}$ is an ergodic measure-preserving Boolean homomorphism. Let $\theta : \mathfrak{C} \to \mathfrak{C}$ be the measure-preserving Boolean homomorphism such that $\theta(a \otimes b) = \phi a \otimes \psi b$ for all $a \in \mathfrak{A}$ and $b \in \mathfrak{B}$ (325Xe). Show that θ is ergodic. (β) Show that if ψ is weakly mixing then θ is weakly mixing. (γ) Show that if ϕ and ψ are mixing then θ is mixing.

p 422 l 25 Exercise 372Yn(i): for 'mixing' read 'weakly mixing'.

p 422 l 34 (372Y) Add new exercises:

(p) (i) Show that there are a Boolean algebra $\mathfrak{A}$ and a Boolean automorphism $\pi : \mathfrak{A} \to \mathfrak{A}$ which is not ergodic, but has fixed-point algebra $\{0, 1\}$. (ii) Show that there are a σ-finite measure algebra $(\mathfrak{A}, \bar{\mu})$ and a measure-preserving Boolean homomorphism $\pi : \mathfrak{A} \to \mathfrak{A}$ which is not ergodic, but has fixed-point algebra $\{0, 1\}$.

(q) For a Boolean algebra $\mathfrak{A}$ and a Boolean homomorphism $\pi : \mathfrak{A} \to \mathfrak{A}$, write T_π for the corresponding operator from $L^\infty = L^\infty(\mathfrak{A})$ to itself, as defined in 363F. (i) Suppose that $\mathfrak{A}$ is a Boolean algebra, $\pi : \mathfrak{A} \to \mathfrak{A}$ is a Boolean homomorphism, $u \in L^\infty$ and $T_\pi u = u$. Show that if either π is ergodic or $\mathfrak{A}$ is Dedekind σ-complete and the fixed-point subalgebra of π is $\{0, 1\}$, then u must be a multiple of $\chi 1$. (ii) Find a Boolean algebra $\mathfrak{A}$, an automorphism $\pi : \mathfrak{A} \to \mathfrak{A}$ with fixed-point algebra $\{0, 1\}$, and a $u \in L^\infty$, not a multiple of $\chi 1$, such that $T_\pi u = u$.

(r) Set $\mathcal{F}_d = \{I : I \subseteq \mathbb{N}, \lim_{n\to\infty} \frac{1}{n}\#(I \cap n) = 1\}$. (i) Show that $\mathcal{F}_d$ is a filter on $\mathbb{N}$. (ii) Show that for a bounded sequence $\langle\alpha_n\rangle_{n\in\mathbb{N}}$ in $\mathbb{R}$, the following are equiveridical: (α) $\lim_{n\to\mathcal{F}_d} \alpha_n = 0$; ($\beta$) $\lim_{n\to\infty} \frac{1}{n+1}\sum_{k=0}^n |\alpha_k| = 0$; ($\gamma$) $\lim_{n\to\infty} \frac{1}{n+1}\sum_{k=0}^n \alpha_k^2 = 0$. ($\mathcal{F}_d$ is called the **(asymptotic) density filter**.)

p 428 l 26 Part (b-iv) of the proof of 373F should be replaced by the following:
By (a),

$$\|u^*\|_{\infty,1} = \max(|u^*\|_\infty, \|u^*\|_1) = \max(|u\|_\infty, \|u\|_1) = \|u\|_{\infty,1}$$

whenever any of these is finite.

p 431 l 13 (proof of 373L): for '$\sup_{c\in\mathfrak{B}^f, c\subseteq 1\setminus e}(-Su)$' read '$\sup_{c\in\mathfrak{B}^f, c\subseteq 1\setminus e}\int_c(-Su)$'.

p 432 l 10 (part (c) of the proof of 373O): for '$u \preccurlyeq u^*$' read '$u^* \preccurlyeq u$'.

p 434 l 4 (part (g) of the proof of 373O): for '$\beta_i = \sum_{j=1}^i \bar\mu a_j$ for each j' read '$\beta_i = \sum_{j=1}^i \bar\mu a_j$ for each i'.

p 436 ll 12-14 (proof of 373Q): for '$\mathcal{T}_{\bar\mu,\bar\mu}$' read '$\mathcal{T}_{\bar\mu,\bar\nu}$' (three times).

p 437 l 8 (proof of 373R): for '355F(iii)' read '355J'.

p 437 l 31 (part (c) of the proof of 373S): for '$\mathcal{T}_{\bar\mu,\bar\mu}$' read '$\mathcal{T}_{\bar\mu,\bar\nu}$'.

p 438 l 21 (proof of 373T): for '$v \in M_{\bar\nu}^{1,0}$' read '$v_0 \in M_{\bar\nu}^{1,0}$',

p 439 l 14 Exercise 373Xg: for '$g \in \mathcal{L}^1(\mu) + \mathcal{L}^\infty(\mu)$' read '$f \in \mathcal{L}^1(\mu) \cup \mathcal{L}^\infty(\mu)$'.

p 443 l 6 (part (d) of the proof of 374B): for '$v \in M_{\bar\mu}^{0,\infty}$' read '$v \in M_{\bar\mu}^{1,\infty}$'.

p 443 l 24 (part (a-i) of the proof of 374C): for '$w \in M_{\bar\mu}^{1,\infty}$' read '$w \in M_{\bar\mu_L}^{1,\infty}$'.

p 443 l 32 (part (a-i) of the proof of 374C): for '$\sup\{\int |u \times Tw| : T \in \mathcal{T}_{\bar\mu_L,\bar\mu_L}\}$' read '$\sup\{\int |u \times Tw| : T \in \mathcal{T}_{\bar\mu_L,\bar\mu}\}$'.

p 448 l 6 The proof of 374L has serious errors, starting with the claim '$2^n\gamma \le \bar\mu[\![u \ge \alpha_n]\!]$' (p 448 l 9); the following is I hope correct.

proof I take three cases separately.

(a) Suppose that $\mathfrak{A}$ is purely atomic; then u, $v \in L^\infty(\mathfrak{A})$ and u^*, $v^* \in L^\infty(\mathfrak{A}_L)$, so neither u^* nor v^* can belong to $L^1_{\bar\mu_L}$ and neither u nor v can belong to $L^1_{\bar\mu}$. Let γ be the common measure of the atoms of $\mathfrak{A}$. For each $n \in \mathbb{N}$, set

$$\alpha_n = \inf\{\alpha : \alpha \ge 0,\ \bar\mu[\![u > \alpha]\!] \le 3^n\gamma\}, \quad \tilde a_n = [\![u > \tfrac{1}{2}\alpha_n]\!].$$

Then $\bar\mu[\![u > \alpha_n]\!] \le 3^n\gamma$; also $\alpha_n > 0$, since otherwise u would belong to $L^1_{\bar\mu}$, so $\bar\mu\tilde a_n \ge 3^n\gamma$. We can therefore choose $\langle a'_n\rangle_{n\in\mathbb{N}}$ inductively such that $a'_n \subseteq \tilde a_n$ and $\bar\mu a'_n = 3^n\gamma$ for each n (using 374Ib). For each $n \ge 1$, set $a''_n = a'_n \setminus \sup_{i<n} a'_i$; then $\bar\mu a''_n \ge \frac{1}{2}\cdot 3^{-n}\gamma$, so we can choose an $a_n \subseteq a''_n$ such that $\bar\mu a_n = 3^{n-1}\gamma$.

