Group Theory ten-minute test questions

Which of the following are semigroups? [AA]
(a) $(\mathbb{N},+)$
(b) $(\mathbb{N},\times)$
(c) $(\mathbb{N},-)$
(d) $(\mathbb{N},\div)$
(e) $(\mathbb{N}\setminus\{0\},+)$

Which of the following are semigroups? [AB]
(a) $(\mathbb{N}\setminus\{0\},\times)$
(b) $(\mathbb{N}\setminus\{0\},-)$
(c) $(\mathbb{Z},+)$
(d) $(\mathbb{Z},\times)$
(e) $(\mathbb{Z},-)$

Which of the following are semigroups? [AC]
(a) $(\mathbb{Z}\setminus\{0\},+)$
(b) $(\mathbb{Z}\setminus\{0\},\times)$
(c) $(\mathbb{R},+)$
(d) $(\mathbb{R},\times)$
(e) $(\mathbb{R},-)$

Which of the following are semigroups? [AD]
(a) $(\mathbb{R},\div)$
(b) $(\mathbb{R}\setminus\{0\},+)$
(c) $(\mathbb{R}\setminus\{0\},\times)$
(d) $(\mathbb{R}\setminus\{0\},\div)$
(e) $(\mathbb{Q},+)$

Which of the following are semigroups? [AE]
(a) $(\mathbb{Q},\times)$
(b) $(\mathbb{Q}\setminus\{0\},\times)$
(c) $(\mathbb{Q}\setminus\{0\},\div)$
(d) $(\mathbb{C},+)$
(e) $(\mathbb{C},\times)$

Which of the following are semigroups? [AF]
(a) $(\mathbb{C},-)$
(b) $(\mathbb{C}\setminus\{0\},+)$
(c) $(\mathbb{C}\setminus\{0\},\times)$
(d) $(\mathbb{R}^n,+)$ where $+$ is vector addition
(e) $(\mathbb{R}^3,\times)$ where $\times$ is cross-product

Which of the following are semigroups with identity? [AG]
(a) $(\mathbb{N},+)$
(b) $(\mathbb{N},\times)$
(c) $(\mathbb{Z},+)$
(d) $(\mathbb{Z},\times)$
(e) $(\mathbb{N}\setminus\{0\},+)$

Which of the following are semigroups with identity? [AH]
(a) $(\mathbb{N}\setminus\{0\},\times)$
(b) $(\mathbb{Z}\setminus\{0\},\times)$
(c) $(\mathbb{R},+)$
(d) $(\mathbb{R},\times)$
(e) $(\mathbb{Q},+)$

Which of the following are semigroups with identity? [AI]
(a) $(\mathbb{Q}, \times)$
(b) $(\mathbb{Q} \setminus \{0\}, \times)$
(c) $(\mathbb{R} \setminus \{0\}, \times)$
(d) $(\mathbb{C}, +)$
(e) $(\mathbb{C}, \times)$

In the proof that a semigroup has at most one identity, which of the following formulae would you expect to see? [AN]
(a) $e = ee' = e'$
(b) $y = ye = yxz = ez = z$
(c) $e = e^2$
(d) $(x^{-1})^{-1} = x$
(e) $e^{-1} = e$

Which of the following are semigroups? [AP]
(a) The set of all functions from $\mathbb{N}$ to itself, with composition
(b) The set of all injective functions from $\mathbb{N}$ to itself, with composition
(c) The set of all surjective functions from $\mathbb{N}$ to itself, with composition
(d) The set of all bijective functions from $\mathbb{N}$ to itself, with composition
(e) The set of all bijective functions from $\mathbb{N}$ to $\mathbb{Z}$, with composition

Which of the following are semigroups? [AQ]
(a) The set of all 3×3 real matrices, with matrix multiplication
(b) The set of all non-zero 3×3 real matrices, with matrix multiplication
(c) The set of all 3×3 real matrices with determinant greater than or equal to 0, with matrix multiplication
(d) The set of all 3×3 real matrices with determinant greater than 0, with matrix multiplication
(e) The set of all non-singular 3×3 real matrices, with matrix multiplication

