

Answers

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I list the answers to former problems from my list. Note that dates often indicate the time when I heard of the answer, rather than when it was known to the answerer.

A. M.Talagrand (9.1.06) has given an example of an exhaustive submeasure which is not uniformly exhaustive.

Z. Let $M \subseteq \mathbb{R}^{[0,1]}$ be the space of Borel measurable real-valued functions on $[0, 1]$. If there is a real-valued-measurable cardinal then $\overline{\Gamma(K)} \subseteq M$ whenever $K \subseteq M$ is a \mathfrak{T}_p -compact set. See FREMLIN N95.

AB. ELLIOTT P18 gives examples of (i) a separable Banach lattice (ii) a Dedekind complete Banach lattice with a weakly Fatou norm for which there is no equivalent Fatou norm.

AR(a). ROSŁANOWSKI & SHELAH P00 show that it is relatively consistent with ZFC to suppose that for every function $f : \mathbb{R} \rightarrow \mathbb{R}$ there is a non-negligible set $A \subseteq \mathbb{R}$ such that $f \upharpoonright A$ is continuous.

AR(b). S.Shelah showed (June 1995) that it is relatively consistent with ZFC to suppose that for every function $f : \mathbb{R} \rightarrow \mathbb{R}$ there is a non-meager set $A \subseteq \mathbb{R}$ such that $f \upharpoonright A$ is continuous. (SHELAH SH473.)

AS. S.Shelah proved (October 1989) that it is relatively consistent with ZFC to suppose that there is an open set $G \subseteq [0, 1]^2$, of planar Lebesgue measure 1, such that if $C \times D \subseteq G$ then either D has outer Lebesgue measure less than 1, or C is Lebesgue negligible. See SHELAH & FREMLIN 93.

AU. BALCAR JECH & PAZÁK P03 have shown that it is consistent to suppose that whenever \mathfrak{A} is a ccc weakly σ -distributive Dedekind complete Boolean algebra, there is a strictly positive Maharam submeasure on \mathfrak{A} . It follows that the question ‘is it consistent to suppose that whenever \mathfrak{A} is a ccc weakly σ -distributive Dedekind complete Boolean algebra with countable Maharam type, then \mathfrak{A} is measurable?’ is equivalent to the Control Measure Problem. We see also that it is consistent to suppose that whenever \mathfrak{A} is a ccc Boolean algebra such that every countably generated order-closed subalgebra of \mathfrak{A} is measurable, then \mathfrak{A} is measurable. (For the hypotheses ensure that \mathfrak{A} is weakly σ -distributive and every Maharam submeasure on \mathfrak{A} is uniformly exhaustive.)

AU(c). A.Kamburelis and W.Głównyński have shown (autumn 1989) that if it is relatively consistent to suppose that there is a two-valued-measurable cardinal, then it is relatively consistent to suppose that there is a cardinal κ with a κ -additive ideal \mathcal{I} of $\mathcal{P}\kappa$ such that the algebra $\mathfrak{A} = \mathcal{P}\kappa/\mathcal{I}$ is ccc, countably generated, atomless, Dedekind complete, weakly σ -distributive, ‘localizable’, not $\{0\}$, not isomorphic to the Lebesgue measure algebra; also we have $\kappa < \mathfrak{m} = \mathfrak{c}$. See GŁÓWCZYŃSKI 91 and KAMBURELIS N89.

AU(d). S.Todorčević (March 1994) has given an example of a Dedekind complete Boolean algebra \mathfrak{A} , satisfying Knaster’s condition, such that every countably-generated closed subalgebra of \mathfrak{A} carries a strictly positive *finitely* additive measure, but \mathfrak{A} does not. See FREMLIN N94A.

FARAH & VELIČKOVIĆ 06 show that a construction of GITIK & SHELAH 01, using consequences of $V=L$ which are true in a wide variety of models, provides an example of a Dedekind complete Boolean algebra \mathfrak{A} satisfying Knaster’s condition such that every countably-generated order-closed subalgebra is measurable, but \mathfrak{A} is not. (See also FREMLIN N05.)

BQ(b). K.Kunen has shown (August 1991) that it is consistent to suppose that $\mathfrak{m} = \mathfrak{c} = \omega_2$ and that there is a metric space X , of weight ω_ω , such that every separable subset of X is countable, but not every subset of X is Borel and moreover X has Borel subsets of every class less than ω_1 .

