Errata and addenda for Volume 4, 2013 edition

I collect here known errors and omissions, with their discoverers, in Volume 4 of my book *Measure Theory* (see my web page, `http://www1.essex.ac.uk/maths/people/fremlin/mt.htm`).

Part I

p 21 l 37 (part (b) of the proof of 412C): for '$\subseteq \{L^\bullet : L \in \mathcal{L},\ L \subseteq H\}$' read '$\subseteq \sup\{L^\bullet : L \in \mathcal{L},\ L \subseteq H\}$'.

p 22 l 35 Part (b) of Proposition 412H is wrong as stated, and should read

(b) Now suppose that μ is semi-finite and that

(‡) $\bigcap_{n\in\mathbb{N}} K_n \in \mathcal{K}$ whenever $\langle K_n\rangle_{n\in\mathbb{N}}$ is a non-increasing sequence in $\mathcal{K}$.

If either $\hat\mu$ or $\tilde\mu$ is inner regular with respect to $\mathcal{K}$ then μ is inner regular with respect to $\mathcal{K}$.

p 24 l 42 Corollary 412L has been elaborated, and is now

412M Proposition Let X be a set and $\mathcal{K}$ a family of subsets of X. Suppose that μ and ν are two complete locally determined measures on X, with domains including $\mathcal{K}$, and both inner regular with respect to $\mathcal{K}$.

(a) If $\mu K \le \nu K$ for every $K \in \mathcal{K}$, then $\mu \le \nu$ in the sense of 234P.

(b) If $\mu K = \nu K$ for every $K \in \mathcal{K}$, then $\mu = \nu$.

412M is now 412L.

p 26 l 25 (proof of 412R): for '$\mathcal{A} = \{E \times F : E \in \Sigma\} \cup \{X \times F : F \in \mathrm{T}\}$' read '$\mathcal{A} = \{E \times Y : E \in \Sigma\} \cup \{X \times F : F \in \mathrm{T}\}$'.

p 27 l 43 (proof of 412Wa) The argument given assumes that f is finite-valued. If $E = \{x : f(x) = \infty\}$ is non-empty, it is negligible; choose a sequence $\langle H_n\rangle_{n\in\mathbb{N}}$ of measurable open sets such that $E \subseteq H_n$ and $\mu H_n \le 2^{-n}\eta$ for every n, and add $\sum_{n=0}^{\infty} \chi H_n$ to the given formula for g; with an adjustment to the formula for η, this will work.

p 30 l 3 Exercise 412Xr (now 412Xu) is wrong as written, and should be

(u) Let X be a topological space and μ a measure on X which is inner regular with respect to the closed sets and outer regular with respect to the open sets. Show that if $f : X \to \mathbb{R}$ is integrable and $\epsilon > 0$ then there is a lower semi-continuous $g : X \to]-\infty, \infty]$ such that $f \le g$ and $\int g - f \le \epsilon$.

p 30 l 17 Exercise 412Xv (now 412Xf) is wrong, and has been replaced by

(f) Let $\langle \mu_i\rangle_{i\in I}$ be a family of measures on a set X, with sum μ. Suppose that $\mathcal{K} \subseteq \operatorname{dom}\mu$ is a family of sets such $K \cup K' \in \mathcal{K}$ for every $K,\ K' \in \mathcal{K}$ and every μ_i is inner regular with respect to $\mathcal{K}$. Show that μ is inner regular with respect to $\mathcal{K}$.

p 30 l 20 Exercise 412Xw has been moved to part (a) of the new Proposition 412M.
Other exercises have been moved: 412Xa-412Xd are now 412Xb-412Xe, 412Xe-412Xs are now 412Xh-412Xv, 412Xt is now 412Xa, 412Xu is now 412Ya, 412Xv is now 412Xf, 412Xx is now 412Xg.

p 30 l 35 (412Y) Add new exercise:

(d) Give an example of a measure space (X, Σ, μ) and a family $\mathcal{K}$ of sets such that

(‡) $\bigcap_{n\in\mathbb{N}} K_n \in \mathcal{K}$ whenever $\langle K_n\rangle_{n\in\mathbb{N}}$ is a non-increasing sequence in $\mathcal{K}$

and the completion of μ is inner regular with respect to $\mathcal{K}$, but μ is not.

412Ya-412Yb are now 412b-412Yc, 412Yc-412Ye are now 412Ye-412Yg.

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p 32 l 14 (statement of 413B): for '$\phi : X \to [0,\infty]$' read '$\phi : \mathcal{P}X \to [0,\infty]$'.

p 32 l 35 (part (a) of the proof of 413C): for '$\mu A < \infty$' read '$\phi A < \infty$'. (K.Yates.)

p 36 l 17 Proposition 413T has been brought forward to 413H. 413H-413O are now 413I-413P.

p 39 l 19 (now 413K): for $F_m = \bigcap_{n\le m} E_{nn}$ read $F_m = \bigcap_{i,j\le m} E_{ji}$. (A.P.Pyshchev.)

p 43 l 7 The following definitions have been brought together as a new paragraph:

413Q Definitions Let P be a lattice and $f : P \to [-\infty,\infty[$ a function.

(a) f is **supermodular** if $f(p \vee q) + f(p \wedge q) \ge f(p) + f(q)$ for all p, $q \in P$.

(b) f is **submodular** if $f(p \vee q) + f(p \wedge q) \le f(p) + f(q)$ for all p, $q \in P$.

(c) f is **modular** if $f(p \vee q) + f(p \wedge q) = f(p) + f(q)$ for all p, $q \in P$.

413P-413S are now 413R-413U.

p 47 l 18 Exercise 413Xk is wrong as stated, and has been dropped. 413Xl-413Xs are now 413Xk-413Xr.

p 47 l 41 (Exercise 413Xo, now 413Xn): for 'non-decreasing sequence $\langle H_i\rangle_{i\in\mathbb{N}}$' read 'non-increasing sequence $\langle H_i\rangle_{i\in\mathbb{N}}$'.

p 48 l 15 (413Y) Add new exercises:

(a) Let $\mathfrak{A}$ be a Boolean algebra, $(S,+)$ a commutative semigroup with identity e and $\phi : \mathfrak{A} \to S$ a function such that $\phi 0 = e$. Show that

$$\mathfrak{B} = \{b : b \in \mathfrak{A},\ \phi a = \phi(a \cap b) + \phi(a \setminus b) \text{ for every } a \in \mathfrak{A}\}$$

is a subalgebra of of $\mathfrak{A}$, and that $\phi(a \cup b) = \phi a + \phi b$ for all disjoint a, $b \in \mathfrak{B}$.

(j) Let (X,Σ,μ) be a semi-finite measure space, λ the c.l.d. product measure on $X \times \mathbb{R}$ when $\mathbb{R}$ is given Lebesgue measure, and λ_* the associated inner measure. Show that for any $f : X \to [0,\infty[$,

$$\underline{\int} f\,d\lambda = \lambda_*\{(x,\alpha) : x \in X,\ 0 \le \alpha < f(x)\} = \lambda_*\{(x,\alpha) : x \in X,\ 0 \le \alpha \le f(x)\}.$$

413Ya-413Yb are now 413Yb-413Yc, 413Yc-413Yf are now 413Ye-413Yh, 413Yg is now 413Yd, 413Yh is now 413Yi.

p 56 l 21 (414X) Add new exercise, formerly part of 416Xf:

(u) Let $(X,\mathfrak{T},\Sigma,\mu)$ be a complete locally determined effectively locally finite τ-additive topological measure space. Show that there is a decomposition $\langle X_i\rangle_{i\in I}$ for μ in which every X_i is expressible as the intersection of a closed set with an open set.

p 58 l 11 In Proposition 415D, we need to suppose from the beginning that X is regular.

p 63 l 3 (part (a-α) of the proof of 415L): for '$\mu F \ge \mu_0(E_0 \setminus E_1) - \epsilon$' read '$\mu_0 F \ge \mu_0(E_0 \setminus E_1) - \epsilon$'.

p 63 l 7 (part (a-α) of the proof of 415L): for '$\mu_0^* L + \mu_0^* K' > \mu^* K$' read '$\mu_0^* L + \mu_0^* K' > \mu_0^* K$'.

p 67 l 24 (part (e-ii) of the proof of 415Q): for 'Let $W \in \mathrm{T}$ be a measurable envelope of $F \in \Sigma$ such that $\nu(W \triangle F^*) = 0$' read 'Let $W \in \mathrm{T}$ be a measurable envelope of $R^{-1}[A]$, and take $F \in \Sigma$ such that $\nu(W \triangle F^*) = 0$'.

p 69 l 11 Exercise 415Xn has been dropped. Add new exercise:

(q) Find a second-countable Hausdorff space X, a subset Y and a quasi-Radon probability measure on Y which is not the subspace measure induced by any quasi-Radon measure on X.