Which of the following are semigroups with identity? [AR]
(a) The set of all functions from $\mathbb{N}$ to itself, with composition
(b) The set of all injective functions from $\mathbb{N}$ to itself, with composition
(c) The set of all surjective functions from $\mathbb{N}$ to itself, with composition
(d) The set of all bijective functions from $\mathbb{N}$ to itself, with composition
(e) The set of all functions from $\mathbb{N}$ to $\mathbb{Z}$, with composition

Which of the following are semigroups with identity? [AS]
(a) The set of all 3×3 real matrices, with matrix multiplication
(b) The set of all singular 3×3 real matrices, with matrix multiplication
(c) The set of all 3×3 real matrices with determinant greater than or equal to 0, with matrix multiplication
(d) The set of all 3×3 real matrices with determinant greater than 0, with matrix multiplication
(e) The set of all non-singular 3×3 real matrices, with matrix multiplication

Which of the following are groups? [AT]
(a) The set of all functions from $\mathbb{N}$ to itself, with composition
(b) The set of all injective functions from $\mathbb{N}$ to itself, with composition
(c) The set of all surjective functions from $\mathbb{N}$ to itself, with composition
(d) The set of all bijective functions from $\mathbb{N}$ to itself, with composition
(e) The set of all injective functions from $\{0, 1, 2, 3, 4\}$ to itself, with composition

Which of the following are groups? [AU]
(a) The set of all 3×3 real matrices, with matrix multiplication
(b) The set of all non-singular 3×3 matrices with integer coefficients, with matrix multiplication
(c) The set of all 3×3 real matrices with determinant greater than or equal to 0, with matrix multiplication
(d) The set of all 3×3 real matrices with determinant greater than 0, with matrix multiplication
(e) The set of all non-singular 3×3 real matrices, with matrix multiplication

Which of the following are groups? [BA]
(a) $(\mathbb{N}, +)$
(b) $(\mathbb{N}, \times)$
(c) $(\mathbb{Z}, +)$
(d) $(\mathbb{Z}, \times)$
(e) $(\mathbb{N}, -)$

Which of the following are groups? [BB]
(a) $(\mathbb{N} \setminus \{0\}, \times)$
(b) $(\mathbb{Z} \setminus \{0\}, \times)$
(c) $(\mathbb{R}, +)$
(d) $(\mathbb{R}, \times)$
(e) $(\mathbb{Q}, +)$

Which of the following are groups? [BC]
(a) $(\mathbb{Q}, \times)$
(b) $(\mathbb{Q} \setminus \{0\}, \times)$
(c) $(\mathbb{R} \setminus \{0\}, \times)$
(d) $(\mathbb{C}, +)$
(e) $(\mathbb{C}, \times)$

In the proof that an element in a semigroup with identity has at most one inverse, which of the following formulae would you expect to see? [BD]
(a) $e = ee' = e'$
(b) $y = ye = yxz = ez = z$
(c) $e = e^2$
(d) $(x^{-1})^{-1} = x$
(e) $e^{-1} = e$

Which of the following are true in any group? (a, x, y are arbitrary elements of the group.) [BE]
(a) $ax = ay \Rightarrow x = y$
(b) $xa = ay \Rightarrow x = y$
(c) $xa = ya \Rightarrow x = y$
(d) $x^{-1} = y^{-1} \Rightarrow x = y$
(e) $xx = yy \Rightarrow x = y$

Which of the following are true in any group? (x, y are arbitrary elements of the group, and e is the identity.) [BF]
(a) $xy = x \Rightarrow y = e$
(b) $xy = x \Rightarrow x = e$
(c) $xy = y \Rightarrow x = e$
(d) $xx = x \Rightarrow x = e$
(e) $xx = e \Rightarrow x = e$

Which of the following are true in any group? (x, y are arbitrary elements of the group, and e is the identity.) [BG]
(a) $(x^{-1})^{-1} = x$
(b) $(xy)^{-1} = x^{-1}y^{-1}$
(c) $(xy)^{-1} = y^{-1}x^{-1}$
(d) $e^{-1} = e$
(e) $(xyx^{-1})^{-1} = xy^{-1}x^{-1}$