BS. Let \mathfrak{t} be the least cardinal such that there is a family $\langle A_\xi \rangle_{\xi < \mathfrak{t}}$ of infinite subsets of \mathbb{N} such that (i) $A_\xi \setminus A_\eta$ is finite whenever $\eta \leq \xi < \mathfrak{t}$ (ii) there is no infinite $A \subseteq \mathbb{N}$ such that $A \setminus A_\xi$ is finite for every $\xi < \mathfrak{t}$. MALLIARIS & SHELAH P13 (see also FREMLIN N14) have shown that $\mathfrak{p} = \mathfrak{t}$.

BZ. If $\mathfrak{m} > \omega_1$, then every ccc countably tight Čech-complete space has a countable π -base. (S.Todorčević, letter of October 1990.)

CG. It is relatively consistent with ZFC to suppose that every universally measurable subset of \mathbb{R} is Δ_2^1 ; see LARSON & SHELAH N20.

CL. There is a non-empty ccc compact Hausdorff space X which is expressible as the union of a totally ordered family of nowhere dense sets. (S.Todorčević, letter of October 1990.)

CO. S.Shelah (June 1995/May 1996) has shown that it is relatively consistent with ZFC to suppose that the smallest cardinal of any covering of the real line by Lebesgue negligible sets is precisely ω_ω . (SHELAH 00.)

DD. If $\mathfrak{m} = \mathfrak{c} = \omega_2$ and the Open Covering Axiom is true, then the Lebesgue measure algebra is not isomorphic to any subalgebra of $\mathcal{P}\mathbb{N}/[\mathbb{N}]^{<\omega}$. (DOW & HART 00.)

DJ. If $\mathfrak{m} > \omega_1$ then every uncountable Boolean algebra has an uncountable set of pairwise incomparable elements. (LOSADA & TODORČEVIĆ P98.)

DK. There is a point-countable family \mathcal{F} of compact subsets of \mathbb{R} such that $\#(\mathcal{F}) = \mathfrak{c}$ and no uncountable subfamily of \mathcal{F} is point-finite. (TODORČEVIĆ 92, Example E.)

DL. S. Todorčević (Dec. 1995; see TODORČEVIĆ 98) has shown that it is relatively consistent with ZFC to suppose that $\mathfrak{m} = \mathfrak{c} = \omega_2$ and that there is a set $X \subseteq \mathbb{N}^{\mathbb{N}}$ such that $X \cap K$ is F_σ for every compact $K \subseteq \mathbb{N}^{\mathbb{N}}$, but X is not analytic.

DO(f). M.Burke (October 2007) has pointed out that if the continuum hypothesis is true, there is a Borel lifting θ of Lebesgue measure on \mathbb{R} such that $x \in \theta E$ whenever $E \subseteq \mathbb{R}$ is a Borel set and x is a Lebesgue density point of E ; see NEUMANN 31, §I.1, or BURKE 93, Theorem 1.1.

DQ. J.Zapletal (ZAPLETAL N12) has shown that there is a forcing notion \mathbb{P} such that (i) there is a \mathbb{P} -name \dot{h} such that

$$\Vdash_{\mathbb{P}} \dot{h} \in \mathbb{N}^{\mathbb{N}}$$

and for every $g \in \mathbb{N}^{\mathbb{N}}$

$$\Vdash_{\mathbb{P}} \exists n \in \mathbb{N}, \dot{h}(n) = \check{g}(n);$$

(ii) for every \mathbb{P} -name \dot{f} for a function from \mathbb{N} to itself, there is a sequence $\langle F_n \rangle_{n \in \mathbb{N}}$ of nowhere dense subsets of $\mathbb{N}^{\mathbb{N}}$ such that

$$\Vdash_{\mathbb{P}} \dot{f} \in \bigcup_{n \in \mathbb{N}} \overline{F_n}.$$

DV. S. Todorčević (March 1994) has shown that if (\mathfrak{A}, μ) is a measure algebra and $\langle a_{\xi\eta} \rangle_{\xi < \eta < \omega_1}$ is a family in \mathfrak{A} such that $\mu a_{\xi\eta} \geq \epsilon > 0$ whenever $\xi < \eta < \omega_1$, and if *either* $\mathfrak{m} > \omega_1$ *or* \mathfrak{A} is everywhere of Maharam type at least ω_2 , then there is a family $\langle c_\xi \rangle_{\xi < \omega_1}$ in \mathfrak{A} such that $\{\xi : c_\xi \neq \mathbf{0}\}$ is uncountable and

$$\inf_{\xi \in I} c_\xi \subseteq \sup_{\max I < \eta < \omega_1} \inf_{\xi \in I} a_{\xi\eta}$$

for every finite non-empty $I \subseteq \omega_1$. (See FREMLIN N94B.)