Other exercises have been rearranged: 415Xh is now 415Xi, 415Xi-415Xm are now 415Xk-415Xo, 415Xo-415Xq are now 415Xr-415Xt, 415Xr is now 415Xj, 415Xs is now 415Xh, 415Xt is now 415Xp.

p 70 l 1 Exercise 415Yf, rewritten, is now 415Xq. 415Yg-415Ym are now 415Yf-415Yl.

p 75 l 19 (part (b) of the proof of 416K) For '$\phi_0 K \le \tilde\nu(H_0 \cup H_1) \le \tilde\nu H_0 + \tilde\nu H_1 \le \phi_0 L + \phi_0 K' + 2\epsilon$' read '$\phi_0 K \le \tilde\nu(H_0 \cup H_1) \le \tilde\nu H_0 + \tilde\nu H_1 \le \phi_0 L + \phi_0 K_1 + 2\epsilon$'.

p 75 l 36 (condition γ in the statement of 416L) For 'open set G containing X' read 'open set G containing x'.

p 76 l 42 (proof of 416M) For '$\phi_1' K \le \phi_0 K' \le \psi G \le \phi K + \epsilon$' read '$\phi_1' K \le \phi_0 K' \le \psi G \le \phi_1 K + \epsilon$'.

p 80 l 44 (part (b-iii) of the proof of 416S) For '$G \supseteq X$' read '$G \supseteq K$'.

p 82 l 40 In Exercise 416Xb, we must assume that the topology of X is Hausdorff.

p 83 l 2 Part (ii) of 416Xf is now 414Xu.

p 83 l 7 Exercises 416Xh, 416Xi, 416Xm and 416Xo have been dropped.

p 83 l 38 (416Xt, now 416Xm): for '$\phi : S_2^* \to [0, \infty[$' read '$\phi : S_2 \to [0, \infty[$'.
Other exercises have been rearranged: 416Xd is now 416Xf, 416Xe is now 416Xd, 416Xf is now 416Xe, 416Xj-416Xk are now 416Xh-416Xi, 416Xl is now 416Xk, 416Xn is now 416Xj, 416Xp-416Xr are now 416Xn-416Xp, 416Xs-416Xt are now 416Xl-416Xm, 416Xu-416Xz are now 416Xq-416Xv.

p 84 l 34 (416Y) Add new exercise:

(f) In 416Qb, show that μ is atomless iff ν is properly atomless in the sense of 326F.

Other exercises have been rearranged: 416Ya-416Yc are now 416Yb-416Yd, 416Yd is now 416Ya, 416Yf-416Yg are now 416Yg-416Yh, 416Yh is now 416Yj.

p 87 l 39 Theorems 417C and 417E have been revised in order to support a definition of 'τ-additive product measure' (417G) without supposing that the factor measures are inner regular with respect to the Borel sets.

p 88 l 5 Clause (v) of the statement of 417C should read 'the support of $\tilde{\lambda}$ is the product of the supports of μ and ν'.

p 91 l 13 Clause (iv) of the statement of 417E should read 'the support of $\tilde{\lambda}$ is the product of the supports of the μ_i'.

p 94 l 27 Corollary 417F has been dropped, and replaced by

417H Corollary Let $(X, \mathfrak{T}, \Sigma, \mu)$ and $(Y, \mathfrak{S}, \mathrm{T}, \nu)$ be two complete locally determined effectively locally finite τ-additive topological measure spaces. Let $\tilde{\lambda}$ be the τ-additive product measure on $X \times Y$, and $\tilde{\Lambda}$ its domain. If $A \subseteq X$, $B \subseteq Y$ are non-negligible sets such that $A \times B \in \tilde{\Lambda}$, then $A \in \Sigma$ and $B \in \mathrm{T}$.

417G-417H are now 417F-417G.

p 97 l 29 There is a catastrophic error in the proof of 417K, but the result can easily be proved from 417H (now 417G) by the method outlined in 252Xc.

p 102 l 4 Exercise 417Xb is covered by 411Xi, and has been dropped.
Add new exercise:

(x) (i) Let $\langle (X_i, \mathfrak{T}_i, \Sigma_i, \mu_i) \rangle_{i \in I}$ be a finite family of effectively locally finite τ-additive topological measure spaces, and let Λ be the σ-algebra of subsets of $X = \prod_{i \in I} X_i$ generated by $\widehat{\bigotimes}_{i \in I} \Sigma_i$ together with the open subsets of X. Show that there is a unique effectively locally finite τ-additive measure λ with domain Λ such that $\lambda(\prod_{i \in I} E_i) = \prod_{i \in I} \mu_i E_i$ whenever $E_i \in \Sigma_i$ for $i \in I$. (ii) Let $\langle (X_i, \mathfrak{T}_i, \Sigma_i, \mu_i) \rangle_{i \in I}$ be a family of τ-additive topological probability spaces, and let Λ be the σ-algebra of subsets of $X = \prod_{i \in I} X_i$ generated by $\widehat{\bigotimes}_{i \in I} \Sigma_i$ together with the open subsets of X. Show that there is a unique τ-additive measure λ with domain Λ extending the usual product measure on $\widehat{\bigotimes}_{i \in I} \Sigma_i$.

p 108 l 14 (part (c) of the proof of 418D): for '$\mu E_1 \ge \mu(E \cap f^{-1}[Y_n] \setminus E_1) - \epsilon$' read '$\mu E_1 \ge \mu(E \cap f^{-1}[Y_n]) - \epsilon$'.

p 117 l 32 Add new result:

418V Proposition Let (X, Σ, μ) be a σ-finite measure space, $\mathfrak{T}$ a topology on X such that μ is inner regular with respect to the Borel sets, $(Y, \mathfrak{S})$ a topological space and $f : X \to Y$ an almost continuous function. Then there is a Borel measurable function $g : X \to Y$ which is equal almost everywhere to f.

p 109 l 6 (statement of Proposition 418Ha): for 'effectively locally finite τ-additive measure on X' read 'effectively locally finite τ-additive topological measure on X'.

p 115 l 3 (part (d-ii) of the proof of 418P): for

$$\text{`}F_k = \{x : x \in \prod_{j\in I} K_j^*,\ x(k) = z,\ f_{jk}x(k) = x(j) \text{ whenever } j \le k\}\text{'}$$

read

$$\text{`}F_k = \{x : x \in \prod_{j\in I} K_j^*,\ x(i) = z,\ f_{jk}x(k) = x(j) \text{ whenever } j \le k\}\text{'}.$$

p 115 l 7 (part (d-ii) of the proof of 418P): for '$x(k) = z$' read '$x(i) = z$'.

p 117 l 33 (418X) Add new exercise:

(a) Let (X, Σ, μ) be a measure space, and $(X, \hat{\Sigma}, \hat{\mu})$ its completion. (i) Show that if Y is a second-countable topological space, a function $f : X \to Y$ is $\hat{\Sigma}$-measurable iff there is a Σ-measurable $g : X \to Y$ such that $f =_{\text{a.e.}} g$. (ii) Show that if X is endowed with a topology, and Y is a topological space, then a function from X to Y is μ-almost continuous iff it is $\hat{\mu}$-almost continuous.

Other exercises have been rearranged: 418Xa-418Xk are now 418Xb-418Xl, 418Xl-418Xs are now 418Xn-418Xu, 418Xt-418Xw are now 418Xw-418Xz, 418Xx is now 418Xm, 418Xy is now 418Xv.

p 119 l 32 Exercise 418Yd is wrong, and has been deleted.

p 120 l 16 Exercise 418Yl is wrong, and has been deleted.
Other exercises have been rearranged: 418Ye-418Yj are now 418Yd-418Yi, 418Yk is now 418Yl, 418Ym is now 418Yj, 418Yn is now 418Ym, 418Yo is now 418Yk, 418Yp-418Yr are now 418Yn-418Yp.

p 131 l 20 (419Y) Add new exercise:

(d) Give an example of a Lebesgue measurable function $\phi : \mathbb{R}^2 \to \mathbb{R}$ such that $\operatorname{dom} \dfrac{\partial\phi}{\partial\xi_1}$ is not measurable.

p 134 l 30 (421C) Add new part, formerly 421Xm:

(f)(i) I will say that a Souslin scheme $\langle E_\sigma\rangle_{\sigma\in S^*}$ is **fully regular** if $E_\sigma \subseteq E_\tau$ whenever σ, $\tau \in S^*$, $\#(\tau) \le \#(\sigma)$ and $\sigma(i) \le \tau(i)$ for every $i < \#(\sigma)$.