Also, of course, $\langle\alpha_n\rangle_{n\in\mathbb{N}}$ is non-increasing. We now see that

$$\langle a_n\rangle_{n\ge1} \text{ is disjoint}, \quad u \ge \frac{1}{2}\alpha_n\chi a_n \text{ for every } n \ge 1,$$

$$u^* \le \|u\|_\infty\chi\,[0,\gamma[^\bullet \vee \sup_{n\in\mathbb{N}} \alpha_n\chi\,[3^n\gamma, 3^{n+1}\gamma[^\bullet.$$

Similarly, there are a non-increasing sequence $\langle\beta_n\rangle_{n\in\mathbb{N}}$ in $[0,\infty[$ and a disjoint sequence $\langle b_n\rangle_{n\ge1}$ in $\mathfrak{A}$ such that

$$\bar\mu b_n = 3^{n-1}\gamma, \quad v \ge \frac{1}{2}\beta_n\chi b_n \text{ for every } n \ge 1,$$

$$v^* \le \|v\|_\infty\chi\,[0,\gamma[^\bullet \vee \sup_{n\in\mathbb{N}} \beta_n\chi\,[3^n\gamma, 3^{n+1}\gamma[^\bullet.$$

We are supposing that

$$\infty = \int u^* \times v^* = \gamma\|u\|_\infty\|v\|_\infty + \sum_{n=0}^{\infty} 2\cdot 3^n\gamma\alpha_n\beta_n$$

$$= \gamma\|u\|_\infty\|v\|_\infty + 2\gamma\alpha_0\beta_0 + 2\gamma\sum_{n=0}^{\infty} 3^{2n+1}(\alpha_{2n+1}\beta_{2n+1} + 3\alpha_{2n+2}\beta_{2n+2})$$

$$\le \gamma\|u\|_\infty\|v\|_\infty + 2\gamma\alpha_0\beta_0 + 24\sum_{n=0}^{\infty} 3^{2n}\gamma\alpha_{2n+1}\beta_{2n+1},$$

so $\sum_{n=0}^{\infty} 3^{2n}\alpha_{2n+1}\beta_{2n+1} = \infty$.

At this point, recall that we are dealing with a purely atomic algebra in which every atom has measure γ. Let A_n, B_n be the sets of atoms included in a_n, b_n for each $n \ge 1$, and $A = \bigcup_{n\ge 1} A_n \cup B_n$. Then $\#(A_n) = \#(B_n) = 3^{n-1}$ for each $n \ge 1$. We therefore have a permutation $\phi : A \to A$ such that $\phi[B_{2n+1}] = A_{2n+1}$ for every n. (The point is that $A \setminus \bigcup_{n\in\mathbb{N}} A_{2n+1}$ and $A \setminus \bigcup_{n\in\mathbb{N}} B_{2n+1}$ are both countably infinite.) Define $\pi : \mathfrak{A} \to \mathfrak{A}$ by setting

$$\pi c = (c \setminus \sup A) \cup \sup_{a\in A, a\subseteq c} \phi a$$

for $c \in \mathfrak{A}$. Then π is well-defined (because A is countable), and it is easy to check that it is a measure-preserving Boolean automorphism (because it is just a permutation of the atoms); and $\pi b_{2n+1} = a_{2n+1}$ for every n. Consequently

$$\int u \times T_\pi v \ge \sum_{n=0}^{\infty} \frac{1}{4}\alpha_{2n+1}\beta_{2n+1}\bar\mu a_{2n+1} = \frac{1}{4}\gamma\sum_{n=0}^{\infty} 3^{2n}\alpha_{2n+1}\beta_{2n+1} = \infty.$$

So we have found a suitable automorphism.

(b) Next, consider the case in which $(\mathfrak{A}, \bar\mu)$ is atomless and of finite magnitude γ. Of course $\gamma > 0$. For each $n \in \mathbb{N}$ set

$$\alpha_n = \inf\{\alpha : \alpha \ge 0,\ \bar\mu[\![u > \alpha]\!] \le 3^{-n}\gamma\}, \quad \tilde a_n = [\![u > \frac{1}{2}\alpha_n]\!].$$

Then $\langle\alpha_n\rangle_{n\in\mathbb{N}}$ is non-decreasing and

$$u^* \le \sup_{n\in\mathbb{N}} \alpha_{n+1}\chi\,[3^{-n-1}\gamma, 3^{-n}\gamma[^\bullet.$$

This time, $\bar\mu\tilde a_n \ge 3^{-n}\gamma$, and we are in an atomless measure algebra, so we can choose $a'_n \subseteq \tilde a_n$ such that $\bar\mu a'_n = 3^{-n}\gamma$; taking $a''_n = a'_n \setminus \sup_{i>n} a'_i$, $\bar\mu a''_n \ge \frac{1}{2}\cdot 3^{-n}\gamma$, and we can choose $a_n \subseteq a''_n$ such that $\bar\mu a_n = 3^{-n-1}\gamma$ for every n. As before, $u \ge \frac{1}{2}\alpha_n\chi a_n$ for every n, and $\langle a_n\rangle_{n\in\mathbb{N}}$ is disjoint.

In the same way, we can find $\langle\beta_n\rangle_{n\in\mathbb{N}}$, $\langle b_n\rangle_{n\in\mathbb{N}}$ such that $\langle b_n\rangle_{n\in\mathbb{N}}$ is disjoint,

$$v^* \le \sup_{n\in\mathbb{N}} \beta_{n+1}\chi\,[3^{-n-1}\gamma, 3^{-n}\gamma[^\bullet, \quad v \ge \sup_{n\in\mathbb{N}} \frac{1}{2}\beta_n\chi b_n$$

and $\bar\mu b_n = 3^{-n-1}\gamma$ for each n. In this case, we have

$$\infty = \int u^* \times v^* \le \sum_{n=0}^{\infty} 2\cdot 3^{-n-1}\gamma\alpha_{n+1}\beta_{n+1},$$

and $\sum_{n=0}^{\infty} 3^{-n}\alpha_n\beta_n$ is infinite.

Now all the principal ideals $\mathfrak{A}_{a_n}$, $\mathfrak{A}_{b_n}$ are homogeneous and of the same Maharam type, so there are measure-preserving isomorphisms $\pi_n : \mathfrak{A}_{b_n} \to \mathfrak{A}_{a_n}$; similarly, setting $\tilde a = 1 \setminus \sup_{n\in\mathbb{N}} a_n$ and $\tilde b = 1 \setminus \sup_{n\in\mathbb{N}} b_n$, there is a measure-preserving isomorphism $\tilde\pi : \mathfrak{A}_{\tilde b} \to \mathfrak{A}_{\tilde a}$. Define $\pi : \mathfrak{A} \to \mathfrak{A}$ by setting

$$\pi c = \tilde\pi(c \cap \tilde b) \cup \sup_{n\in\mathbb{N}} \pi_n(c \cap a_n)$$

for every $c \in \mathfrak{A}$; then π is a measure-preserving automorphism of $\mathfrak{A}$, and $\pi b_n = a_n$ for each n. In this case,

$$\int u \times T_\pi v \ge \frac{1}{4}\sum_{n=0}^{\infty} 3^{-n-1}\gamma\alpha_n\beta_n = \infty,$$

and again we have a suitable automorphism.

(c) Thirdly, consider the case in which $\mathfrak{A}$ is atomless and not totally finite; take κ to be the common Maharam type of all the principal ideals $\mathfrak{A}_a$ where $0<\bar{\mu}a<\infty$. In this case, set

$$\alpha_n=\inf\{\alpha:\bar{\mu}[\![u>\alpha]\!]\le 3^n\},\quad \beta_n=\inf\{\alpha:\bar{\mu}[\![v>\alpha]\!]\le 3^n\}$$

for each $n\in\mathbb{Z}$. This time

$$u^*\le\sup_{n\in\mathbb{Z}}\alpha_n\chi\left[3^n,3^{n+1}\right[^\bullet,\quad v^*\le\sup_{n\in\mathbb{Z}}\beta_n\chi\left[3^n,3^{n+1}\right[^\bullet,$$

so

$$\infty=\int u^*\times v^*=2\sum_{n=-\infty}^{\infty}3^n\alpha_n\beta_n\le 8\sum_{n=-\infty}^{\infty}3^{2n}\alpha_{2n}\beta_{2n}.$$

For each $n\in\mathbb{Z}$, $3^n\le\bar{\mu}[\![u>\frac{1}{2}\alpha_n]\!]$, so there is an a''_n such that

$$a''_n\subseteq[\![u>\tfrac{1}{2}\alpha_n]\!],\quad \bar{\mu}a''_n=3^n.$$

Set $a'_n=a''_n\setminus\sup_{-\infty<i<n}a''_i$; then $\bar{\mu}a'_n\ge\dfrac{1}{2}\cdot 3^n$ for each n; choose $a_n\subseteq a'_n$ such that $\bar{\mu}a_n=3^{n-1}$. Then $\langle a_n\rangle_{n\in\mathbb{N}}$ is disjoint and $u\ge\dfrac{1}{2}\alpha_n\chi a_n$ for each n.