Which of the following are true? [BH]
(a) $\mathbb{N}$ is a subgroup of $(\mathbb{Z}, +)$
(b) $\mathbb{Z}$ is a subgroup of $(\mathbb{R}, +)$
(c) $\mathbb{Z} \setminus \{0\}$ is a subgroup of $(\mathbb{R} \setminus \{0\}, \times)$
(d) $]0, \infty[$ is a subgroup of $(\mathbb{R} \setminus \{0\}, \times)$
(e) $\left]\frac{1}{2}, 2\right[$ is a subgroup of $(\mathbb{R} \setminus \{0\}, \times)$

Let G be the group of all bijective functions from $\mathbb{N}$ to itself. Which of the following are subgroups of G? [BI]
(a) $\{g : g \in G,\, g(0) = 0\}$
(b) $\{g : g \in G,\, g(1) = 0\}$
(c) $\emptyset$
(d) $\{g : g \in G,\, g(0) = 0,\, g(1) = 1\}$
(e) $\{g : g \in G,\, g(0) = 1,\, g(1) = 0\}$

In the group S_3, which of the following are equal to the product $(\overleftarrow{12})(\overleftarrow{13})$? [CA]
(a) $(\overleftarrow{132})$
(b) $(\overleftarrow{123})$
(c) $(\overleftarrow{23})$
(d) $(\overleftarrow{21})$
(e) $(\overleftarrow{321})$

The group A_5 has 60 elements. If H is a subgroup of A_5 with 4 members, how may left cosets does H have in A_5? [CB]
(a) 15
(b) 12
(c) 4
(d) 60
(e) 1

The group S_4 has 24 elements. If H is a subgroup of S_4 with 8 elements, and $x \in S_4$, how many different left cosets of H contain x? [CC]
(a) 1
(b) 8
(c) we don't have enough information to decide
(d) 3
(e) 24

If, in the group S_3, we set $\rho = (\overleftarrow{123})$ and $\sigma = (\overleftarrow{12})$, then $\rho^3 = \sigma^2 = e$ and $\rho\sigma = \sigma\rho^2$. Let K be the subgroup $\{e, \sigma\}$. Which of the following are left cosets of K? [CD]
(a) $\{e, \sigma\}$
(b) $\{\rho, \sigma\rho^2\}$
(c) $\{\rho^2, \sigma\rho\}$
(d) $\{\rho, \sigma\rho\}$
(e) $\{\rho^2, \sigma\rho^2\}$

In the lemmas leading to the proof of Lagrange's theorem, which of the following would you expect to see? [CE]
(a) if $x \in aH$ then $aH = xH$
(b) the function $h \mapsto ah : H \to aH$ is bijective
(c) $xH = Hx$
(d) $x \in xH$
(e) if $x \in aH$ then $a \in xH$

If H is a subgroup of a group G, which of the following are enough to ensure that H is a normal subgroup? [DA]

(a) $aha^{-1} \in H$ for every $a \in G$, $h \in H$
(b) $aha^{-1} \in H$ for every $a \in H$, $h \in H$
(c) $a^{-1}ha \in H$ for every $a \in G$, $h \in H$
(d) $aH \subseteq Ha$ for every $a \in G$
(e) $ab^{-1} \in H$ for all a, $b \in H$

In the proof that the set G/H of cosets of a normal subgroup H of a group G is a group, which of the following facts would you expect to use in some way? [DB]

(a) $A(BC) = (AB)C$ for all subsets A, B, C of G
(b) $HH = H$
(c) $eH = H$
(d) $H^{-1} = H$
(e) $\#(H)$ is a factor of $\#(G)$

Which of the following functions ϕ are homomorphisms between the given groups G and H? [DC]

(a) $G = (\mathbb{R}, +)$, $H = (\mathbb{R} \setminus \{0\}, \times)$, $\phi(x) = \exp(x)$ for $x \in \mathbb{R}$
(b) $G = (]0, \infty[\,, \times)$, $H = (\mathbb{R}, +)$, $\phi(x) = \ln x$ for $x > 0$
(c) $G = H = (\mathbb{R} \setminus \{0\}, \times)$, $\phi(x) = x^2$ for $x \in \mathbb{R} \setminus \{0\}$
(d) $G = H = S_3$ with composition, $\phi(a) = a^2$ for $a \in S_3$
(e) $G = H$ the group of non-singular 2×2 matrices with matrix multiplication, $\phi(\mathbf{A}) = \mathbf{A}^{-1}$ for such matrices $\mathbf{A}$