(ii) Let $\mathcal{E}$ be a family of sets such that $E \cup F$ and $E \cap F$ belong to $\mathcal{E}$ for all E, $F \in \mathcal{E}$. Then every member of $\mathcal{S}(\mathcal{E})$ can be expressed as the kernel of a regular Souslin scheme in $\mathcal{E}$.

p 140 l 36 Exercise 421Xm has been moved to 421Cf.

p 144 l 21 (part (h) of the proof of 422D): for '$R' = S \circ R$' read '$R' = R \circ S$'.

p 147 l 21 (part (c) of the proof of 422K): for '$\overline{R[I_{\phi\restriction n}]} \cap \overline{S[I_{\psi\restriction n}]}$ is empty' read '$\overline{R[I_{\phi\restriction n}] \cap S[I_{\psi\restriction n}]}$ is empty'.

p 147 l 33 The exercises for §422 have been rearranged: 422Xd is now 422Xf, 422Xe-422Xf are now 422Xd-422Xe, 422Yb-422Ye are now 422Yc-422Yf, 422Yf is now 422Yb.

p 152 l 23 423S is now 423J.

p 154 l 25 Add new result:

423O Corollary Let X be an analytic Hausdorff space, Y a set and T a σ-algebra of subsets of Y which is closed under Souslin's operation. Suppose that $W \in \mathcal{S}(\mathcal{B}(X)\widehat{\otimes}\mathrm{T})$ where $\mathcal{B}(X)$ is the Borel σ-algebra of X. Then $W[X] \in \mathrm{T}$ and there is a T-measurable function $f : W[X] \to X$ such that $(f(y), y) \in W$ for every $y \in W[X]$.

423J-423M are now 423K-423N, 423N-423R are now 423P-423T.

p 155 l 14 (proof of 423O, now 423Q): for '$\mathrm{T} = \mathcal{S}(\mathcal{B}(Y))$' read '$\mathrm{T} \supseteq \mathcal{S}(\mathcal{B}(Y))$'.

p 155 l 35 (Remark 423Qb, now 423Sb) for 'analytic subset of $\mathbb{N}^{\mathbb{N}} \setminus A$' read 'analytic subset of $\mathbb{N}^{\mathbb{N}} \setminus A_0$'.

p 156 l 41 (Exercise 423Xf) for '$f : W[X] \to Y$' read '$f : W[X] \to X$'.

p 163 l 12 (Exercise 424Ya) Add new part:

(iii) Show that Σ is the Borel σ-algebra of $\mathcal{C}$ when $\mathcal{C}$ is given its Fell topology.

p 174 l 14 (Theorem 431D) Add new part, formerly 431Xc:

(b) If $\langle E_\sigma\rangle_{\sigma\in S^*}$ is fully regular, then $\mu A = \sup\{\mu(\bigcap_{n\ge1} E_{\psi\restriction n}) : \psi \in \mathbb{N}^{\mathbb{N}}\}$, and if moreover μ is totally finite, $\mu A = \sup\{\inf_{n\ge1} \mu E_{\psi\restriction n} : \psi \in \mathbb{N}^{\mathbb{N}}\}$.

p 176 l 18 Exercise 431Xc is now 431Db; 431Xd-431Xe are now 431Xc-431Xd.

p 176 l 32 (431Y) 431Yc has been moved to 431G; 431Yb is now 431Yc. Add new exercises:

(b) Let (X, Σ, μ) be a totally finite measure space, Y a set and T a σ-algebra of subsets of Y. Suppose that $A \in \mathcal{S}(\Sigma\widehat{\otimes}\mathrm{T})$. Show that $\{y : \mu A^{-1}[\{y\}] > \alpha\}$ belongs to $\mathcal{S}(\mathrm{T})$ for every $\alpha \in \mathbb{R}$.

(d) Let $r \ge 1$ be an integer and $f : \mathbb{R}^r \to \mathbb{R}$ a Borel measurable function. Show that the domain of its first partial derivative $\frac{\partial f}{\partial \xi_1}$ is coanalytic, therefore Lebesgue measurable, but may fail to be Borel.

p 181 l 28 (432X) Exercise 432Xj has been revised, and now reads

(j) Let P be a lattice, and $c : P \to \mathbb{R}$ an order-preserving functional. Show that the following are equiveridical: (i) c is submodular; (ii) $(p, q) \mapsto 2c(p \vee q) - c(p) - c(q)$ is a pseudometric on P; (iii) setting $c_r(p) = c(p \vee r) - c(r)$, $c_r(p \vee q) \le c_r(p) + c_r(q)$ for all p, q, $r \in P$.

Add new exercises:

(k) Let X be a topological space, $c : X \to [0, \infty]$ a Choquet capacity, and $f : [0, \infty] \to [0, \infty]$ a non-decreasing function. (i) Show that if f is continuous then fc is a Choquet capacity. (ii) Show that if $f\restriction [0, \infty[$ is concave and c is submodular, then fc is submodular.

(l) Let X be a Hausdorff space, c a Choquet capacity on X, and $\mathcal{K}$ a non-empty downwards-directed family of compact subsets of X. Show that $c(\bigcap \mathcal{K}) = \inf_{K\in\mathcal{K}} c(K)$.

p 189 l 21 (part (b) of the proof of 434F): for '$\nu K \ge \mu_Y Y - \epsilon$' read '$\mu_Y K \ge \mu_Y Y - \epsilon$'.

p 195 l 42 (Proposition 434P): add new part

(f) A quasi-dyadic space is ccc.

p 197 l 29 (part (f) of the proof of 434Q): in this part of the proof, you need to take it that every ξ is supposed to be a member of C, so that '$\xi > \gamma$' should be read as '$\xi \in C \setminus (\gamma + 1)$', etc.

p 198 l 13 (part (a-α) of the proof of 434R): for 'with union $W[\{x\}]$' read 'with union $W[\{x_0\}]$'.

p 200 l 14 Add new result:

434U Proposition Let X and Y be compact Hausdorff spaces and $f : X \to Y$ a continuous open map. If μ is a completion regular topological measure on X, then the image measure μf^{-1} on Y is completion regular.

p 200 l 30 The exercises for §434 have been re-arranged: 434Xe-434Xy are now 434Xf-434Xz, 434Xz is now 434Xe, 434Yc is now 434Yd, 434Ye-434Yi are now 434Yf-434Yj, 434Yp is now 434Yq, 434Yq is now 434Yp, 434Ys is now 434Yc, 434Yt is now 434Ye.

p 202 l 19 Exercises 434Yd and 434Yj are wrong, and should be deleted.

p 203 l 27 In exercise 434Yt (now 434Ye), we need to suppose that $\mathcal{A}$ is closed under $\cap$.

p l Add sentence to 437Jd:

Writing δ_x for the Dirac measure on X concentrated at x, $x \mapsto \delta_x : X \to P_{\mathrm{top}}$ is a homeomorphism between X and $\{\delta_x : x \in X\}$.

p 228 l 6 (statement Corollary 437L) Add new fact: In particular, the narrow topology on M_τ is completely regular.

p 240 l 38 Exercises 437Xt and 437Xu have been run together as 437Xt. Add new exercises:

(y) Let X and Y be analytic spaces, and $P = P_{\mathrm{R}}(X)$ the space of Radon probability measures on X with its narrow topology. (i) Let V be an analytic subset of $(P \times Y) \times X$. Show that $\{(\mu, y) : \mu \in P,\ y \in Y,\ \mu V[\{(\mu, y)\}] > \alpha\}$ is analytic for every $\alpha \in \mathbb{R}$. (ii) Let W be a coanalytic subset of $(P \times Y) \times X$. Show that $\{(\mu, y) : \mu \in P,\ y \in Y,\ W[\{(\mu, y)\}]$ is not μ-negligible$\}$ is coanalytic.