Similarly, there is a disjoint sequence $\langle b_n\rangle_{n\in\mathbb{N}}$ such that

$$\bar{\mu}b_n=3^{n-1},\quad v\ge\frac{1}{2}\beta_n\chi b_n$$

for each $n\in\mathbb{N}$.

Set $d^*=\sup_{n\in\mathbb{Z}}a_n\cup\sup_{n\in\mathbb{Z}}b_n$. Then

$$\tilde{a}=d^*\setminus\sup_{n\in\mathbb{Z}}a_{2n},\quad \tilde{b}=d^*\setminus\sup_{n\in\mathbb{Z}}b_{2n}$$

both have magnitude ω and Maharam type κ. So there is a measure-preserving isomorphism $\tilde{\pi}:\mathfrak{A}_{\tilde{b}}\to\mathfrak{A}_{\tilde{a}}$ (332J). At the same time, for each $n\in\mathbb{Z}$ there is a measure-preserving isomorphism $\pi_n:\mathfrak{A}_{b_{2n}}\to\mathfrak{A}_{a_{2n}}$. So once again we can assemble these to form a measure-preserving automorphism $\pi:\mathfrak{A}\to\mathfrak{A}$, defined by the formula

$$\pi c=(c\setminus d^*)\cup\tilde{\pi}(c\cap\tilde{b})\cup\sup_{n\in\mathbb{Z}}\pi_n(c\cap b_{2n}).$$

Just as in (a) and (b) above,

$$\int u\times T_\pi v\ge\sum_{n=-\infty}^{\infty}\frac{1}{4}\cdot 3^{2n-1}\alpha_{2n}\beta_{2n}=\infty.$$

Thus we have a suitable π in any of the cases allowed by 374H.

p 450 l 36 Part (ii) of Exercise 374Xh is wrong, and should be deleted.

p 453 l 28 Add new result:

375E Theorem Let $(\mathfrak{A},\bar{\mu})$ be a semi-finite measure algebra, $(\mathfrak{B},\bar{\nu})$ any measure algebra, and $T:L^0(\mathfrak{A})\to L^0(\mathfrak{B})$ an order-continuous positive linear operator. Then T is continuous for the topologies of convergence in measure.

375E-375K are now 375F-375L.

p 456 l 25 (part (e) of the proof of 375I, now 375J): for '$d\mapsto\phi(d\cap a^\bullet)$' read '$d\mapsto b\cap\phi(d\cap a^\bullet)$'.

p 457 l 31 (375X) Add new exercise:

(h) Let $\mathfrak{A}$ and $\mathfrak{B}$ be Dedekind σ-complete Boolean algebras, and $T:L^0(\mathfrak{A})\to L^0(\mathfrak{B})$ a Riesz homomorphism. Show that there are a sequentially order-continuous ring homomorphism $\pi:\mathfrak{A}\to\mathfrak{B}$ and a $w\in L^0(\mathfrak{A})^+$ such that $Tu=w\times T_\pi u$ for every $u\in L^0(\mathfrak{A})$, where $T_\pi:L^0(\mathfrak{A})\to L^0(\mathfrak{B})$ is defined as in 364Yg.

p 457 l 32 Exercise 375Yd is wrong, and has been replaced by the reverse exercise:

(d) Let $\mathfrak{A}$ be the measure algebra of Lebesgue measure on $[0,1]$, and set $L^0=L^0(\mathfrak{A})$. Show that there is a positive linear operator $T:L^0\to L^0$ such that $T[L^0]$ is not order-closed in L^0.

Add new exercises:

(a) Let $\mathfrak{A}$ and $\mathfrak{B}$ be Dedekind σ-complete Boolean algebras, and $T : L^0(\mathfrak{A}) \to L^0(\mathfrak{B})$ a linear operator. (i) Show that if T is order-bounded, then $\langle Tu_n\rangle_{n\in\mathbb{N}}$ order*-converges to 0 in $L^0(\mathfrak{B})$ (definition: 367A) whenever $\langle u_n\rangle_{n\in\mathbb{N}}$ order*-converges to 0 in $L^0(\mathfrak{A})$. (ii) Show that if $\mathfrak{B}$ is ccc and weakly (σ,∞)-distributive and $\langle Tu_n\rangle_{n\in\mathbb{N}}$ order*-converges to 0 in $L^0(\mathfrak{B})$ whenever $\langle u_n\rangle_{n\in\mathbb{N}}$ order*-converges to 0 in $L^0(\mathfrak{A})$, then T is order-bounded.

(d) Show that the following are equiveridical: (i) there is a probability space (X,Σ,μ) such that $\Sigma = \mathcal{P}X$ and $\mu\{x\} = 0$ for every $x \in X$; (ii) there are localizable measure algebras $(\mathfrak{A},\bar\mu)$ and $(\mathfrak{B},\bar\nu)$ and a positive linear operator $T : L^0(\mathfrak{A}) \to L^0(\mathfrak{B})$ which is not order-continuous.

(f) Let $\mathfrak{A}$, $\mathfrak{B}$ be Dedekind σ-complete Boolean algebras of which $\mathfrak{B}$ is weakly σ-distributive. Let $\phi : \mathfrak{A} \to \mathfrak{B}$ be a σ-subhomomorphism such that $\pi a \ne 0$ whenever $a \in \mathfrak{A} \setminus \{0\}$. Show that $\mathfrak{A}$ is weakly σ-distributive.

375Ya-375Yb are now 375Yb-375Yc, 375Yc is now 375Yf, 375Ye-375Yf are now 375Yh-375Yi.

p 468 l 28 In Lemma 376L, the proof as written assumes that U is a Banach lattice, and the simplest fix is to put this into the hypothesis.

p 470 l 34 (part (b) of the proof of 376P): for 'by 368Pc, V is weakly (σ,∞)-distributive' read 'by 368Pc, $U^\times$ is weakly (σ,∞)-distributive'.

p 475 l 1 Exercise 376Xn (now 376Xo) is in a muddle, and should read

(o) Suppose, in 376Xk (formerly 376Xj), that $U = L^\tau$ for some extended Fatou norm on $L^0(\mu)$ and that $V = L^1(\nu)$, so that $V^\# = L^\infty(\nu)$. Set $k_y(x) = k(x,y)$ whenever this is defined, $w_y = k_y^\bullet$ whenever $k_y \in \mathcal{L}^0(\mu)$. Show that $w_y \in L^{\tau'}$ for almost every $y \in Y$, and that the norm of T in $\mathsf{B}(L^\tau; L^\infty)$ is $\operatorname{ess\,sup}_y \tau'(w_y)$.

p 475 l 35 Oops! Exercise 376Yg should end 'Show that T satisfies the conditions (ii) and (iii) of 376J but does not belong to $\mathsf{L}^\times(L^\infty(\mu); L^\infty(\nu))$'.

p 475 l 39 Exercise 376Yi adds nothing to 376Ma, and 376Yk is wrong; these have been deleted. 376Yn has been moved to 376Xi. Other exercises have been renamed: 376Xi-376Xn are now 376Xj-376Xo, 376Yl is now 376Yk and 376Ym is now 376Yi.

p 477 l 31 There is a new section §377, 'Function spaces of reduced products', continuing from the new §328.

p 478 l 43 (381Bd) Delete the phrase 'and as periodic with period 1', which contradicts 381Bc.

p 480 l 2 (part (c) of the proof of 381E): for '$\pi d = \sup_{b\in B} \pi(d\setminus b)$' read '$\phi d = \sup_{b\in B} \phi(d\setminus b)$', and in the next line again read 'ϕ' for 'π'.

p 480 l 9 (part (g) of the proof of 381E): the reference to (f) should be to (e). And to justify the second assertion add

If $c = \operatorname{supp}\pi^n$ then πc supports π^n so $c \subseteq \pi c$. Consequently $\pi^i c \subseteq \pi^{i+1} c$ for every $i \in \mathbb{N}$ and $c \subseteq \pi c \subseteq \pi^n c$. But as $\pi^n c = c$, by (a), $\pi c = c$.