DY(c). LOUVEAU & VELIČKOVIĆ 99 have shown that $\ell^1 \not\leq \mathcal{Z}$. T.Mátrai (June 2009) has shown that $\mathcal{F} \not\leq \mathcal{Z}$.

ED. See AU(c) above. Because $\mathcal{P}\kappa/\mathcal{I}$ there is countably generated, etc., it is isomorphic to \mathcal{B}/\mathcal{J} for some σ -ideal \mathcal{J} of the algebra \mathcal{B} of Borel subsets of \mathbb{R} .

EG(a) S.Shelah showed (March 1991) that if κ is a real-valued-measurable cardinal then $\text{cf}([\kappa]^\omega) = \kappa$. See FREMLIN N92, S7B.

EG(c) If κ is a real-valued-measurable cardinal, and ν is a normal κ -additive probability on $\mathcal{P}\kappa$, then $\nu(\{\lambda : \lambda < \kappa \text{ is a weakly } \Pi_1^1\text{-indescribable cardinal}\}) = 1$ (K.Kunen, Dec. 1989).

EG(j) S.Shelah remarked (October 2004) that if we start with GCH and a two-valued-measurable cardinal κ , and add λ random reals where $\lambda > \kappa$ has cofinality ω_1 , then in the extension we have $\mathfrak{c} = \lambda$, $2^{\omega_1} = 2^\lambda = \lambda^+$.

EH. A.Dow & J.Steprāns (October 1991; see DOW & STEPRĀNS 93) have shown that the measure algebra of $\{0, 1\}^{\mathfrak{c}}$ is σ - n -linked for every $n \in \mathbb{N}$

EJ(a). S.Shelah has given an example of a complete ω_1 -saturated measurable set with negligibles which has no lifting. See BURKE N96.

EL. There can be a set $A \subseteq [0, 1]$ such that $\mathcal{P}A \cap \mathcal{N}_L$ is a proper ω_1 -saturated ideal of $\mathcal{P}A$, where \mathcal{N}_L is the ideal of Lebesgue negligible sets. (SHELAH 03.)

Note that it is also possible that there is an $A \subseteq [0, 1]$ such that $\mathcal{M} \cap \mathcal{P}A$ is a proper ω_1 -saturated ideal of $\mathcal{P}A$, where \mathcal{M} is the ideal of meager subsets of $[0, 1]$; see KOMJÁTH 89).

M.R.Burke has pointed out (February 1995) that if $2^{\omega_1} = \mathfrak{c}$ and $S \subseteq \mathbb{R}$ is of cardinal ω_1 and neither null nor meager, then $\mathcal{A} = \mathcal{P}S$ is a family of sets of cardinal \mathfrak{c} such that there is no $B \subseteq \mathbb{R}$ such that A , $A \setminus B$, $A \cap B$ have the same outer measure for every $A \in \mathcal{A}$, nor can $A \setminus B$, $A \cap B$ both be non-meager for every non-meager $A \in \mathcal{A}$.

EM(a). Let f be a continuous non-negative real-valued function defined on the square $[0, 1]^2$. J.M.Dowden (October 1990) and others have shown that

$$\int_0^1 \int_0^1 \int_0^1 \int_0^1 f(x_1, y_1) f(x_2, y_1) f(x_2, y_2) dx_1 dx_2 dy_1 dy_2 \geq \left(\int_0^1 \int_0^1 f(x, y) dx dy \right)^3.$$

(a)' A computer search (looking for functions defined on the set $\{0, 1\}^3$) has shown that there are continuous non-negative real-valued functions f defined on the cube $[0, 1]^3$ such that

$$\begin{aligned} \int_0^1 \int_0^1 \int_0^1 \int_0^1 \int_0^1 \int_0^1 f(x_2, y_1, z_1) f(x_1, y_2, z_1) f(x_1, y_1, z_2) dx_1 dx_2 dy_1 dy_2 dz_1 dz_2 \\ < \left(\int_0^1 \int_0^1 \int_0^1 f(x, y, z) dx dy dz \right)^3. \end{aligned}$$

(I learnt this from R.Bumby, May 1991.)