(z) Let X be a second-countable topological space, $P = P_\tau(X)$ the space of τ-additive probability measures on X with its narrow topology and $\mathcal{C}$ the space of closed subsets of X with the Fell topology. For $\mu \in P$ write $\operatorname{supp}\mu$ for the support of μ. (i) Show that $\{(x, C) : C \in \mathcal{C},\ x \in C\}$ is a Borel subset of $X \times \mathcal{C}$. (ii) Show that $\{(\mu, x) : \mu \in P,\ x \in \operatorname{supp}\mu\}$ is a Borel subset of $P \times X$. (iii) Show that $\mu \mapsto \operatorname{supp}\mu : P \to \mathcal{C}$ is Borel measurable. (Cf. 424Ya.)

p 241 l 37 (Exercise 437Yi) Parts (ii) and (iii) of Exercise 437Yi are mistakes, and should be deleted. The remaining part (i) has been elaborated by adding new material, and the exercise now reads

(i) Let X be a Hausdorff space, and $M_{\mathrm{R}}^{\infty+}$ the set of all Radon measures on X. Define addition and scalar multiplication (by positive scalars) on $M_{\mathrm{R}}^{\infty+}$ as in 234G, 234Xf and 416De and $\leq$ by the formulae of 234P or 416Ea. (i) Show that $M_{\mathrm{R}}^{\infty+}$ is a Dedekind complete lattice. (ii) Show that if $A \subseteq M_{\mathrm{R}}^{\infty+}$ is upwards-directed and non-empty, it is bounded above iff $\{G : G \subseteq X$ is open, $\sup_{\nu \in A} \nu G < \infty\}$ covers X, and in this case $\operatorname{dom}(\sup A) = \bigcap_{\nu \in A} \operatorname{dom}\nu$ and $(\sup A)(E) = \sup_{\nu \in A} \nu E$ for every $E \in \operatorname{dom}(\sup A)$. (iii) Show that if $\mu,\ \nu \in M_{\mathrm{R}}^{\infty+}$ then $\nu = \sup_{n \in \mathbb{N}} \nu \wedge n\mu$ iff every μ-negligible set is ν-negligible. (iv) Show that if $\mu,\ \nu \in M_{\mathrm{R}}^{\infty+}$ then ν is uniquely expressible as $\nu_s + \nu_{ac}$ where $\nu_s,\ \nu_{ac} \in\in M_{\mathrm{R}}^{\infty+}$, $\mu \wedge \nu_s = 0$ and $\nu_{ac} = \sup_{n \in \mathbb{N}} \nu_{ac} \wedge n\mu$. (v) Show that if $\mu,\ \nu \in M_{\mathrm{R}}^{\infty+}$ then $\operatorname{dom}(\mu \vee \nu) = \operatorname{dom}\mu \cap \operatorname{dom}\nu$ and $(\mu \vee \nu)(E) = \sup\{\mu F + \nu(E \setminus F) : F \in \operatorname{dom}\mu \cap \operatorname{dom}\nu,\ F \subseteq E\}$ for every $E \in \operatorname{dom}(\mu \vee \nu)$. (vi) Show that if $\mu,\ \nu \in M_{\mathrm{R}}^{\infty+}$ then $\operatorname{dom}(\mu \wedge \nu) = \{E \cup F : E \in \operatorname{dom}\mu,\ F \in \operatorname{dom}\nu\}$. (vii) Show that if $\mu,\ \nu \in M_{\mathrm{R}}^{\infty+}$ then $\mu \wedge \nu = 0$ iff there is a set $E \subseteq X$ which is μ-negligible and ν-conegligible. (viii) Show that there is a Dedekind complete Riesz space V such that the positive cone of V is isomorphic to $M_{\mathrm{R}}^{\infty+}$.

p 243 l 10 Exercise 437Yw is now 437Xu.

p 258 l 24 (438Y) Add new exercise:

(m) Let (X, Σ, μ) be a complete locally determined measure space, and $\theta = \frac{1}{2}(\mu^* + \mu_*)$ the outer measure described in 413Xd. Show that if the measure μ_θ defined by Carathéodory's method is not equal to μ, then there is a set $A \subseteq X$ such that $0 < \mu^* A < \infty$ and the subspace measure on A induced by μ measures every subset of A.

p 269 l 29 (proof of 439R): for '$\tilde{\nu}_Y$' read '$\tilde{\mu}_Y$'.

p 271 l 4 (Exercise 439Xc) for 'complete locally determined' read 'with locally determined negligible sets'.

p 271 l 24 Exercise 439Xi is now 439Fb, so has been dropped. 439Xn (now 439Xp) has been revised, and reads

(p) (i) Suppose that X is a completely regular space and there is a continuous function f from X to a realcompact completely regular space Z such that $f^{-1}[\{z\}]$ is realcompact for every $z \in Z$. Show that X is realcompact (definition: 436Xg). (ii) Show that the spaces X of 439K and X^2 of 439Q are realcompact.

Other exercises have been re-named: 439Xm is now 439Xo, 439Xo is now 439Xm, 439Xp is now 439Xi.

p 272 l 22 Exercise 439Yf (now 439Xn) has been re-phrased, and reads

(n) Show that a semi-finite Borel measure on ω_1, with its order topology, must be purely atomic.

A second part has been added to 439Yj:

(j)(ii) Show that $\mathbb{R}^{\mathfrak{c}}$ is not a Prokhorov space.

Add new exercise:

(k) Show that the Sorgenfrey line is not a Prokhorov space.

Other exercises have been re-arranged; 439Yd-439Ye are now 439Ye-439Yf, 439Yk is now 439Yd.

p 275 l 45 (part (c-iii) of the proof of 441C): for '$\lceil A:U\rceil\le\lceil b^{-1}\bullet b\bullet A:U\rceil=\lceil A:U\rceil$' read '$\lceil A:U\rceil=\lceil b^{-1}\bullet b\bullet A:U\rceil\le\lceil b\bullet A:U\rceil$'.

p 277 l 5 (proof of 441E): for '$uv^{-1}\in G\setminus M$' read '$u^{-1}v\in G\setminus M$'.

p 279 l 37 The exercises for §441 have been rearranged: 441Xc-441Xs are now 441Xd-441Xt, 441Xt is now 441Xc.

441Yb-441Ye are now 441Yc-441Yf, 441Yf is now 441Yk, 441Yg-441Yi are now 441Yh-441Yj, 441Yj-441Yl are now 441Yl-441Yn, 441Ym is now 441Yb, 441Yn-441Yq are now 441Yo-441Yr.

p 281 l 11 (441X) Add new exercise:

(u) Let X be a set with its zero-one metric and G the group of permutations of X with its topology of pointwise convergence. Let $\mathcal{W}\subseteq\mathcal{P}(X^2)$ be the set of total orderings of X. Show that $\mathcal{W}$ is compact for the usual topology of $\mathcal{P}(X^2)$. For $g\in G$ and $W\in\mathcal{W}$ write $g\bullet W=\{(g(x),g(y)):(x,y)\in W\}$; show that $\bullet$ is a continuous action of G on $\mathcal{W}$. Show that there is a unique G-invariant Radon probability measure μ on $\mathcal{W}$ such that $\mu\{W:(x_i,x_{i+1})\in W$ for every $i<n\}=\frac{1}{(n+1)!}$ whenever $x_0,\dots,x_n\in X$ are distinct.

p 281 l 29 (Exercise 441Ye, now 441Yf) The formula given for $T^\top T$ is wrong; it should be $\begin{pmatrix}\frac{2}{1-z^2}&0&0\\0&2&2z\\0&2z&2\end{pmatrix}$.

p 281 l 37 (441Y) Add new exercise:

(g) Let $\mathbb{H}$ be the division ring of the **quaternions**, that is, $\mathbb{R}^4$ with its usual addition and with multiplication defined by the rule

$$\begin{aligned}(\xi_0,\xi_1,\xi_2,\xi_3)\times(\eta_0,\eta_1,\eta_2,\eta_3)=(&\xi_0\eta_0-\xi_1\eta_1-\xi_2\eta_2-\xi_3\eta_3,\xi_0\eta_1+\xi_1\eta_0+\xi_2\eta_3-\xi_3\eta_2,\\&\xi_0\eta_2-\xi_1\eta_3+\xi_2\eta_0+\xi_4\eta_1,\xi_0\eta_3+\xi_1\eta_2-\xi_2\eta_1+\xi_3\eta_0).\end{aligned}$$

For Lebesgue measurable $E\subseteq\mathbb{H}\setminus\{0\}$, set $\nu E=\int_E\frac{1}{\|x\|^4}dx$. Show that (i) $\|x\times y\|=\|x\|\|y\|$ for all x, $y\in\mathbb{H}$ (ii) $\mathbb{H}\setminus\{0\}$ is a group (iii) ν is a (two-sided) Haar measure on $\mathbb{H}\setminus\{0\}$.

p 289 l 15 (442Y) Add new exercise:

(d) Let (X,ρ) be a metric space, and μ, ν two non-zero quasi-Radon measures on X such that $\mu B(x,\delta)=\mu B(y,\delta)$ and $\nu B(x,\delta)=\nu B(y,\delta)$ for all $\delta>0$ and x, $y\in X$. Show that μ is a multiple of ν.

p l (451X) There is an error in Exercise 451Xk; part (iv) should be deleted. R.O.Cavalcante

p 434 l 18 (451Y) Add new exercise:

(u) Let μ be a measure on $\mathbb{R}$ which is quasi-Radon for the right-facing Sorgenfrey topology. Show that μ is weakly α-favourable.