p 480 l 21 (part (k) of the proof of 381E): for the final '$\pi_2\pi\pi_2^{-1}$' read '$\pi_2\phi\pi_2^{-1}$'.

p 480 l 31 (part (b) of the proof of 381G) The remarks printed are unhelpful. The result is immediate from 381Ei.

p 481 l 6 (proof of 381H): for '$\operatorname{supp}\pi^n d$' read '$\operatorname{supp}\pi^n$'.

p 481 l 7 (proof of 381H): for '$c_n \ne 0$' read '$c_j \ne 0$'.

p 481 l 15 The former exercise 381Xe is now part (a) of 381I, as follows:

(a) Let G be a subgroup of $\operatorname{Aut}\mathfrak{A}$. Let H be the set of those $\pi \in \operatorname{Aut}\mathfrak{A}$ such that for every non-zero $a \in \mathfrak{A}$ there are a non-zero $b \subseteq a$ and a $\phi \in G$ such that $\pi c = \phi c$ for every $c \subseteq b$. Then H is a full subgroup of $\operatorname{Aut}\mathfrak{A}$, the smallest full subgroup of $\mathfrak{A}$ including G.

p 482 l 22 I have rewritten the last part of the proof of 381I as follows:

(ii)⇔(iv) The point is that, for $n \in \mathbb{Z}$ and $b \in \mathfrak{A}$,

$$\begin{aligned}\phi c = \pi^n c \text{ for every } c \subseteq b &\iff \pi^{-n}\phi c = c \text{ for every } c \subseteq b\\ &\iff b \cap \operatorname{supp}(\pi^{-n}\phi) = 0.\end{aligned}$$

So we have

$$\begin{aligned}\text{(ii)} &\iff \forall\, a \in \mathfrak{A} \setminus \{0\}\ \exists\, n \in \mathbb{Z},\ b \text{ such that } 0 \ne b \subseteq a \text{ and } b \cap \operatorname{supp}(\pi^{-n}\phi) = 0\\ &\iff \forall\, a \in \mathfrak{A} \setminus \{0\}\ \exists\, n \in \mathbb{Z},\ a \setminus \operatorname{supp}(\pi^{-n}\phi) \ne 0\\ &\iff \inf_{n\in\mathbb{Z}} \operatorname{supp}(\pi^{-n}\phi) = 0,\end{aligned}$$

as required.

p 484 l 3 (proof of 381M): for '$a \cap \sup_{n\ge1}(\pi^n a \setminus \sup_{1\le i<n} \pi^i)(a)$' read '$a \cap \sup_{n\ge1}(\pi^n a \setminus \sup_{1\le i<n} \pi^i a)$'.

p 484 l 35 (part (c) of the proof of 381N): for '$j = n$' read '$j = 1$'.

p 484 l 38 (part (c) of the proof of 381N): for '$0 \ne c \subseteq b$' read '$0 \ne b' \subseteq b$'.

p 484 l 39 (part(c) of the proof of 381N): for '$\pi^i\pi^{n-i}d = \pi_a\pi_a^j d$' read '$\pi^{n-i}\pi^i d = \pi_a^j\pi_a d$'.

p 486 l 27 (part (a) of the proof of 381P) for '$a \cap \pi^n a$ witnesses' read 'b witnesses'.

p 486 l 35 (part (b-ii) of the proof of 381P) for 'b witnesses' read '$a \cap \pi^n a$ witnesses'.

p 487 l 8 In the statement of 381Qe we need to suppose that $\mathfrak{A}$ is Dedekind σ-complete (in order to have a well-defined induced automorphism π_a). And for 'automorphism on $\operatorname{Aut}\mathfrak{A}_a$' read 'automorphism on $\mathfrak{A}_a$'.

p 488 l 9 (part (e) of the proof of 381Q) for 'π_k' read 'π^k' (twice).

p 489 l 4 In Lemma 381Sa, $a \cup b$ is actually the support of $(\overleftarrow{a\ {}_\pi\ b})$.
Add new part:

(d) If G is a countably full subgroup of $\operatorname{Aut}\mathfrak{A}$, $a_1, \dots, a_n \in \mathfrak{A}$ are disjoint, and $\pi_1, \dots, \pi_{n-1} \in G$, then $(\overleftarrow{a_1\ {}_{\pi_1}\ a_2\ {}_{\pi_2}\ \dots\ {}_{\pi_{n-1}}\ a_n}) \in G$.

p 489 l 42 In Exercise 381Xb, part (iii) should read

Show that if *either* I is finite *or* I is countable and $\mathfrak{A}$ is Dedekind σ-complete *or* $\mathfrak{A}$ is Dedekind complete, then π is surjective iff every π_i is.

And add new part

(iv) Show that π is order-continuous, or sequentially order-continuous, iff every π_i is.

p 490 l 5 Exercise 381Xf has been deleted, and other exercises have been re-arranged; 381Xe is now 381Ia, 381Xg-381Xi are now 381Xi-381Xk, 381Xj is now 381Xg, 381Xk-381Xm are now 381Xm-381Xo, 381Xn is now 381Xe, 381Xo is now 381Xf, 381Xp is now 381Xl, 381Ya is now 381Yb, 381Yb is now 381Ya.

p 489 l 42 In Exercise 381Xb, part (iii) should read

show that if *either* I is finite *or* I is countable and $\mathfrak{A}$ is Dedekind σ-complete *or* $\mathfrak{A}$ is Dedekind complete, then π is surjective iff every π_i is.

p 490 l 13 (381X) New exercises have been added:

(h) Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra, G a subgroup of $\operatorname{Aut}\mathfrak{A}$ and $\phi \in \operatorname{Aut}\mathfrak{A}$. Show that ϕ belongs to the full subgroup of $\operatorname{Aut}\mathfrak{A}$ generated by G iff $\inf_{\pi\in G} \operatorname{supp}(\pi\phi) = 0$.

(p) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra, G a subgroup of $\operatorname{Aut}\mathfrak{A}$ and G^* the countably full subgroup of $\operatorname{Aut}\mathfrak{A}$ generated by G. Suppose that every member of G has a support. Show that every member of G^* has a support.

p 490 l 34 (exercise 381Xn, now 381Xe): (ii) ought to read 'show that π is periodic, with period $n \ge 1$, iff $\mu X > 0$, $f^n(x) = x$ for almost every x and $\{x : f^i(x) = x\}$ is negligible for $1 \le i < n$'.

p 491 l 9 (exercise 381Yb, now 381Ya): add

(ii) Give an example to show that, in 381H, we can have $\pi c_\omega \ne c_\omega$.

p 491 l 6 (exercise 381Ya, now 381Yb): for 'full semigroup generated by ϕ' read 'full semigroup generated by π'.

p 491 l 13 (381Y) Add new exercise:

(e)(i) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra, and a, b two elements of $\mathfrak{A}$. Suppose that $\pi : \mathfrak{A}_a \to \mathfrak{A}_b$ is a Boolean isomorphism such that there is no disjoint sequence $\langle c_n \rangle_{n\in\mathbb{N}}$ of non-zero elements of $\mathfrak{A}_{a\cap b}$ such that $\pi c_n = c_{n+1}$ for every $n \in \mathbb{N}$. Show that there is a Boolean automorphism of $\mathfrak{A}$ extending π. (ii) Let $(\mathfrak{A}, \bar\mu)$ be a measure algebra, and a, $b \in \mathfrak{A}$ two elements of $\mathfrak{A}$ such that $\bar\mu(a \cap b) < \infty$. Show that any measure-preserving isomorphism from $\mathfrak{A}_a$ to $\mathfrak{A}_b$ extends to a measure-preserving automorphism of $\mathfrak{A}$. (Compare 332L.)

p 492 l 23 A smoother proof of Lemma 382C runs as follows:

If π exchanges a and πa then of course a is a separator for π. If π has a separator, then every power of π has a separator, so 382B tells us that π has a transversal. If a is a transversal for π then $a \cup \pi a = \sup_{n\in\mathbb{Z}} \pi^n a = 1$ and $\pi b = b$ whenever $b \subseteq a \cap \pi a$, so π exchanges $a \setminus \pi a$ and $\pi a \setminus a$.