Let G, H be groups and $\phi : G \to H$ a homomorphism. Which of the following are necessarily true for all a, b, $c \in G$? [DD]

(a) $\phi(ab) = \phi(b)\phi(a)$
(b) $\phi(abc) = \phi(a)\phi(b)\phi(c)$
(c) $\phi(aba^{-1}) = \phi(a)\phi(b)\phi(a)^{-1}$
(d) $\phi(aba^{-1}) = \phi(b)\phi(a)\phi(b)^{-1}$
(e) $\phi(aba^{-1}) = \phi(a)^{-1}\phi(b)\phi(a)$

If G and H are groups and $\phi : G \to H$ is a homomorphism, which of the following are equal to the kernel of ϕ? [EA]

(a) $\{x : x \in G,\ \phi(x) = e_H\}$
(b) $\{x : x \in H,\ \phi(x) = e_H\}$
(c) $\{x : x \in G,\ \phi(x) = e_G\}$
(d) $\{x : x \in G,\ \phi^{-1}(x) = e_H\}$
(e) $\{x : x \in H,\ \phi^{-1}(x) = e_G\}$

If G and H are groups and $\phi : G \to H$ is a homomorphism with kernel K and image $\phi[G]$, which of the following are necessarily true? [EB]

(a) $K \lhd G$
(b) K is a subgroup of G
(c) $\phi[G] \lhd H$
(d) $\phi[G]$ is a subgroup of H
(e) $K \lhd H$

If G and H are groups and $\phi : G \to H$ is a homomorphism, which of the following are necessarily true? [EC]

(a) ϕ^{-1} is a homomorphism
(b) if ϕ is bijective, then $\phi^{-1} : H \to G$ is a homomorphism
(c) if ϕ is injective, then $\phi^{-1} : \phi[G] \to G$ is a homomorphism
(d) if ϕ is surjective, then ϕ^{-1} is a homomorphism
(e) $\phi^{-1} : \phi[G] \to G$ is a homomorphism

Which of the following is the First Isomorphism Theorem? [FA]

(a) if G and H are groups, and $\phi : G \to H$ a homomorphism with kernel K, then $G/K \cong \phi[G]$
(b) if G and H are groups, and $\phi : G \to H$ a homomorphism with kernel K, then $H/K \cong \phi[H]$
(c) if G and H are groups, and $\phi : G \to H$ a homomorphism with kernel K, then $G \cong \phi[G]K$
(d) if G and H are groups, and $\phi : G \to H$ a homomorphism with kernel K, then $G/\phi[G] \cong K$
(e) if G and H are groups, and $\phi : G \to H$ a homomorphism with kernel K, then $G \setminus K \cong \phi[G]$

Let G and H be groups, and $\phi : G \to H$ a homomorphism with kernel K. Which of the following are true? [FB]

(a) for $x, y \in G$, $xK = yK$ iff $\phi(x) = \phi(y)$
(b) if $x, y \in G$ and $x^{-1}y \in K$, then $xK = yK$
(c) $xK \mapsto \phi(x)$ defines an isomorphism from G/K to H
(d) $xK \mapsto \phi(x)$ defines a homomorphism from G/K to H
(e) $xK \mapsto \phi(x)$ defines a function from G/K to $\phi[G]$

How many different subgroups of $\mathbb{Z}$ contain the number 12? [FC]

(a) 6
(b) 12
(c) none
(d) 1
(e) 4

Which of the following are equal to $(\overleftarrow{123})(\overleftarrow{234})$? [GA]

(a) $(\overleftarrow{12})(\overleftarrow{34})$
(b) $(\overleftarrow{43})(\overleftarrow{12})$
(c) $(\overleftarrow{124})$
(d) $(\overleftarrow{421})$
(e) $(\overleftarrow{14})(\overleftarrow{23})$

What is the order of the permutation $(\overleftarrow{12})(\overleftarrow{345})(\overleftarrow{6\,7\,8\,9\,10\,11})$? [GB]

(a) 12
(b) 6
(c) infinite
(d) 36
(e) 1

Which of the following are the inverse of $(\overleftarrow{123})(\overleftarrow{4567})$? [GC]