p 200 l 14 Add new result:

434U Proposition Let X and Y be compact Hausdorff spaces and $f:X\to Y$ a continuous open map. If μ is a completion regular topological measure on X, then the image measure μf^{-1} on Y is completion regular.

p 448 l 5 (452X) Add new exercise:

(v) Let $\langle X_i\rangle_{i\in I}$ be a family of compact Hausdorff spaces with product X, and μ a completion regular topological measure on X. Show that all the marginal measures of μ are completion regular.

p 456 l 32 (proof of 453Ma): for

$$`\iff F \subseteq g_\phi^{-1}[F^*] \text{ for every closed set } F \subseteq X$$
$$\iff F \subseteq \phi F \text{ for every closed set } F \subseteq X'$$

read

$$`\iff g_\phi^{-1}[F^*] \subseteq F \text{ for every closed set } F \subseteq X$$
$$\iff \phi F \subseteq F \text{ for every closed set } F \subseteq X.'$$

(M.R.Burke.)

p 464 l 6 (part (b) of the proof of 454H): for '$\int_{Z_n}\int_{X_n} f(z,\xi_{n+1})\nu_z(d\xi_{n+1})\tilde\mu_n(dz)$' read '$\int_{Z_n}\int_{X_n} f(z,\xi_n)\nu_z(d\xi_n)\tilde\mu_n(dz)$'.

p 465 l 3 (part (b) of the proof of 454J): for '$\nu\{x : \phi(x)(i_r) \le \alpha_r$ for $r \le n\}$' read '$\mu\{\omega : \phi(\omega)(i_r) \le \alpha_r$ for $r \le n\}$'.

p 468 l 34 Proposition 454T, in a different form, is now 434U.
Add new results:

454T Convergence of distributions (a) Let I be a set. Write M for the set of distributions on $\mathbb{R}^I$. I will say that the **vague topology** on M is the topology generated by the functionals $\nu \mapsto \int f d\nu$ as f runs over the space $C_b(\mathbb{R}^I)$ of bounded continuous real-valued functions on $\mathbb{R}^I$.

(b) The vague topology on M is Hausdorff.

454U Theorem Let (Ω,Σ,μ) be a probability space, and I a set. Let M be the set of distributions on $\mathbb{R}^I$; for a family $\boldsymbol{X} = \langle X_i\rangle_{i\in I}$ of real-valued random variables on Ω, let $\nu_{\boldsymbol{X}}$ be its distribution. Then the function $\boldsymbol{X} \mapsto \nu_{\boldsymbol{X}} : \mathcal{L}^0(\mu)^I \to M$ is continuous for the product topology on $\mathcal{L}^0(\mu)^I$ corresponding to the topology of convergence in measure on $\mathcal{L}^0(\mu)$ and the vague topology on M.

454V Distributions of processes in $L^0(\mathfrak{A})^I$ (a)(i) If $\mathfrak{A}$ is a Dedekind σ-complete Boolean algebra, I is a set, and $u \in L^0(\mathfrak{A})^I$, we have a sequentially order-continuous Boolean homomorphism $E \mapsto [\![u \in E]\!] : \mathcal{B}\mathfrak{a}(\mathbb{R}^I) \to \mathfrak{A}$ defined by saying that

$$[\![u \in \{x : x \in \mathbb{R}^I,\, x(i) \le \alpha\}]\!] = [\![u(i) \le \alpha]\!]$$

whenever $i \in I$ and $\alpha \in \mathbb{R}$.

(ii) If $h : \mathbb{R}^I \to \mathbb{R}$ is a Baire measurable function, there is a function $\bar h : L^0(\mathfrak{A})^I \to L^0(\mathfrak{A})$ defined by saying that $[\![\bar h(u) \in E]\!] = [\![u \in h^{-1}[E]]\!]$ for every Borel set $E \subseteq \mathbb{R}$.

(b) Suppose that $(\mathfrak{A},\bar\mu)$ is a probability algebra, I is a set and $u \in L^0(\mathfrak{A})^I$. Then there is a unique complete probability measure ν on $\mathbb{R}^I$, measuring every Baire set and inner regular with respect to the zero sets, such that

$$\nu\{x : x \in \mathbb{R}^I,\, x(i) \in E_i \text{ for every } i \in J\} = \bar\mu(\inf_{i\in J}[\![u(i) \in E_i]\!])$$

whenever $J \subseteq I$ is finite and $E_i \subseteq \mathbb{R}$ is a Borel set for every $i \in J$.

(c) I call ν the **(joint) distribution** of u.

(d) Let $(\mathfrak{A},\bar\mu)$ and $(\mathfrak{A}',\bar\mu')$ be probability algebras, and $u \in L^0(\mathfrak{A})^I$, $u' \in L^0(\mathfrak{A}')^I$ families with the same distribution. Suppose that $\langle h_j\rangle_{j\in J}$ is a family of Baire measurable functions from $\mathbb{R}^I$ to $\mathbb{R}$. Then $\langle \bar h_j(u)\rangle_{j\in J}$ and $\langle \bar h_j(u')\rangle_{j\in J}$ have the same distribution.

(e) If $(\mathfrak{A},\bar\mu)$ is a probability algebra, I a set, and we write ν_u for the distribution of $u \in L^0(\mathfrak{A})^I$, $u \mapsto \nu_u$ is continuous for the product topology on $L^0(\mathfrak{A})^I$ corresponding to the topology of convergence in measure on $L^0(\mathfrak{A})$ and the vague topology on the space of distributions on $\mathbb{R}^I$.

p 469 l 26 Exercise 454Xj is wrong as written, and has been replaced by

(j) Let $\langle X_i\rangle_{i\in I}$ be an independent family of normal random variables. Show that its distribution is a quasi-Radon measure on $\mathbb{R}^I$.

Other exercises have been rearranged: 454Xk-454Xm are now 454Xl-454Xn, 454Xn is now 454Xk.

p 498 l 2 (part (b-i-α) of the proof of 455P): for '$\epsilon \in]0,1[$' read '$\epsilon \in \left]0,\frac{1}{2}\right[$'.

p 503 l 35 (part (a) of the proof of 455T) for 'g is a conditional expectation of f on $\ddot{\Sigma}_\tau$' read 'g is a conditional expectation of f on $\ddot{\Sigma}^+_\tau$'.

p 504 l 21 (455U) The hypothesis

$\tau : C_{\text{dlg}} \to [0,\infty]$ is a stopping time adapted to $\langle\ddot{\Sigma}_t\rangle_{t\ge0}$

has been weakened to

$\tau : C_{\text{dlg}} \to [0,\infty]$ si a stopping time adapted to $\langle\hat{\ddot{\Sigma}}_t\rangle_{t\ge0}$.

p 506 l 42 Part (iv) of Exercise 455Xi is wrong, and has been deleted.

p 548 l 21 (part (a) of the proof of 458G): for 'If H is an atom of Λ and $\mu H > 0$, then there are integers k_i, for $i \in J$, such that $2^{-n}k_i \le g_i(x) < 2^{-n}(k_i + 1)$ for every $i \in J$ and $x \in H$' read 'If H is an atom of Λ and $\mu H > 0$, then there are integers k_{iH}, for $i \in J$, such that $2^{-n}k_{iH} \le g_i(x) < 2^{-n}(k_{iH} + 1)$ for every $i \in J$ and $x \in H$'; and similarly in the next two sentences.

p 548 l 29 (part (a) of the proof of 458G): for

$$|\phi_\Lambda(F,\langle E_i\rangle_{i\in J}) - \int_F \prod_{i\in J} g_i d\mu| \le \max(\int |\prod_{i\in J} g'_{in} - \prod_{i\in J} g_i|d\mu, \int |\prod_{i\in J} g''_{in} - \prod_{i\in J} g_i|d\mu)$$
$$\le \max(\int \sum_{i\in J} |g'_{in} - g_i|d\mu, \int \sum_{i\in J} |g''_{in} - g_i|d\mu)$$

read

$$|\phi_\Lambda(F,\langle E_i\rangle_{i\in J}) - \int_F \prod_{i\in J} g_i d\mu| \le \int \prod_{i\in J} g''_{in} - \prod_{i\in J} g'_i d\mu \le \int \sum_{i\in J} g''_{in} - g'_i d\mu.$$

p 549 l 31 (458I) There is a confusion in the definition of 'relative distribution' which I have resolved by simply declaring