p 494 l 33 (part (a) of the proof of 382G): for '$\sup_{n\in\mathbb{N}}(b_n \cap \pi^k b_n \setminus \sup_{i<m} a_i)$' read '$\sup_{n\in\mathbb{N}}(b_n \cap \pi^k b_n \setminus \sup_{i<n} a_i)$'.

p 495 l 21 (proof of 382H): for '$\sup_{n,j\in\mathbb{Z}} \pi^j a_n \cup \sup_{j\in\mathbb{Z}} \pi^j a_0$' read '$\sup_{n\in\mathbb{N},j\in\mathbb{Z}} \pi^j a_n \cup \sup_{j\in\mathbb{Z}} \pi^j a_0$'.

p 496 l 5 (proof of 382H): for

$$\sup_{j\le 0} \pi^j a_0 \cup \sup_{n\ge 2} \pi a_n \cup \sup_{\substack{n\ge 2 \\ -\lfloor n/2\rfloor < j \le 0}} \pi^j a_n$$

read

$$\sup_{j\le 0} \pi^j a_0 \cup \sup_{\substack{n\ge 2 \\ -\lfloor n/2\rfloor < j \le 0}} \pi^j a_n.$$

p 496 l 34 (part (c) of the proof of 382I): for '$k \ge 1$' read '$n \ge 1$'.

p 497 l 3 (statement of Lemma 382J): for 'subgroup of $\mathfrak{A}$' read 'subgroup of $\operatorname{Aut}\mathfrak{A}$'.

p 497 l 37 The proof of Lemma 382K has more than its share of minor errors; my apologies. The first problem is an omission. We need to check that every member of G has a support. So part (a) of the proof is now

If $\phi \in G$, there is a partition $\langle a_n \rangle_{n\in\mathbb{Z}}$ of unity such that $\phi a = \pi^n a$ whenever $n \in \mathbb{Z}$ and $a \subseteq a_n$ (381Ib). If $a \subseteq a_0$, then $\phi a = a$, so $1 \setminus a_0$ supports ϕ. On the other hand, if $a \setminus a_0 \ne 0$, there is an $n \ne 0$ such that $a \cap a_n \ne 0$. As $\operatorname{supp} \pi^n = 1$, there is a non-zero $d \subseteq a \cap a_n$ such that $0 = d \cap \pi^n d = d \cap \phi d$. Thus $1 \setminus a_0 = \sup\{d : d \cap \phi d = 0\}$ is the support of ϕ (381Ei).

The former part (a) is now part (b).

p 498 l 25 (part (d) of the proof of 382K): in 'ϕ_2 exchanges $\sup_{j,l\ge 1} d_{lj} \subseteq \sup_{j,l\ge 1} d''_{lj}$ with $\sup_{j,l\ge 1} \pi^j d_{lj} \subseteq \sup_{j,l\ge 1} d'_{lj}$', the d''_{lj} and d'_{lj} are the wrong way round.

p 499 l 33 (part (f-i) of the proof of 382K, now (g-i)): for 'g_1^x exchanges i and l'' read 'g_1^x exchanges i' and l''.

p 499 l 35 (part (f-i) of the proof of 382K, now (g-i)): for '$g_1^x(i') = l$ for some $l' \in \,]i, l[$' read '$g_1^x(i') = l'$ for some $l' \in \,]i, l[$'.

p 500 l 7 (part (f-ii) of the proof of 382K, now (g-ii)): for 'exchanged by C_0' read 'exchanged by g_1^x'.

p 500 l 15 (part (f-iii) of the proof of 382K, now (g-iii)): for

$$V'_l = \{i + l : i \in C_0,\ i + j \notin C_0 \text{ if } -l \le j < l\},$$

$$V''_l = \{i : i - l \in C_0,\ i + j \notin C_0 \text{ if } -l < j \le l\},$$

read

$$V_l' = \{i : i - l \in C_0,\, i + j \notin C_0 \text{ if } -l < j \le l\} = \{i + l : i \in C_0,\, C_0 \cap \,]i, i + 2l] = \emptyset\},$$

$$V_l'' = \{i : i + l \in C_0,\, i + j \notin C_0 \text{ if } -l \le j < l\} = \{i - l : i \in C_0,\, C_0 \cap [i - 2l, i[\, = \emptyset\}.$$

p 500 l 38 (part (f-v) of the proof of 382K, now (g-v)): for 'the greatest element of V_l'' less than i' read 'the greatest element of V_l'' less than i''.

p 502 l 12 (proof of 382P): for '$\mathfrak{A}_{1\setminus c}$' read '$\mathfrak{A}_{1\setminus a}$'.

p 504 l 11 Exercise 382Xb is wrong, and should be deleted. Rename 382Xc-382Xj as 382Xc-382Xi.

p 504 l 31 Exercise 382Xk is wrong, and should be replaced by

(j) Let $(\mathfrak{A}, \bar\mu)$ be an atomless probability algebra. Show that if $\pi : \mathfrak{A} \to \mathfrak{A}$ is an ergodic measure-preserving automorphism it has no transversal.

Rename 382Xl as 382Xk.

p 504 l 42 Add new exercise:

(l) Let $\mathfrak{A}$ be a Dedekind σ-complete Boolean algebra and $\pi \in \operatorname{Aut}\mathfrak{A}$. Show that π has a separator iff there is a sequence $\langle a_n\rangle_{n\in\mathbb{N}}$ in $\mathfrak{A}$ such that π is supported by $\sup_{n\in\mathbb{N}} a_n \triangle \pi a_n$.

p 505 l 9 (382Y) Add new exercise:

(b) Devise an expression of the ideas of parts (f)-(h) of the proof of 382K which does not involve the Stone representation.

Rename 382Yb-382Yc as 382Yc-382Yd.

p 509 l 20 (383X) Add new exercises:

(b) Let (X, Σ, μ) be a measure space and $(\mathfrak{A}, \bar\mu)$ its measure algebra. Let S be the set of functions which are isomorphisms between conegligible measurable subsets of X with their subspace measures. (i) Show that the composition of two members of S belongs to S. (ii) Show that there is a map $f \mapsto \pi_f : S \to \operatorname{Aut}_{\bar\mu}\mathfrak{A}$ defined by saying that $\pi_f(E^\bullet) = f^{-1}[E]^\bullet$ for every $E \in \Sigma$, and that $\pi_{fg} = \pi_g\pi_f$, $\pi_f^{-1} = \pi_{f^{-1}}$ for all f, $g \in S$. (iii) Show that $\{\pi_f : f \in S\}$ is a countably full subgroup of $\operatorname{Aut}_{\bar\mu}\mathfrak{A}$.

(c) Let (X, Σ, μ) be a measure space and $(\mathfrak{A}, \bar\mu)$ its measure algebra. Let Φ be the group of measure space automorphisms of (X, Σ, μ). For $f \in \Phi$, let $\pi_f \in \operatorname{Aut}_{\bar\mu}\mathfrak{A}$ be the corresponding automorphism, defined by setting $\pi_f(E^\bullet) = (f^{-1}[E])^\bullet$ for every $E \in \Sigma$. (i) Show that $f \mapsto \pi_f^{-1}$ is a group homomorphism from Φ to $\operatorname{Aut}_{\bar\mu}\mathfrak{A}$. (ii) Show that if $F \subseteq \Phi$ and the subgroup of Φ generated by F is Ψ, then the subgroup of $\operatorname{Aut}_{\bar\mu}\mathfrak{A}$ generated by $\{\pi_f : f \in F\}$ is $\{\pi_f : f \in \Psi\}$. (iii) Show that if (X, Σ, μ) is countably separated and $F \subseteq \Phi$ is a countable subgroup, then the full subgroup of $\operatorname{Aut}_{\bar\mu}\mathfrak{A}$ generated by $\{\pi_f : f \in F\}$ is $\{\pi_g : g \in F^*\}$, where

$$F^* = \{g : g \in \Phi,\, g(x) \in \{f(x) : x \in F\} \text{ for every } x \in X\}.$$