EO, EP. Let (X, μ) be a Radon probability space and $\kappa < \mathfrak{m}$ a cardinal. Let $\langle E_\xi \rangle_{\xi < \kappa}$ be a stochastically independent family of measurable sets with $\sup_{\xi < \kappa} \mu E_\xi = \epsilon \leq \frac{1}{2}$, and let $E, E' \subseteq X$ be measurable sets with $E \times E' \subseteq \bigcup_{\xi < \kappa} E_\xi \times E_\xi$. Then $\mu E + \mu E' \leq 2\epsilon$. (FREMLIN N91.)

EQ. K.Kunen has pointed out (August 1991) that if it is consistent to suppose that there is a two-valued-measurable cardinal, then it is consistent to suppose that there is a model N of ZFC with an atomlessly-measurable cardinal κ such that if $M \subseteq N$ is any inner model, and $N = M[G]$ where $G \subseteq M$ is an M -generic filter in a measure algebra of M , then $\kappa \leq \mathfrak{c}$ in M (so that, in particular, N is not directly obtainable by Solovay's construction from a model in which κ is two-valued-measurable).

S.Shelah has offered a variety of constructions of atomlessly-measurable cardinals.

ES. V.Konjagin found an example (August 1992) of a compact metric space with non-zero finite one-dimensional Hausdorff measure from which there is no Lipschitz surjection onto any non-trivial interval.

EV. A. V. Arhangel'skii has given an example of a quasi-dyadic compact Hausdorff space which is not dyadic. (March 1993.)

EW. P.Biryukov has pointed out that ANZAI 51 gives an example of a measure-preserving automorphism π of the measure algebra \mathfrak{A} of Lebesgue measure on $[0, 1]$ which has been shown by RYZHIKOV 93 not to be the product of two involutions.

Note also that the odometer transformation $f : \{0, 1\}^{\mathbb{N}} \rightarrow \{0, 1\}^{\mathbb{N}}$ (FREMLIN 02, 387H) is the product of two Borel measurable measure-preserving involutions h and hf where $h(x)(n) = 1 - x(n)$ for every x and n .

EX. BISHOP & HAKOBYAN 08 give an example of an open set $\Omega \subseteq \mathbb{R}^2$ such that its central set (the set of centres of maximal open balls included in Ω) has Hausdorff dimension 2. (March 2007.)

FE. R.Pol (POL 00) has shown that if X and Y are compact Hausdorff spaces, and $f : X \times Y \rightarrow \mathbb{R}$ is separately continuous, then it is Borel measurable. (May 2000.)

FH. P.Komj ath (September 2000) has pointed out that there is always a non-decreasing sequence $\langle A_n \rangle_{n \in \mathbb{N}}$, with union \mathbb{R} , such that no A_n includes any arithmetic progression of length $n + 1$; see FREMLIN N00.

P.Komj ath (KOMJ ATH N01) has shown that if we add ω_4 Cohen reals to a model of GCH, we obtain a model in which there is no function $f : \mathbb{R} \rightarrow \mathbb{N}$ such that $\lim_{h \downarrow 0} \max(f(x+h), f(x-h)) = \infty$ for every

$x \in \mathbb{R}$; so that if $\langle D_n \rangle_{n \in \mathbb{N}}$ is any non-decreasing sequence of sets with union \mathbb{R} , there is some n such that D_n includes arithmetic progressions of length 3 and arbitrarily small diameter. (October 2001.)

FI. BEŠLAGIČ VAN DOUWEN MERRILL & WATSON 87 give a construction of a sequentially compact Hausdorff space without isolated points of cardinal ω_1 . (Communicated by S.Watson.)

FQ. SHELAH & STEPRĀNS P03 have shown that it is relatively consistent with ZFC to suppose that the uniformity of Lebesgue measure is ω_2 while the uniformity of linear Hausdorff measure on \mathbb{R}^2 is ω_1 .

FR. G.Plebanc has shown that if $\mathfrak{m} = \mathfrak{c}$ then there is a compact Hausdorff space of cardinal \mathfrak{c} with a family of $2^{\mathfrak{c}}$ mutually singular Radon probability measures.

See <http://www.essex.ac.uk/maths/people/fremlin/probFR.ps>.

