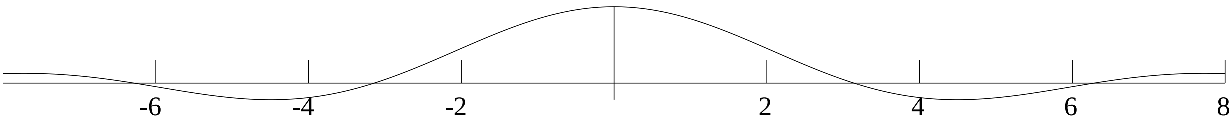
Version of 22.2.11

1. Consider the formula $\frac{\sin x}{x}$.

The natural domain for the corresponding real function is $\{x : x \neq 0\}$.



$\frac{\sin x}{x}$ for $x \neq 0$.

Since $\lim_{x\to 0} \sin x = \lim_{x\to 0} x = 0$ and

$$\lim_{x\to 0} \frac{\frac{d}{dx}\sin x}{\frac{d}{dx}x} = \lim_{x\to 0} \frac{\cos x}{1} = \frac{\cos 0}{1} = \frac{1}{1} = 1,$$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$, by L'Hôpital's rule.

Since $-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$ for every $x > 0$, and $\lim_{x\to\infty}(-\frac{1}{x}) = \lim_{x\to\infty} \frac{1}{x} = 0$, $\lim_{x\to\infty} \frac{\sin x}{x} = 0$, by the Sandwich Theorem.

Since $\frac{1}{x} \leq \frac{\sin x}{x} \leq -\frac{1}{x}$ for every $x < 0$, and $\lim_{x\to -\infty} \frac{1}{x} = \lim_{x\to -\infty}(-\frac{1}{x}) = 0$, $\lim_{x\to -\infty} \frac{\sin x}{x} = 0$, by the Sandwich Theorem.

There is a hole in the function at $x = 0$ which we can fill by setting

$$f(x) = \frac{\sin x}{x} \text{ for real } x \neq 0,$$
$$= 1 \text{ if } x = 0.$$