(l) Let $(\mathfrak{A}, \bar\mu)$ be a probability algebra. For π, $\phi \in \operatorname{Aut}_{\bar\mu}\mathfrak{A}$ set

$$\rho(\pi, \phi) = \sup_{a\in\mathfrak{A}} \bar\mu(\pi a \triangle \phi a), \quad \sigma(\pi, \phi) = \bar\mu(\operatorname{supp}(\pi^{-1}\phi)).$$

(i) Show that ρ and σ are metrics on $\operatorname{Aut}_{\bar\mu}\mathfrak{A}$, and that $\rho \le \sigma \le \frac{3}{2}\rho$. (*Hint*: 382Eb.) (ii) Show that $\rho(\psi\pi, \psi\phi) = \rho(\pi\psi, \phi\psi) = \rho(\pi, \phi)$, $\rho(\pi^{-1}, \phi^{-1}) = \rho(\pi, \phi)$, $\rho(\pi\psi, \phi\theta) \le \rho(\pi, \phi) + \rho(\psi, \theta)$, $\sigma(\psi\pi, \psi\phi) = \sigma(\pi\psi, \phi\psi) = \sigma(\pi, \phi)$, $\sigma(\pi^{-1}, \phi^{-1}) = \sigma(\pi, \phi)$, $\sigma(\pi\psi, \phi\theta) \le \sigma(\pi, \phi) + \sigma(\psi, \theta)$ for all π, ϕ, ψ, $\theta \in \operatorname{Aut}_{\bar\mu}\mathfrak{A}$. (iii) Show that $\operatorname{Aut}_{\bar\mu}\mathfrak{A}$ is complete under ρ and σ.

383Xb-383Xi are now 383Xd-383Xk.

p 510 l 16 Exercise 383Yc is covered by the new 328J, and has been deleted.

p 519 l 37 (384X) Add new exercise:

(f) Show that if X is any set such that $\#(X) \ne 6$, the group G of all permutations of X has no outer automorphisms.

384Xf is now 384Xg.

p 529 l 8 (Remark 385Sb): add

It follows that if $(\mathfrak{A},\bar\mu)$ is an atomless homogeneous probability algebra it has a two-sided Bernouilli shift.

p 532 l 4 The exercises to §385 have been rearranged: 385Xf-385Xh are now 385Xg-385Xi, 385Xi-385Xm are now 385Xk-385Xo, 385Xn-385Xp are now 385Xq-385Xs, 385Xq, in revised form, is now 385Xf, 385Xr is now 385Xj, 385Xs is now 385Xp.

385Yb-385Yg are now 385Yc-385Yh, 385Yh is now 385Yb.

p 532 l 29 I have not been able to decide whether every automorphism which is conjgate to its inverse is expressible as a product of at most two involutions. Exercise 385Xl (now 385Xn) therefore now reads

(n) Let $(\mathfrak{A},\bar\mu)$ be a probability algebra and $\pi:\mathfrak{A}\to\mathfrak{A}$ a two-sided Bernoulli shift. (i) Show that π^{-1} is a two-sided Bernoulli shift. (ii) Show that π and π^{-1} are conjugate in $\operatorname{Aut}_{\bar\mu}\mathfrak{A}$. (iii) Show that π is expressible as the product of at most two involutions.

p 533 l 5 There is an error in Exercise 385Xq: the lattice operation $\wedge$ is not in general continuous for the entropy metric. This exercise now reads

(f) Let $(\mathfrak{A},\bar\mu)$ be a probability algebra, and write $\mathcal{A}$ for the set of partitions of unity in $\mathfrak{A}$ not containing 0, ordered by saying that $A\le B$ if B refines A. (i) Show that $\mathcal{A}$ is a Dedekind complete lattice, and can be identified with the lattice of purely atomic closed subalgebras of $\mathfrak{A}$. Show that for A, $B\in\mathcal{A}$, $A\vee B$, as defined in 385F, is $\sup\{A,B\}$ in $\mathcal{A}$. (ii) Show that $H(A\vee B)+H(A\wedge B)\le H(A)+H(B)$ for all A, $B\in\mathcal{A}$, where $\vee$, $\wedge$ are the lattice operations on $\mathcal{A}$. (iii) Set $\mathcal{A}_1=\{A:A\in\mathcal{A},\,H(A)<\infty\}$. For A, $B\in\mathcal{A}_1$ set $\rho(A,B)=2H(A\vee B)-H(A)-H(B)$. Show that ρ is a metric on $\mathcal{A}_1$ (the **entropy metric**). (iv) Show that $H:\mathcal{A}_1\to[0,\infty[$ is 1-Lipschitz for ρ. (v) Show that the lattice operation $\vee$ is uniformly ρ-continuous on $\mathcal{A}_1$. (vi) Show that $H:\mathcal{A}_1\to[0,\infty[$ is order-continuous. (vii) Show that if $\mathfrak{B}$ is any closed subalgebra of $\mathfrak{A}$, then $A\mapsto H(A|\mathfrak{B})$ is order-continuous and 1-Lipschitz on $\mathcal{A}_1$. (viii) Show that if $\pi:\mathfrak{A}\to\mathfrak{A}$ is a measure-preserving Boolean homomorphism, $A\mapsto h(\pi,A):\mathcal{A}_1\to[0,\infty[$ is 1-Lipschitz for ρ.

p 539 l 24 Definition 386G has been dropped, in favour of expressing the results here in terms of the function q of 385A.

386H-386O are now 386G-386N.

p 541 l 22 (proof of 386K, now 386J) for '$\rho(c_i,B_k)$' read '$\rho(c_j,B_k)$'.

p 544 l 14 (part (c) of the proof of 386N, now 386M) for '$\rho(c_b,\mathfrak{C})$' read '$\rho(b,\mathfrak{C})$'.

p 546 l 17 (387A) Add new part:

(d) Let $\mathfrak{B}$, $\mathfrak{C}$ be closed subalgebras of $\mathfrak{A}$ such that $\pi[\mathfrak{B}]\subseteq\mathfrak{B}$ and $\pi[\mathfrak{C}]\subseteq\mathfrak{C}$. I will write $\operatorname{Hom}_{\bar\mu,\pi}(\mathfrak{B};\mathfrak{C})$ for the set of Boolean homomorphisms $\phi:\mathfrak{B}\to\mathfrak{C}$ such that

$$\bar\mu\phi b=\bar\mu b,\quad \pi\phi b=\phi\pi b$$

for every $b\in\mathfrak{B}$. On $\operatorname{Hom}_{\bar\mu,\pi}(\mathfrak{B};\mathfrak{C})$ the **weak uniformity** will be the uniformity generated by the pseudometrics

$$(\phi,\psi)\mapsto\bar\mu(\phi b\,\triangle\,\psi b)$$

for $b\in\mathfrak{B}$; the **weak topology** on $\operatorname{Hom}_{\bar\mu,\pi}(\mathfrak{B};\mathfrak{C})$ will be the associated topology.

p 546 l 18 387B has been rewritten, and is now

387B Elementary facts Suppose that $(\mathfrak{A},\bar\mu)$ is a probability algebra, $\pi\in\operatorname{Aut}_{\bar\mu}\mathfrak{A}$ and that $\langle b_i\rangle_{i\in I}$ is a Bernoulli partition for π. Write $\mathfrak{B}_0$ for the closed subalgebra of $\mathfrak{A}$ generated by $\{b_i:i\in I\}$, $\mathfrak{B}$ for the closed subalgebra generated by $\{\pi^jb_i:i\in I,\,j\in\mathbb{Z}\}$, and B for $\{b_i:i\in I\}\setminus\{0\}$.

(a) $\pi{\restriction}\mathfrak{B}$ is a two-sided Bernoulli shift with root algebra $\mathfrak{B}_0$ and entropy $H(B)=h(\pi,B)\le h(\pi)$.

(b) If $H(B)>0$ then $\mathfrak{A}$ is atomless.