Let (X,Σ,μ) be a probability space, T a σ-subalgebra of Σ, and $f \in \mathcal{L}^0(\mu)$. Then a **relative distribution** of f over T will be a family $\langle\nu_x\rangle_{x\in X}$ of Radon probability measures on $\mathbb{R}$ such that $x \mapsto \nu_x H$ is a conditional expectation of $\chi f^{-1}[H]$ on T, for every Borel set $H \subseteq \mathbb{R}$.

p 550 l 27 (part (a) of the proof of 458K): for '$\int_F \prod_{i=0}^n \nu_{ix} H_i \mu(dx)$' read '$\int_F \prod_{i\in J} \nu_{ix} H_i \mu(dx)$'.

p 555 l 37 (part (b) of the proof of 458P): for '$\lambda = \bar\nu\theta'$' read '$\lambda = \bar\mu'\theta$'.

p 556 l 18 (($\dagger$) of the definition of 'relative product measure'): for 'the functional $F \mapsto \mu_i(E \cap \pi_i^{-1}[F]) :$ T $\to [0,1]$' read 'the functional $F \mapsto \mu_i(E_i \cap \pi_i^{-1}[F]) :$ T $\to [0,1]$'.

p 559 l 4 (458U) The first sentence of the proof needs expanding, as the definition in 458Qb must be applied to $f_i \times \chi(\pi_i^{-1}F)$ rather than to f_i.

p 561 l 1 The exercises in 458Y have been rearranged; 458Yb-458Yh are now 458Yc-458Yi, 458Yi is now 458Yb.

Part II

p 49 l 25 Proposition 465C has been rearranged, with extra fragments, and now reads

465C Proposition Let (X,Σ,μ) be a semi-finite measure space.

(a) Let $A \subseteq \mathbb{R}^X$ be a stable set.

(i) Any subset of A is stable.

(ii) $\overline{A}$, the closure of A in $\mathbb{R}^X$ for the topology of pointwise convergence, is stable.

(iii) $\gamma A = \{\gamma f : f \in A\}$ is stable, for any $\gamma \in \mathbb{R}$.

(iv) If $g \in \mathcal{L}^0 = \mathcal{L}^0(\Sigma)$, then $A + g = \{f + g : f \in A\}$ is stable.

(v) If $g \in \mathcal{L}^0$, then $A \times g = \{f \times g : f \in A\}$ is stable.

(vi) Let $h:\mathbb{R}\to\mathbb{R}$ be a continuous non-decreasing function. Then $\{hf:f\in A\}$ is stable.

(b)(i) Suppose that $A\subseteq\mathbb{R}^X$, $E\in\Sigma$, $n\ge 1$ and $\alpha<\beta$ are such that $0<\mu E<\infty$ and $(\mu^{2n})^*D_n(A,E,\alpha,\beta)<(\mu E)^{2n}$. Then

$$\lim_{k\to\infty}\frac{1}{(\mu E)^{2k}}(\mu^{2k})^*D_k(A,E,\alpha,\beta)=0.$$

(ii) If A, $B\subseteq\mathbb{R}^X$ are stable, then $A\cup B$ is stable.

(iii) If $A\subseteq\mathcal{L}^0$ is finite it is stable.

(iv) If $A\subseteq\mathbb{R}^X$ is stable, so is $\{f^+:f\in A\}\cup\{f^-:f\in A\}$.

(c) Let A be a subset of $\mathbb{R}^X$.

(i) If $\hat\mu$, $\tilde\mu$ are the completion and c.l.d. version of μ, then A is stable with respect to one of the measures μ, $\hat\mu$, $\tilde\mu$ iff it is stable with respect to the others.

(ii) Let ν be an indefinite-integral measure over μ. If A is stable with respect to μ, it is stable with respect to ν and with respect to $\nu\restriction\Sigma$.

(iii) If A is stable, and $Y\subseteq X$ is such that the subspace measure μ_Y is semi-finite, then $A_Y=\{f\restriction Y:f\in A\}$ is stable in $\mathbb{R}^Y$ with respect to the measure μ_Y.

(iv) A is stable iff $A_E=\{f\restriction E:f\in A\}$ is stable in $\mathbb{R}^E$ with respect to the subspace measure μ_E whenever $E\in\Sigma$ has finite measure.

(v) A is stable iff $A_n=\{\mathrm{med}(-n\chi X,f,n\chi X):f\in A\}$ is stable for every $n\in\mathbb{N}$.

(d) Suppose that μ is σ-finite, (Y,T,ν) is another measure space and $\phi:Y\to X$ is inverse-measure-preserving. If $A\subseteq\mathbb{R}^X$ is stable with respect to μ, then $B=\{f\phi:f\in A\}$ is stable with respect to ν.

p 54 l 31 (part (a) of the proof of 465H): for '$M\bigl(\frac{(k-n)!}{k!}-\frac{1}{k^n}\bigr)$' read '$\frac{Mk!}{(k-n)!}\bigl(\frac{(k-n)!}{k!}-\frac{1}{k^n}\bigr)$'.

p 63 l 37 (part (b-iii) of the proof of 465N): the argument won't do, as h^{-1} isn't defined on the whole of $\mathbb{R}$. However there is an easy fix using the new 465C(c-vi).

p 66 l 4 There is a catastrophic error in the proof presented for part (b) of Theorem 465P, and I have no reason to suppose that the claimed result is true. I have therefore rewritten the theorem in the following less general form.

Theorem Let (X,Σ,μ) be a semi-finite measure space, with measure algebra $(\mathfrak{A},\bar\mu)$.

(a) Suppose that $A\subseteq\mathcal{L}^0(\Sigma)$ and that $Q=\{f^\bullet:f\in A\}\subseteq L^0(\mu)$, identified with $L^0=L^0(\mathfrak{A})$. Then Q is stable iff every countable subset of A is stable.

(b) Suppose that μ is complete and strictly localizable and Q is a stable subset of $L^\infty(\mu)$, identified with $L^\infty(\mathfrak{A})$ (363I). Then there is a stable set $B\subseteq\mathcal{L}^\infty(\Sigma)$ such that $Q=\{f^\bullet:f\in B\}$.

p 71 l 21 (proof of 465T): replace the displayed formulae

$$D_k(A,X,\alpha,\beta)=\bigcup_{f\in A}\{w:w\in X^{2k},f(w(2i))<\alpha,$$
$$f(w(2i+1))>\beta\text{ for }i<k\},$$

$$\tilde\mu^{2k}D_k(A,E,\alpha,\beta)=\tilde\mu^{2k}D_k(A',E,\alpha,\beta),$$

$$\begin{aligned}(\tilde\mu^{2k})^*D_k(A,E,\alpha,\beta)&\le\tilde\mu^{2k}D_k(A,E,\alpha',\beta')=\tilde\mu^{2k}D_k(A',E,\alpha',\beta')\\&\le\tilde\mu^{2k}D_k(A',E,\alpha',\beta')<(\mu E)^{2k}\end{aligned}$$

with

$$D'_k(A,X,\alpha,\beta)=\bigcup_{f\in A}\{w:w\in X^{2k},f(w(2i))<\alpha,$$
$$f(w(2i+1))>\beta\text{ for }i<k\},$$

$$\tilde\mu^{2k}D'_k(A,E,\alpha,\beta)=\tilde\mu^{2k}D'_k(A',E,\alpha,\beta),$$

$$(\tilde{\mu}^{2k})^* D_k(A, E, \alpha, \beta) \le \tilde{\mu}^{2k} D'_k(A, E, \alpha', \beta') = \tilde{\mu}^{2k} D'_k(A', E, \alpha', \beta') \le \tilde{\mu}^{2k} D_k(A', E, \alpha', \beta') < (\mu E)^{2k}.$$

p 72 l 40 (part (d) of the proof of 465U): for '$\mu(V \cap \prod_{j<m} E'_{kj}) > 0$' read '$\mu^m(V \cap \prod_{j<m} E'_{kj}) > 0$'.

p 74 l 13 465Xh is now covered by 465C(c-v), so has been dropped. Add new exercises:

(h) Let (X, Σ, μ) be a semi-finite measure space and A a subset of $\mathbb{R}^X$. Suppose that for every $\epsilon > 0$ there is a stable set $B \subseteq \mathbb{R}^X$ such that for every $f \in A$ there is a $g \in B$ such that $\|f - g\|_\infty \le \epsilon$. Show that A is stable.