(a) $(\overleftarrow{7654})(\overleftarrow{321})$
(b) $(\overleftarrow{132})(\overleftarrow{4765})$
(c) $(\overleftarrow{321})(\overleftarrow{7654})$
(d) $(\overleftarrow{132})(\overleftarrow{46})(\overleftarrow{67})$
(e) $(\overleftarrow{312})(\overleftarrow{7654})$

Which of the following are equal to $(\overleftarrow{1234})$? [HA]

(a) $(\overleftarrow{14})(\overleftarrow{13})(\overleftarrow{12})$
(b) $(\overleftarrow{12})(\overleftarrow{13})(\overleftarrow{14})$
(c) $(\overleftarrow{41})(\overleftarrow{31})(\overleftarrow{21})$
(d) $(\overleftarrow{13})(\overleftarrow{24})(\overleftarrow{23})$
(e) $(\overleftarrow{23})(\overleftarrow{14})(\overleftarrow{13})(\overleftarrow{12})$

Which of the following are even permutations? [HB]
(a) $(\overleftarrow{1234})$
(b) $(\overleftarrow{12345})$
(c) $(\overleftarrow{12})(\overleftarrow{345})$
(d) $(\overleftarrow{12})(\overleftarrow{34})$
(e) $(\overleftarrow{12})(\overleftarrow{34})(\overleftarrow{56})$

Which of the following are even permutations? [HC]
(a) $(\overleftarrow{12})(\overleftarrow{23})(\overleftarrow{345})$
(b) $(\overleftarrow{123})(\overleftarrow{23})(\overleftarrow{345})$
(c) $(\overleftarrow{12})(\overleftarrow{234})(\overleftarrow{345})$
(d) $(\overleftarrow{124})(\overleftarrow{23})(\overleftarrow{3456})$
(e) $(\overleftarrow{123})(\overleftarrow{321})$

In the proof that $\epsilon_{\rho\sigma} = \epsilon_\rho \epsilon_\sigma$ and the lemmas leading to it, as given in lectures, which of the following were used? [HD]
(a) $(\overleftarrow{ij})(\overleftarrow{i \ldots j \ldots}) = (\overleftarrow{i \ldots})(\overleftarrow{j \ldots})$
(b) every permutation is expressible as a product of transpositions
(c) $\epsilon_{\rho^{-1}} = \epsilon_\rho$
(d) $\epsilon_{\rho\sigma} = \epsilon_{\sigma\rho}$
(e) every 4-cycle is an odd permutation

Writing K for $\{e, (\overleftarrow{12})(\overleftarrow{34}), (\overleftarrow{13})(\overleftarrow{24}), (\overleftarrow{14})(\overleftarrow{23})\}$, which of the following are true? [IA]
(a) $K \lhd A_4$
(b) $K \lhd S_4$
(c) K is a subgroup of A_5
(d) K is a normal subgroup of A_5
(e) K is a subgroup of S_5

,
Which of the following are true? [IB]
(a) Whenever $\rho \in S_n$ and $\sigma \in S_n$ is a k-cycle, $\rho\sigma\rho^{-1}$ is a k-cycle
(b) Whenever $\rho \in S_n$ and $\sigma \in S_n$ is a k-cycle, $\rho^{-1}\sigma\rho$ is a k-cycle
(c) Whenever $\rho \in S_n$ and $\sigma \in S_n$ is a k-cycle, $\sigma\rho\sigma^{-1}$ is a k-cycle
(d) Whenever $\rho \in S_n$ and $\sigma \in S_n$ is a product of two transpositions, then $\rho\sigma\rho^{-1}$ is a product of two transpositions
(e) Whenever $\rho \in S_n$ and $\sigma \in S_n$ is a k-cycle, $\rho\sigma\rho$ is a k-cycle

Which of the following are true? [IC]
(a) S_4 has exactly four normal subgroups
(b) S_n has exactly three normal subgroups for every $n \geq 5$
(c) A_3 has exactly two normal subgroups
(d) S_2 has exactly three normal subgroups
(e) A_n has exactly two normal subgroups for every $n \geq 3$

Which of the following are true? (Count the identity permutation as a product of no 3-cycles). [JA]
(a) every even permutation is a product of 3-cycles
(b) every permutation is a product of 3-cycles
(c) the product of two transpositions is always a 3-cycle
(d) the product of two transpositions is always the product of at most two 3-cycles
(e) $(\overleftarrow{12})(\overleftarrow{34})$ is the product of two 3-cycles