(c) Suppose now that $\langle c_i\rangle_{i\in I}$ is another Bernoulli partition for π with $\bar\mu c_i=\bar\mu b_i$ for every i; let $\mathfrak{C}$ be the closed subalgebra of $\mathfrak{A}$ generated by $\{\pi^jc_i:i\in I,\,j\in\mathbb{Z}\}$. Then we have a unique $\phi \in \mathrm{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$ such that $\phi b_i = c_i$ for every $i \in I$, and ϕ is an isomorphism between $(\mathfrak{B}, \bar{\mu}\restriction\mathfrak{B}, \pi\restriction\mathfrak{B})$ and $(\mathfrak{C}, \bar{\mu}\restriction\mathfrak{C}, \pi\restriction\mathfrak{C})$.

p 551 l 8 (part (e) of the proof of 387C): for '$\bar{\mu}(\sup B) \geq 1 - 2\delta$' read '$\bar{\mu}(\sup B') \geq 1 - 2\delta$'.

p 552 l 29 (part (i) of the proof of 387C): for

This means that there must be some $b \in B$ and $d' \in D_n(C, \pi)$ such that $d \cap f(d) \cap b \cap f(d') \neq 0$ and $\#(I_{bd'}) \geq n(1 - \beta - 4\delta)$; of course $b = f(d)$ and $d' = d$ (because f is injective), so that $\#(I_{f(d),d})$ must be at least $n(1 - \beta - 4\delta)$

read

This means that there must be some $b \in B$ and $d' \in D_n(C, \pi)$ such that $d \cap f(d) \cap b \cap d' \neq 0$ and $\#(I_{bd'}) \geq n(1 - \beta - 4\delta)$; of course $d' = d$ and $b = f(d)$, so that $\#(I_{f(d),d})$ must be at least $n(1 - \beta - 4\delta)$.

p 555 l 20 Add new result:

387F Lemma Let $(\mathfrak{A}, \bar{\mu})$ be a probability algebra, π a measure-preserving automorphism of $\mathfrak{A}$, and $\mathfrak{B}$, $\mathfrak{C}$ closed subalgebras of $\mathfrak{A}$ such that $\pi[\mathfrak{B}] = \mathfrak{B}$ and $\pi[\mathfrak{C}] = \mathfrak{C}$.

(a) Suppose that $\phi \in \mathrm{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$.

(i) $\pi^j \phi = \phi \pi^j$ for every $j \in \mathbb{Z}$.

(ii) $\phi[\mathfrak{B}]$ is a closed subalgebra of $\mathfrak{C}$ and $\pi[\phi[\mathfrak{B}]] = \phi[\mathfrak{B}]$; ϕ is an isomorphism between $(\mathfrak{B}, \bar{\mu}\restriction\mathfrak{B}, \pi\restriction\mathfrak{B})$ and $(\phi[\mathfrak{B}], \bar{\mu}\restriction\phi[\mathfrak{B}], \pi\restriction\phi[\mathfrak{B}])$.

(iii) If $\psi \in \mathrm{Hom}_{\bar{\mu},\pi}(\phi[\mathfrak{B}];\mathfrak{C})$ then $\psi\phi \in \mathrm{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$.

(iv) If $\langle b_i \rangle_{i \in I}$ is a Bernoulli partition for $\pi\restriction\mathfrak{B}$, then $\langle \phi b_i \rangle_{i \in I}$ is a Bernoulli partition for $\pi\restriction\mathfrak{C}$.

(b) $\mathrm{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$ is complete under its weak uniformity.

(c) Let $B \subseteq \mathfrak{B}$ be such that $\mathfrak{B}$ is the closed subalgebra of itself generated by $\bigcup_{i \in \mathbb{Z}} \pi^i[B]$. Then the weak uniformity of $\mathrm{Hom}_{\bar{\mu},\pi}(\mathfrak{B};\mathfrak{C})$ is the uniformity defined by the pseudometrics $(\phi, \psi) \mapsto \bar{\mu}(\phi b \mathbin{\triangle} \psi b)$ as b runs over B.

387F-387L are now 387G-387M.

p 569 l 12 (387X) Add new exercises:

(c) Let $(\mathfrak{A}, \bar{\mu})$ be a measure algebra and $\pi \in \mathrm{Aut}_{\bar{\mu}}\,\mathfrak{A}$. Show that $(\phi, \psi) \mapsto \psi\phi : \mathrm{Hom}_{\bar{\mu},\pi}(\mathfrak{A};\mathfrak{A}) \times \mathrm{Hom}_{\bar{\mu},\pi}(\mathfrak{A};\mathfrak{A}) \to \mathrm{Hom}_{\bar{\mu},\pi}(\mathfrak{A};\mathfrak{A})$ is continuous for the weak topology on $\mathrm{Hom}_{\bar{\mu},\pi}(\mathfrak{A};\mathfrak{A})$.

(d) Let $(\mathfrak{A}, \bar{\mu})$ be a probability algebra, and write ι for the identity map on $\mathfrak{A}$; regard $\mathrm{Aut}_{\bar{\mu}}\,\mathfrak{A}$ as a subset of $\mathrm{Hom}_{\bar{\mu},\iota}(\mathfrak{A};\mathfrak{A})$ with its weak topology. Show that $\pi \mapsto \pi^{-1} : \mathrm{Aut}_{\bar{\mu}}\,\mathfrak{A} \to \mathrm{Aut}_{\bar{\mu}}\,\mathfrak{A}$ is continuous.

387Xc-387Xe are now 387Xc-387Xg.

p l (388X) Add new exercises:

(c) Let $\mathfrak{A}$ be a Boolean algebra and $\pi : \mathfrak{A} \to \mathfrak{A}$ a relatively von Neumann automorphism with fixed-point subalgebra $\mathfrak{C}$ and a dyadic cycle system $\langle d_{mi} \rangle_{m \in \mathbb{N}, i < 2^m}$ such that $\{d_{mi} : m \in \mathbb{N}, i < 2^m\} \cup \mathfrak{C}$ τ-generates $\mathfrak{A}$. Show that for any $n \in \mathbb{N}$ the fixed-point subalgebra of π^{2^n} is the subalgebra of $\mathfrak{A}$ generated by $\{d_{ni} : i < 2^n\} \cup \mathfrak{C}$.

(d) Let $(\mathfrak{A}, \bar{\mu})$ be a probability algebra, and $\pi : \mathfrak{A} \to \mathfrak{A}$ a measure-preserving automorphism. (i) Show that π is weakly von Neumann iff it has a factor (definition: 387Ac) which is a von Neumann automorphism. (ii) Show that if π is a relatively von Neumann automorphism then no non-trivial factor of π can be weakly mixing.

p 580 l 36 Exercise 388Xe (now 388Xg) is wrong in part, and should read

(g) Let $\mathfrak{A}$ be a Boolean algebra and $\pi : \mathfrak{A} \to \mathfrak{A}$ a von Neumann automorphism. (i) Show that π^2 is not ergodic. (ii) Show that π^2 is relatively von Neumann. (iii) Show that π^n is von Neumann for every odd $n \in \mathbb{Z}$. (iv) Show that if $\mathfrak{A}$ is a probability algebra (when endowed with a suitable measure), π is ergodic.

Other exercises have been moved: 388Xc-388Xe are now 388Xe-388Xg.

p 581 l 15 (388Y) Add new exercise:

(f) Give an example of a ccc Dedekind complete Boolean algebra $\mathfrak{A}$ and a von Neumann automorphism $\pi \in \mathrm{Aut}\,\mathfrak{A}$ which is not ergodic.

Other exercises have been moved: 388Yd is now 388Yh, 388Yf is now 388Yd.

p 583 l 1 Since the second printing of this volume, M.Talagrand has solved the Control Measure Problem. Accordingly Chapter 39 has been substantially revised, with a new section on Talagrand's example (§394); the second half of the former §392, with fragments from the former §393, has been put into a new section on Maharam algebras (§393), and §§394-395 have become §§395-396. The notes following are restricted to corrections, without attempting to describe the new material.

p 584 l 30 (391E): for '$\sum_{i\in I}\phi a_i \le \sum_{i\in I}\nu a_i$' read '$\sum_{i\in I}\phi a_i \ge \sum_{i\in I}\nu a_i$.

p 585 l 11 Theorem 391F is sloppily written, and ought to read

Let $\mathfrak{A}$ be a Boolean algebra, not $\{0\}$, and $\psi : A \to [0,1]$ a functional, where $A \subseteq \mathfrak{A}$. Then the following are equiveridical:

(i) there is a non-negative finitely additive functional $\nu : \mathfrak{A} \to [0,1]$ such that $\nu 1 = 1$ and $\nu a \ge \psi a$ for every $a \in \mathfrak{A}$;

(ii) whenever $\langle a_i\rangle_{i\in I}$ is a finite indexed family in A, there is a set $J \subseteq I$ such that $\#(J) \ge \sum_{i\in I}\psi a_i$ and $\inf_{i\in J} a_i \ne 0$.