FS. (i) For any κ there are ω_1 -saturated measurable spaces with negligibles (X, Σ, \mathcal{I}) and (Y, T, \mathcal{J}) such that the skew product is not κ -saturated; see FREMLIN N01. (ii) There is an ω_1 -saturated measurable space with negligibles (Y, T, \mathcal{J}) such that (Y, T) is a standard Borel space and $(\mathbb{R}, \Sigma, \mathcal{N}) \times (Y, T, \mathcal{J})$ is not \mathfrak{c} -saturated, where Σ is the algebra of Lebesgue measurable sets and \mathcal{N} the ideal of negligible sets.

FT. A non-negative linear functional $\phi : \ell^\infty \rightarrow \mathbb{R}$ is a **medial limit** if (i) $\phi(x) = \lim_{n \rightarrow \infty} x(n)$ whenever $x \in \ell^\infty$ and the limit exists (ii) whenever $\langle f_n \rangle_{n \in \mathbb{N}}$ is a uniformly bounded sequence of Lebesgue measurable real-valued functions on $[0, 1]$ and $g(t) = \phi(\langle f_n(t) \rangle_{n \in \mathbb{N}})$ for every $t \in [0, 1]$, then g is Lebesgue measurable and $\int g(t) dt = \phi(\langle \int f_n(t) dt \rangle_{n \in \mathbb{N}})$. P.Larson has shown (June 2009) that it is relatively consistent with ZFC to suppose that there are no medial limits. See FREMLIN 08, 538S¹.

FW. S.Geschke has pointed out that the ill-founded iterations of KANOVEI 99 provide non-isomorphic homogeneous Dedekind complete Boolean algebras \mathfrak{A} , \mathfrak{B} such that each can be regularly embedded in the other.

I.Farah has observed that if \mathfrak{A} is the regular open algebra of the partially ordered set \mathcal{H}_{E_0} discussed on p. 93 in FARAH 96, and \mathfrak{B} is the regular open algebra of $\{0, 1\}^{\omega_2}$, then each of \mathfrak{B} , \mathfrak{A} can be regularly embedded in the other, both are homogeneous, but they are not isomorphic.

GF. For a group X and a set $A \subseteq X$ write $A^{(2)}$ for $\{x^2 : x \in A\}$. There is a locally compact Polish topological group X with Haar measure μ such that $\mu X^{(2)} > 0$ but there is an open subgroup Y of X with $\mu Y^{(2)} = 0$.

GF. There is a σ -finite-cc Boolean algebra which is not σ -bounded-cc; see TODORČEVIĆ 14.

GJ. J. Cancino-Manríquez (CANCINO-MANRÍQUEZ P22) has shown that it is relatively consistent with ZFC that there should be no Hausdorff ultrafilters.

GN. If ν is the image of the usual measure on $\{0, 1\}^{\mathbb{N}}$ under the function $z \mapsto \sum_{n=0}^{\infty} 4 \cdot 5^{-n-1} z(n)$, there is a non-Lebesgue-negligible set $A \subseteq \mathbb{R}$ such that $\nu(A+x) = 0$ for every $x \in \mathbb{R}$.

GP. If X is any non-locally-compact Polish abelian group, there are a coanalytic set $H \subseteq X$ and a Radon probability measure ν on X such that $\nu(xH) = 0$ for every $x \in X$, but if $E \supseteq H$ is analytic and ν_1 is a Radon probability measure on X there is an $x \in X$ such that $\nu_1(xE) > 0$. (ELEKES & VINYÁNSZKY P14.)

GQ. Let F_2 be the free group on two letters x, y . If H is a group, $u \in F_2$ and $a, b \in H$, write $u(a, b)$ for the element of H got by substituting a, b for x, y in u . Let $*$ be the binary operation on $F_2 \times F_2$ defined by setting $w * v = (w_1(v_1, v_2), w_2(v_1, v_2))$ for $w = (w_1, w_2), v = (v_1, v_2) \in F_2 \times F_2$. Then $F_2 \times F_2$ is a semigroup with identity (x, y) . Let G be the group of invertible elements of $F_2 \times F_2$. J.Mycielski has shown that G is the sub-semigroup of $F_2 \times F_2$ generated by $\{(y, x), (x, xy), (x^{-1}, y)\}$. (FREMLIN & MYCIELSKI N08.)

GV. If $n \geq 1$, ρ is any metric on \mathbb{R}^n which defines the usual topology, and $\mu_{H_n}^{(\rho)}$ the corresponding n -dimensional Hausdorff measure, then $\mu_{H_n}^{(\rho)}(\mathbb{R}^n) > 0$. (BAGNARA GENNAIOLI LECCESSE & LUONGO P22.)

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¹Later editions only.

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