Which of the following are true? [JB]

(a) if $H \lhd A_n$ contains a 3-cycle, then $H = A_n$
(b) if $H \lhd S_n$ contains a 3-cycle, then $H = S_n$
(c) if H is a subgroup of A_n containing a 3-cycle, then $H = A_n$
(d) if H is a subgroup of A_n containing all the 3-cycles of S_n, then $H = A_n$
(e) if H is a subgroup of S_n containing all the 3-cycles of S_n, then $H = S_n$

Which of the following are true? [JC]

(a) if $(\overleftarrow{123\dots k})$ is one of the cycles in the disjoint-cycle representation of ρ, where $k \geq 4$, and $\sigma = (\overleftarrow{123})$, then $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 3-cycle
(b) if ρ is any permutation, and $\sigma = (\overleftarrow{123})$, then $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 3-cycle
(c) if the disjoint-cycle representation of ρ has a 4-cycle, and σ is a 3-cycle, then $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 3-cycle
(d) if the disjoint-cycle representation of ρ has a 4-cycle, then there is a 3-cycle σ such that $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 3-cycle
(e) if ρ is any permutation, then there is a 3-cycle σ such that $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 3-cycle

Suppose that G is a group, H is a subgroup of G, $h \in H$ and $a \in G$. Which of the following must be true? [JD]

(a) if $H \lhd G$ then $hah^{-1}a^{-1} \in H$
(b) $hah^{-1}a^{-1} \in H$
(c) if $H \lhd G$ then $ah^{-1}a^{-1}h \in H$
(d) if $a \in H$ then $hah^{-1}a^{-1} \in H$
(e) if $hah^{-1}a^{-1} \in H$ then $a \in H$

Which of the following are true? [KA]

(a) if $(\overleftarrow{123})$, $(\overleftarrow{4\dots k})$ are two of the cycles in the disjoint-cycle representation of ρ, where $k \geq 5$, and $\sigma = (\overleftarrow{124})$, then $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 3-cycle
(b) if $(\overleftarrow{123})$, $(\overleftarrow{4\dots k})$ are two of the cycles in the disjoint-cycle representation of ρ, where $k \geq 5$, and $\sigma = (\overleftarrow{124})$, then $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 5-cycle
(c) if ρ is not itself a 3-cycle, but the disjoint-cycle representation of ρ has a 3-cycle, and σ is a 3-cycle, then $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 3-cycle
(d) if ρ is not itself a 3-cycle, but the disjoint-cycle representation of ρ has a 3-cycle, then there is a 3-cycle σ such that $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 5-cycle
(e) if ρ is any permutation, then there is a 3-cycle σ such that $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 5-cycle

Which of the following are true? [KB]

(a) if $(\overleftarrow{12})$, $(\overleftarrow{34})$ and $(\overleftarrow{56})$ are three of the cycles in the disjoint-cycle representation of ρ, and $\sigma = (\overleftarrow{135})$, then $\rho\sigma\rho^{-1}\sigma^{-1}$ is a product of two disjoint 3-cycles
(b) if ρ is a product of three or more disjoint transpositions, then there is a 3-cycle σ such that $\rho\sigma\rho^{-1}\sigma^{-1}$ has a 3-cycle in its disjoint-cycle representation
(c) if $\rho \in S_n$ is a product of two or more disjoint transpositions, then there is a 3-cycle $\sigma \in S_n$ such that $\rho\sigma\rho^{-1}\sigma^{-1}$ has a 3-cycle in its disjoint-cycle representation
(d) if $\rho, \sigma \in S_n$ then $\rho\sigma\rho^{-1}\sigma^{-1} \in A_n$
(e) if $\rho \in S_6$ is a product of three disjoint transpositions and $\sigma \in S_6$ is a 3-cycle, then $\rho\sigma\rho^{-1}\sigma^{-1}$ has a 3-cycle in its disjoint-cycle representation

Let G be a group and H, K two subgroups of G. Which of the following must be true? [KC]