(i) Let (X, Σ, μ) be a semi-finite measure space and $\langle E_n \rangle_{n\in\mathbb{N}}$ a sequence in Σ. (i) Suppose that whenever $F \in \Sigma$ and $\mu F < \infty$ there is a $k \ge 1$ such that $\sum_{n=0}^\infty (\mu(F \cap E_n)\, \mu(F \setminus E_n))^k$ is finite. Show that $\langle \chi E_n \rangle_{n\in\mathbb{N}}$ is stable. (ii) Suppose that $\mu X = 1$, that $\langle E_n \rangle_{n\in\mathbb{N}}$ is independent, and that $\sum_{n=0}^\infty ((1 - \mu E_n)\mu E_n)^k = \infty$ for every $k \ge 1$. Show that $\langle \chi E_n \rangle_{n\in\mathbb{N}}$ is not stable.

p 74 l 20 Exercise 465Xk is wrong as stated, and has been changed to

(k) Let (X, Σ, μ) be a semi-finite measure space and $A \subseteq \mathbb{R}^X$ a stable set such that $\{f(x) : f \in A\}$ is bounded for every $x \in X$. Let $\overline{A}$ be the closure of A for the topology of pointwise convergence. Show that $\{f^\bullet : f \in \overline{A}\}$ is just the closure of $\{f^\bullet : f \in A\} \subseteq L^0(\mu)$ for the topology of convergence in measure.

Other exercises have been rearranged: 465Xe is now 465Cd, 465Xi-465Xj are now 465Ya-465Yb, 465Xl is now 465Xj, 465Xm-465Xu are now 465Xl-465Xt, 465Ya-465Yi are now 465Yc-465Yk.

p 75 l 30 Exercise 465Yg (now 465Yi) has been corrected, and now reads

(i) Let $(X, \mathfrak{T}, \Sigma, \mu)$ be an effectively locally finite τ-additive topological measure space. Show that a countable R-stable subset of $\mathbb{R}^X$ is stable.

p 99 l 34 (part (g) of the proof of 471D): for '$A_i = U(x_i, \frac{1}{2}\rho(x_i, x_{1-i}))$' read '$A_i = A \cap U(x_i, \frac{1}{2}\rho(x_i, x_{1-i}))$'.

p 100 l 42 (part (a-iii) of the proof of 471G): for 'γC_{mi}', 'γC_{mj}' read 'γ_{mi}', 'γ_{mj}'.

p 101 l 26 (part (a-vi) of the proof of 471G): for '$\le \lim_{n\in I, n\to\infty} \operatorname{diam} D_{ni} \le \lim_{n\in I, n\to\infty} \operatorname{diam} C_{ni} + 4\alpha_i + 4\zeta_i$' read '$\le \liminf_{n\in I, n\to\infty} \operatorname{diam} D_{ni} \le \liminf_{n\in I, n\to\infty} \operatorname{diam} C_{ni} + 4\alpha_i + 4\zeta_i$'.

p 103 l 25 Corollary 471H has been strengthened, and now ends '$\theta_{r\infty}$ is an outer regular Choquet capacity on X'.

p 104 l 36-41 (proof of 471J): for 'ϕ' read 'f', throughout.

p 108 l 7 (part (c) of the proof of 471P): for '$\bigcup_{n\in\mathbb{N}} \tilde{A}_{2^{-n}} \setminus A$' read '$\bigcap_{n\in\mathbb{N}} \tilde{A}_{2^{-n}} \setminus A$'.

p 111 l 6 (471X) Add new exercises:

(c) Let $r \ge 1$ be an integer, and give $\mathbb{R}^r$ the metric $((\xi_1, \dots, \xi_r), (\eta_1, \dots, \eta_r)) \mapsto \max_{i\le r} |\xi_i - \eta_i|$. Show that Lebesgue measure on $\mathbb{R}^r$ is Hausdorff r-dimensional measure for this metric.

(e) Show that all the outer measures $\theta_{r\delta}$ described in 471A are outer regular Choquet capacities.

Other exercises have been renamed: 471Xb is now 471Xd, 471Xc-471Xh are now 471Xf-471Xk, 471Xi is now 471Xb.

p 112 l 11 (471Y) Add new exercise:

(f) Let ρ be a metric on $\mathbb{R}$ inducing the usual topology. Show that the corresponding Hausdorff dimension of $\mathbb{R}$ is at least 1.

p 112 l 26 Exercises 471Yh-471Yi are wrong as stated, and hav been replaced by

(j) Let ρ be the metric on $\{0,1\}^{\mathbb{N}}$ defined in 471Xa. Show that for any integer $k \ge 1$ there are a $\gamma_k > 0$ and a bijection $f : [0,1]^k \to \{0,1\}^{\mathbb{N}}$ such that whenever $0 < r \le 1$, $\mu_{H,rk}$ is Hausdorff rk-dimensional measure on $[0,1]^k$ (for its usual metric) and $\tilde{\mu}_{Hr}$ is Hausdorff r-dimensional measure on $\{0,1\}^{\mathbb{N}}$, then $\mu^*_{H,rk} A \le \gamma_k \tilde{\mu}^*_{Hr} f[A] \le \gamma_k^2 \mu^*_{H,rk} A$ for every $A \subseteq [0,1]^k$.

(k) Let (X,ρ) be a metric space, and $r>0$. Give $X\times\mathbb{R}$ the metric σ where $\sigma((x,\alpha),(y,\beta))=\max(\rho(x,y),|\alpha-\beta|)$. Write μ_L, μ_r and μ_{r+1} for Lebesgue measure on $\mathbb{R}$, r-dimensional Hausdorff measure on (X,ρ) and $(r+1)$-dimensional Hausdorff measure on $(X\times\mathbb{R},\sigma)$ respectively. Let λ be the c.l.d. product of μ_r and μ_L. (i) Show that if $W\subseteq X\times\mathbb{R}$ then $\overline{\int}\mu_r^*W^{-1}[\{\alpha\}]d\alpha\le\mu_{r+1}^*W$. (ii) Show that if $I\subseteq\mathbb{R}$ is a bounded interval, $A\subseteq X$ and μ_r^*A is finite, then $\mu_{r+1}^*(A\times I)=\mu_r^*A\cdot\mu_L I$. (iii) Give an example in which there is a compact set $K\subseteq X\times\mathbb{R}$ such that $\mu_{r+1}K=1$ and $\lambda K=0$. (iv) Show that if μ_r is σ-finite then $\mu_{r+1}=\lambda$.

Other exercises have been moved: 471Yf-471Yg are now 471Yg-471Yh, 471Yj is now 471Yi, 471Yk has been dropped, 471Yl is now 442Yd.

p 176 l 14 (statement of 476A(a-i)): for '$\mu\restriction\mathcal{C}:\mathcal{C}\to\mathbb{R}$' read '$\mu\restriction\mathcal{C}:\mathcal{C}\to[0,\infty[$'.

p 178 l 11 Add new fragment to Lemma 476E:

(b)(ii) For any $A\subseteq X$, $\nu^*\psi(A)\le\nu^*A\le2\nu^*\psi(A)$.

p 179 l 27 (part (e-ii) of the proof of 476E) For '$\nu^*(B(x,\delta)\cap\psi(A))=\nu^*(B(x,\delta)\cap A)$' read

$$\text{`}\mu^*(B(x,\delta)\cap\psi(A))\le\mu^*(B(x,\delta)\cap A)\le2\mu^*(B(x,\delta)\cap\psi(A))$$

where μ is Lebesgue measure on $\mathbb{R}^r$'.

p 184 l 8 (part (b) of the proof of 476K): for '$\int_F(1+(x|e_0))\nu(dx)\ge\int_{F_1}(1+(x|e_0))\nu(dx)$ for every $F\in\mathcal{F}$' read '$\int_F(1+(x|e_0))\nu(dx)\le\int_{F_1}(1+(x|e_0))\nu(dx)$ for every $F\in\mathcal{F}$'.

p 184 l 38 (proof of 476L) for 'ν^*A_1' read '$\nu_X^*A_1$'.

p 217 l 24 (Proposition 478J) Add new part:

(d) Let $G\subseteq\mathbb{R}^r$ be an open set, $K\subseteq G$ a compact set and $g:G\to\mathbb{R}$ a smooth function. Then there is a smooth function $f:G\to\mathbb{R}$ with compact support included in G such that f agrees with g on an open set including K.

p 274 l 50 (441A) Add new part:

(c) If a group G acts on a set X and a measure μ on X is G-invariant, then $\int f(a\bullet x)\mu(dx)$ is defined and equal to $\int f\,d\mu$ whenever f is a virtually measurable $[-\infty,\infty]$-valued function defined on a conegligible subset of X and $\int f\,d\mu$ is defined in $[-\infty,\infty]$.

p 339 l 1 The proof of Theorem 491F is confused, with a potentially catastrophic misquotation of a result from Volume 2, so I have re-written it.

p 340 l 32 (part (a-ii) of the proof of 491H): for the second '$|\int f(yxz)(\lambda*\nu)(dx)-\alpha|\le\epsilon$ for every y, $z\in X$' read '$|\int f(yxz)(\nu*\lambda)(dx)-\alpha|\le\epsilon$ for every y, $z\in X$'.