The proof still works if we define the functional ϕ by the formula

$$\begin{aligned}\phi a &= 1 - \psi(1\setminus a) \text{ if } a \in \mathfrak{A} \text{ and } 1\setminus a \in A,\\ &= 1 \text{ for other } a \in \mathfrak{A}.\end{aligned}$$

(J.M.)

p l Add new result:

391L Proposition (a) If $\mathfrak{A}$ is a measurable algebra, all its principal ideals and σ-subalgebras are, in themselves, measurable algebras.

(b) The simple product of countably many measurable algebras is a measurable algebra.

(c) If $\mathfrak{A}$ is a measurable algebra, $\mathfrak{B}$ is a Boolean algebra and $\pi : \mathfrak{A} \to \mathfrak{B}$ is a surjective order-continuous Boolean homomorphism, then $\mathfrak{B}$ is a measurable algebra.

p 589 l 2 (391Xi): for '$\delta(A) = \sup\{\delta(I) : I$ is a non-empty finite subset of $A\}$' read '$\delta(A) = \inf\{\delta(I) : I$ is a non-empty finite subset of $A\}$'.

p 590 l 25 In the statement of part (c) of Proposition 392C, read 'non-negative additive functional' for 'positive linear functional'.

p 591 l 10 (part (b) of the proof of 392D): for 'there is an $E \subseteq M$ such that $\#(R[E]) \le \#(E) \le k$' read 'there is a non-empty $E \subseteq M$ such that $\#(R[E]) \le \#(E) \le k$'.

p 595 l 10 (part (a) of the proof of 392M): for '$\{m : (a_m \triangle a) \cap a \ne 0\}$' read '$\{m : (a_m \triangle a) \cap b_i \ne 0\}$'.

p 595 l 24 In the statement of Lemma 392N, read 'non-increasing' for 'non-decreasing'. (B.Balcar)

p 597 l 18 (part (b)(β) of the proof of 392O): for '$k \ge n$' read '$k \in \mathbb{N}$'.

p 597 l 20 (part (b)(β) of the proof of 392O): for '$\langle\sup_{i\ge n} a_i\rangle_{n\in\mathbb{N}} \to^* a$' read '$\langle\sup_{i\ge n} a_i\rangle_{n\in\mathbb{N}} \to^* \bar a$'.

p 599 l 31 (part (a)-(b) of the proof of 393B): for '$(b*c) \triangle (b'*c') \subseteq (b \triangle b')*(c \triangle c')$' read '$(b*c) \triangle (b'*c') \subseteq (b \triangle b') \cup (c \triangle c')$'.

p 602 l 2 (proof of 393F): for 'every $n \in \mathbb{N}$, $b \in \mathfrak{B}_n$' read 'every $n \in \mathbb{N}$, $b \in \mathfrak{B}_n \setminus \{0\}$'.

p 609 l 17 (part (b) of 393T): for 'every point of x' read 'every point of X_n'.

p 612 l 33 (Notes to §393): for 'cannot depend on any of the special axioms' read 'cannot depend on most of the special axioms'. (I.Farah.)

p 624 l 8 (part (h) of the proof of 394N, now 395N): for '$u - \theta b_\zeta \theta(a\setminus b_\zeta)$' read '$\theta a - u \le \theta(a \setminus b_\zeta)$'.

p 625 l 28 (394Q, now 395Q) I regret to say that I mis-stated Kawada's theorem. Rather than

Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra such that $\mathrm{Aut}\,\mathfrak{A}$ is ergodic and fully non-paradoxical. Then $\mathfrak{A}$ is measurable.

it should read

Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra such that $\operatorname{Aut}\mathfrak{A}$ has a subgroup which is ergodic and fully non-paradoxical. Then $\mathfrak{A}$ is measurable.

(The result as stated is of course true.)

p 626 l 7 (Exercise 394Xc, now 395Xc): for 'full local semigroup generated by G' read 'full subgroup generated by G'.

p 626 l 20 (Exercise 394Xg, now 395Xg): part (iii) is wrong as stated, and should be deleted.

p 626 l 30 Exercises 394Yb and 394Yc have been put together, with a new third part, as follows:

(c) Let $\mathfrak{A}$ be a Dedekind complete Boolean algebra and G a fully non-paradoxical subgroup of $\operatorname{Aut}\mathfrak{A}$ with fixed-point subalgebra $\mathfrak{C}$. (i) Show that $\mathfrak{A}$ is ccc iff $\mathfrak{C}$ is ccc. (*Hint*: if $\mathfrak{C}$ is ccc, $L^\infty(\mathfrak{C})$ has the countable sup property (363Yb).) (ii) Show that $\mathfrak{A}$ is weakly (σ,∞)-distributive iff $\mathfrak{C}$ is. (iii) Show that $\mathfrak{A}$ is a Maharam algebra iff $\mathfrak{C}$ is.

Exercise 394Yd is now 395Yb.

p 626 l 35 (Exercise 394Yd, now 395Yd): for 'ergodic subgroup of $\mathfrak{A}$' read 'ergodic subgroup of $\operatorname{Aut}\mathfrak{A}$'.

p 627 l 15 (Notes to §394, now 395): for '449J' (proof of Tarski's theorem) read '449L'.

p 637 l 18 (3A3B) Add new part:

(h) Any subspace of a Hausdorff space is Hausdorff.

p 640 l 30 (statement of part (a) of 3A3P): for '$F \subseteq P$' read '$F \subseteq X$'; and similarly eight lines later.

p 642 l 14 (3A4C, uniform continuity) Add new parts:

(e) Two metrics ρ, σ on a set X are **uniformly equivalent** if they give rise to the same uniformity

(f) If U and V are linear topological spaces, and $T : U \to V$ is a continuous linear operator, then T is uniformly continuous for the uniformities associated with the topologies of U and V.

p 645 l 32 The second half of §3A5 has been rearranged; 3A5I-3A5L are now 3A5J-3A5M, 3A5M is now 3A5I.

p 646 l 24 (3A5M, now 3A5I): A hypothesis is missing in the last sentence, which should read 'if U is a Banach space, V is a normed space and $A \subseteq \mathsf{B}(U;V)$, then A is relatively compact for the strong operator topology iff $\{Tu : T \in A\}$ is relatively compact in V for every $u \in U$'.

p 646 l 26 Add new paragraph:

3A5N Bounded sets in linear topological spaces Let U be a linear topological space over $\substack{\mathbb{R}\\ \mathbb{C}}$.

(a) A set $A \subseteq U$ is **bounded** if for every neighbourhood G of 0 there is an $n \in \mathbb{N}$ such that $A \subseteq nG$.

(b) If $A \subseteq U$ is bounded, then

(i) every subset of A is bounded;

(ii) the closure of A is bounded;

(iii) αA is bounded for every $\alpha \in \substack{\mathbb{R}\\ \mathbb{C}}$;

(iv) $A \cup B$ and $A + B$ are bounded for every bounded $B \subseteq U$;

(v) if V is another linear topological space, and $T : U \to V$ is a continuous linear operator, then $T[A]$ is bounded.

(c) If $A \subseteq U$ is relatively compact, it is bounded.

(d) If U is a normed space, and $A \subseteq U$, then the following are equiveridical:

(i) A is bounded in the sense of (a) above for the norm topology of U;

(ii) A is bounded in the sense of 2A4C, that is, $\{\|u\| : u \in A\}$ is bounded above in $\mathbb{R}$;

(iii) A is bounded for the weak topology of U.