MA365 flashcards p. 1 recto 17.6.2003	$\lim_{z\to 0} \frac{\sin z}{z}$	$\operatorname{Res}_{z_0} f$ (definition)
Maclaurin series for $\sin z$	Maclaurin series for $\cos z$	isolated singularity of an analytic function
removable singularity	pole of order n	essential singularity
$\operatorname{Res}_{z_0} f$ (formula)	$\operatorname{Res}_{z_0} f$ (& Laurent series)	Laurent series near a pole
Maclaurin series for $\operatorname{Ln}(1+z)$	Maclaurin series for $\exp z$	Maclaurin series for $\frac{1}{1-z}$
if $z = e^{i\theta}$ what is dz?	$\int_\Gamma \frac{1}{z} dz$ where Γ is a circle with centre 0 taken anticlockwise once	if $z = e^{i\theta}$ what is $\cos\theta$?

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

MA365 flashcards p. 2 recto 17.6.2003	Set of values of Ln	z^w if w is not an integer
if $z = e^{i\theta}$ what is $\sin\theta$?	$\arctan'(z)$	$1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots$
$\sin\theta$	$\cos\theta$	$\mathcal{R}\mathrm{e}\, e^{i\theta}$
$\mathcal{I}\mathrm{m}\, e^{i\theta}$	$\mathrm{Res}_{z_0} \frac{f}{g}$ provided $z_0 \in \mathrm{dom}\, f \cap \mathrm{dom}\, g$ and $g(z_0) = 0$, $g'(z_0) \neq 0$	Jordan's Lemma
$\mathrm{Ln}(\exp z)$	$\exp(\mathrm{Ln}\, z)$	$\mathrm{Ln}'(z)$
Möbius transformation	Möbius transformation taking z_j to w_j	

MA365 flashcards p. 3 recto 17.6.2003		
$\mathrm{Ln}(z)$ (domain & formula)	$\mathrm{Ln}(\frac{1}{z})$	$z^{1/2}$
$(1+z)^w$ (Binomial series)	$\frac{d}{dz} z^w$ if w is not an integer	$\int_\Gamma g'(z)dz$ where Γ starts at a and ends at b

$\frac{1}{2\pi i}\int_\Gamma f(z)dz$ where Γ is a small circle centre z_0 taken anticlockwise once	1	MA365 flashcards p. 1 verso 17.6.2003
f is defined around, but not at, z_0	$1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \ldots$	$z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \ldots$
an isolated singularity which is neither removable nor a pole	$\lim_{z\to z_0} z^n f(z)$ is defined and not 0	$\lim_{z\to z_0} f(z)$ is defined in $\mathbb{C}$
if the Laurent series in an annulus $\{z : 0 < \lvert z - z_0\rvert < R\}$ is $\sum_{n=0}^\infty c_n(z-z_0)^n$ $+\sum_{n=1}^\infty \frac{d_n}{(z-z_0)^n}$ then z_0 is a pole of order n if $d_n \neq 0$, $d_k = 0$ for $k > n$	if the Laurent series of f in an annulus $\{z : 0 < \lvert z - z_0\rvert < R\}$ is $f(z) = \sum_{n=0}^\infty c_n(z-z_0)^n$ $+\sum_{n=1}^\infty \frac{d_n}{(z-z_0)^n}$ then $\mathrm{Res}_{z_0} f = d_1$	$\frac{1}{k!}\lim_{z\to z_0} \frac{d^k}{dz^k}(z-z_0)^{k+1}f(z)$ whenever the limit exists in $\mathbb{C}$
$1 + z + z^2 + z^3 + z^4 + \ldots$	$1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \ldots$	$z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \ldots$
$\frac{1}{2}(z + \frac{1}{z})$	$2\pi i$	$dz = i\, e^{i\theta}\, d\theta$

$\exp(w \operatorname{Ln} z)$ if $z \in \operatorname{dom}(\operatorname{Ln})$	$\{w : \lvert \mathcal{I}\mathrm{m}\, w \rvert < \pi\}$	MA365 flashcards p. 2 verso 17.6.2003
e	$\frac{1}{1+z^2}$	$\frac{1}{2i}(z - \frac{1}{z})$
$\cos\theta$	$\frac{1}{2}(e^{i\theta} + e^{-i\theta})$	$\frac{1}{2i}(e^{i\theta} - e^{-i\theta})$
provided $\lambda > 0$ *and* $\sup_{\lvert z\rvert=R, \mathcal{I}\mathrm{m}\, z \geq 0} \lvert f(z)\rvert \to 0$ as $R \to \infty$, then $\lim_{R\to\infty} \int_{\Gamma_R} e^{i\lambda z} f(z) dz = 0$ where Γ_R is the semicircle from R to $-R$ in the upper half-plane	$\frac{f(z_0)}{g'(z_0)}$	$\sin\theta$
$\frac{1}{z}$ if $z \in \operatorname{dom}(\operatorname{Ln})$	z if $z \in \operatorname{dom}(\operatorname{Ln})$	z if $\lvert \mathcal{I}\mathrm{m}\, z\rvert < \pi$
	$\frac{(w-w_1)(w_2-w_3)}{(w-w_2)(w_1-w_3)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_2)(z_1-z_3)}$	$w = \frac{az+b}{cz+d}$ where $ad \neq bc$

MA365 flashcards
p. 3 verso
17.6.2003

the square root of z
with positive real part
if $z \in \mathrm{dom}(\mathrm{Ln})$

$-\mathrm{Ln}(z)$ if $z \in \mathrm{dom}(\mathrm{Ln})$

$\{z : z \text{ is not real} \leq 0\}$
$\ln(|z|) + i \arg z$

$g(b) - g(a)$
provided g is analytic
and defined at every point
of Γ

wz^{w-1}
if $z \in \mathrm{dom}(\mathrm{Ln})$

$$1 + wz + \frac{w(w-1)}{2!}z^2 + \frac{w(w-1)(w-2)}{3!}z^3 + \ldots$$

if *either* $w \in \mathbb{N}$
or $|z| < 1$