p 344 l 40 (Example 491Ma): for '$\mu L\ge1-\mu K+\epsilon$' read '$\mu L\ge\mu X-\mu K+\epsilon$'.

p 350 l 10 Add new fragments to 491Xc:

(i) Show that there is a $J\subseteq I$ such that $d^*(J)=d^*(I\setminus J)=d^*(I)$. (iii) Show that if $d(I)$ is defined and $0\le\alpha\le d(I)$ there is a $J\subseteq I$ such that $d(J)$ is defined and equal to α.

p 350 l 11 Add new fragment to 491Xd:

(i)(α) Show that if I, $K\subseteq\mathbb{N}$ are such that $d(I)$ and $d(K)$ are defined, there is a $J\subseteq\mathbb{N}$ such that $d(J)$ is defined and $d(J)=d^*(J\cap I)=d^*(K\cap I)$.

p 350 l 25 Add new fragment to 491Xg:

(iv) Show that a sequence $\langle t_i\rangle_{i\in\mathbb{N}}$ in $[0,1]$ is equidistributed iff $\lim_{n\to\infty}\frac{1}{n+1}\sum_{i=0}^n f(t_i)$ is defined and equal to $\mathrm{R}\!\!\int_0^1 f$ for every Riemann integrable function $f:[0,1]\to\mathbb{R}$.

p 350 l 25 (491X) Add new exercise:

(h) Let X be a topological space, μ a probability measure on X measuring every zero set, and $\langle x_i\rangle_{i\in\mathbb{N}}$ an equidistributed sequence in X. Show that $\lim_{n\to\infty}\dfrac{1}{n+1}\sum_{i=0}^n f(x_i)$ is defined and equal to $\int f\,d\mu$ for every bounded $f:X\to\mathbb{R}$ which is continuous almost everywhere.

491Xh is now 491Xi, 491Xi is now 491Ye.

p 351 l 31 Part (ii) of Exercise 491Xw is covered by 491R, and has been dropped.

p 351 l 41 The exercises in 491Y have been rearranged: 491Yb is now 491Yd.

p 352 l 1 (491Y) Add new exercise:

(c) Let $\mathfrak{A}$ be a Boolean algebra, $A \subseteq \mathfrak{A} \setminus \{0\}$ a non-empty set and $\alpha \in [0,1]$. Show that the following are equiveridical: (i) there is a finitely additive functional $\nu : \mathfrak{A} \to [0,1]$ such that $\nu a \ge \alpha$ for every $a \in A$ (ii) for every sequence $\langle a_n \rangle_{n\in\mathbb{N}}$ in A there is a set $I \subseteq \mathbb{N}$ such that $d^*(I) \ge \alpha$ and $\inf_{i\in I\cap n} a_i \ne 0$ for every $n \in \mathbb{N}$.

491Yc-491Yh are now 491Yf-491Yk.

p 352 l 25 (Exercise 491Yh, now 491Yk) for 'the measure νg^{-1} on Z' read 'the measure νg^{-1} on X'. 491Yi-491Yr are now 491Yl-491Yu, 491Ys is now 491Yb.

p 401 l 23 (statement of Lemma 495C): for '$n_i \in I$' read '$n_i \in \mathbb{N}$'.

p 401 l 30-31 (proof of 495C): in 'Let $\mathcal{E}$ be the subring of $\mathcal{P}X$ generated by $\operatorname{dom} q$' and '$H_q = \bigcup_{q\subseteq q'\in Q,\operatorname{dom} q'=\mathcal{E}} H_{q'}$' replace $\mathcal{E}$ by some other symbol.

p 401 l 31 (proof of 495C): in '$H_q = \bigcup_{q\subseteq q'\in Q,\operatorname{dom} q'=\mathcal{E}'} H_{q'}$ is the union of a finite disjoint family in T' replace 'finite' by 'countable'.

p 403 l 0 Add new result:

495F Proposition Let (X,Σ,μ) be an atomless measure space, $\langle X_i \rangle_{i\in I}$ a countable partition of X into measurable sets and $\gamma > 0$, Let ν be the Poisson point process of (X,Σ,μ) with intensity γ; for $i \in I$ let ν_i be the Poisson point process of (X_i,Σ_i,μ_i) with intensity γ, where μ_i is the subspace measure on X_i and Σ_i its domain. For $S \subseteq X$ set $\phi(S) = \langle S \cap X_i \rangle_{i\in I} \in \prod_{i\in I} \mathcal{P}X_i$. Then ϕ is an isomorphism between ν and the product measure $\prod_{i\in I} \nu_i$ on $\prod_{i\in I} \mathcal{P}X_i$.

Proposition 495F is now 495G, 495G-495P are now 495J-495S.

p 403 l 24 Exercises 495Xc(i) and 495Xa has been moved to the main exposition, as follows:

495H Lemma Let (X,Σ,μ) be an atomless σ-finite measure space, and $\gamma > 0$; let ν be the Poisson point process on X with intensity γ. Suppose that $f : X \to \mathbb{R}$ is a Σ-measurable function such that $\mu f^{-1}[\{\alpha\}] = 0$ for every $\alpha \in \mathbb{R}$. Then $\nu\{S : S \subseteq X,\ f{\restriction}S \text{ is injective}\} = 1$.

495I Proposition Let (X,Σ,μ) be an atomless countably separated measure space and $\gamma > 0$. Let ν' be a complete probability measure on $\mathcal{P}X$ such that $\nu'\{S : S \subseteq X,\ S \cap E = \emptyset\}$ is defined and equal to $e^{-\gamma\mu E}$ whenever $E \in \Sigma$ has finite measure. Then ν' extends the Poisson point process ν on X with intensity γ.

p 410 l 46 Add new part:

(b) If $f \in \mathcal{L}^1(\mu) \cap \mathcal{L}^2(\mu)$, $\int Q_f^2 d\nu$ is defined and equal to $\gamma \int f^2 d\mu + (\gamma \int f d\mu)^2$.

Parts (b) and (c) are now (c)-(d).

p 455 l 36 (part (a) of 4A2E): add new fragment

(iv) Any continuous image of a ccc topological space is ccc.

p l Add new remark to 4A2Ub: $\mathbb{N}^{\mathbb{N}}$ is homeomorphic to $\mathbb{R} \setminus \mathbb{Q}$.

p 442 l 32 There is a blunder in Proposition 498B, which now reads

498B Proposition Let $(X,\mathfrak{T},\Sigma,\mu)$ be an atomless Radon measure space, $(Y,\mathfrak{S},\mathrm{T},\nu)$ an effectively locally finite τ-additive topological measure space and $\tilde\lambda$ the τ-additive product measure on $X \times Y$. Then if $W \subseteq X \times Y$ is closed and $\tilde\lambda W > 0$ there are a non-scattered compact set $K \subseteq X$ and a closed set $F \subseteq Y$ of positive measure such that $K \times F \subseteq W$.

p 476 l 41 In 4A3O, parts (d) and (e) have been exchanged.

p 477 l 13 4A3W has been brought forward to be 4A3Q. 4A3Q-4A3V are now 4A3R-4A3W.

p 477 l 21 In Proposition 4A3R (now 4A3S), parts (a-i) and (a-ii) have been rewritten, and are now

(a) Let $A \subseteq X$ be any set.

(i) There is a largest open set $G \subseteq X$ such that $A \cap G$ is meager.

(ii) $H = X \setminus \overline{G}$ is the smallest regular open set such that $A \setminus H$ is meager; $H \subseteq \overline{A}$.

p 478 l 1 (4A3R, now 4A3S, part (a-v) of the proof): for '$A \setminus G'$' read '$G' \cap A$'.

p 481 l 20 Part (ii) of Exercise 4A3Xa has been upstaged by the new 4A3Ya, so has been dropped.

p 481 l 37 (4A3Y) Add new exercise:

(a) Give an example of a Hausdorff space X with a countable network and a metrizable space Y such that $\mathcal{B}(X \times Y) \neq \mathcal{B}(X)\widehat{\otimes}\mathcal{B}(Y)$.

(c) Give an example of compact Hausdorff spaces X, Y and a function $f : X \to Y$ which is $(\mathcal{B}a(X), \mathcal{B}a(Y))$-measurable but not Borel measurable.

4A3Ya is now 4A3Yb, 4A3Yb-4A3Yc are now 4A3Yb-4A3Ye.

p 502 l 9 In the reference

Frolík Z. [61] 'On analytic spaces', Bull. Acad. Polon. Sci. 8 (1961) 747-750

the volume and page number should be Bull. Acad. Polon. Sci. 9 (1961) 721-726. P.Monteiro