Problem EW

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Answer to original problem P.Biryukov has pointed out that Anzai 51 gives an example of a measure-preserving automorphism π of the measure algebra $\mathfrak{A}$ of Lebesgue measure on $[0,1]$ which has been shown by Ryzhikov 93 not to be the product of two involutions. In this example, π is not conjugate to its inverse in the group $\operatorname{Aut}\mathfrak{A}$.

The odometer transformation $f:\{0,1\}^{\mathbb{N}}\to\{0,1\}^{\mathbb{N}}$ (Fremlin 02, 387H) is the product of two Borel measurable measure-preserving involutions h and hf where $h(x)(n)=1-x(n)$ for every x and n. (From the formulae in Fremlin 02, 387H, it is easy to check that $hfh=f^{-1}$, so that hf is an involution.) Note that if we write π, ϕ for the automorphisms of the measure algebra $\mathfrak{A}$ of the usual measure on $\{0,1\}^{\mathbb{N}}$ corresponding to f and h respectively, then the involutions ϕ and $\pi\phi$ do not belong to the full subgroup G generated by π (use Fremlin 02, 387C). In fact B.Miller has shown that π is not the product of two involutions in G.

References

Anzai H. [51] ‘On an example of a measure-preserving transformation which is not conjugate to its inverse’, Proc. Japan Acad. Sci. 27 (1951) 517-522.

Ryzhikov V.V. [93] ‘Factorization of an automorphism of a complete Boolean algebra into a product of three involutions’, Mat. Zametki (=Math. Notes of Russian Acad. Sci.) 54 (1993) 79-84.