(a) $H \cap K$ is a subgroup of G
(b) if $H \lhd G$ then $H \cap K \lhd K$
(c) if $H \lhd G$ then $H \cap K \lhd H$
(d) if $H \lhd G$ and $K \lhd G$ then $H \cap K \lhd G$
(e) if $H \lhd G$ and $K \lhd G$ then $H \cap K$ is a subgroup of G

Which of the following are true? [LA]
(a) for any two transpositions τ_1, $\tau_2 \in S_n$ there is a $\sigma \in S_n$ such that $\tau_2 = \sigma\tau_1\sigma^{-1}$
(b) for any two 3-cycles ρ_1, $\rho_2 \in S_n$ there is a $\sigma \in S_n$ such that $\rho_2 = \sigma\rho_1\sigma^{-1}$
(c) for any two 3-cycles ρ_1, $\rho_2 \in A_n$ there is a $\sigma \in A_n$ such that $\rho_2 = \sigma\rho_1\sigma^{-1}$
(d) for any two transpositions τ_1, $\tau_2 \in S_n$ there is a $\sigma \in S_n$ such that $\tau_2 = \sigma^{-1}\tau_1\sigma$
(e) for any two transpositions τ_1, $\tau_2 \in S_n$ there is a $\sigma \in S_n$ such that $\tau_2 = \tau_1\sigma\tau_1^{-1}$

Which of the following are true? [LB]
(a) if $(\overleftarrow{12\ldots k})$ is one of the cycles in the disjoint-cycle representation of ρ, where $k \geq 3$, and $\tau = (\overleftarrow{12})$, then $\rho\tau\rho^{-1}\tau^{-1}$ is a 3-cycle
(b) if a permutation ρ has a cycle of length 3 or more in its disjoint-cycle representation, there is a transposition τ such that $\rho\tau\rho^{-1}\tau^{-1}$ is a 3-cycle
(c) if a permutation $\rho \in S_n$ has a cycle of length 3 or more in its disjoint-cycle representation, and $\tau \in S_n$ is a transposition, then $\rho\tau\rho^{-1}\tau^{-1}$ is a 3-cycle
(d) if $(\overleftarrow{12\ldots k})$ is one of the cycles in the disjoint-cycle representation of ρ, where $k \geq 3$, and $\tau = (\overleftarrow{12})$, then $\rho\tau\rho^{-1}\tau^{-1}$ is a k-cycle
(e) if $(\overleftarrow{12\ldots k})$ is one of the cycles in the disjoint-cycle representation of ρ, where $k \geq 3$, and $\sigma = (\overleftarrow{123})$, then $\rho\sigma\rho^{-1}\sigma^{-1}$ is a 3-cycle

Let X be a regular polygon with $n \geq 3$ sides. If ρ is a rotation of X through an angle of $2\pi/n$ and σ is a reflection about an axis through the centre of X and the midpoint of one of its edges, which of the following are true? [MA]
(a) $\rho^n = e$
(b) $\sigma^2 = e$
(c) every isometry of X is of the form $\sigma^i\rho^j$ where i, $j \in \mathbb{N}$
(d) $\sigma\rho = \rho\sigma$
(e) $\rho\sigma\rho^{-1} = \sigma^{-1}$

Let X be a regular polygon with $n \geq 3$ sides. If ρ is a rotation of X through an angle of $2\pi/n$ and σ is a reflection about an axis through the centre of X and one of its vertices, which of the following are true? [MB]
(a) ρ has order n
(b) $(\sigma\rho^i)^2 = e$ for every $i \in \mathbb{Z}$
(c) $\sigma\rho\sigma^{-1} = \rho^{-1}$
(d) $\sigma\rho = \rho\sigma$
(e) $\rho\sigma\rho^{-1} = \sigma\rho^{-2}$

Let X be a regular polygon with $n \geq 3$ sides. If ρ is a rotation of X through an angle of $2\pi/n$ and σ is any reflection taking X to itself, which of the following are necessarily true? [MC]
(a) every isometry of X is of the form $\sigma^i\rho^{2j}$ where i, $j \in \mathbb{N}$
(b) $\sigma^4 = e$
(c) $\sigma\rho\sigma^{-1} = \rho^{-1}$
(d) $\sigma\rho \neq \rho\sigma$
(e) $\rho\sigma\rho^{-1} = \sigma$

Which of the following is isomorphic to the isometry group of the regular tetrahedron? [NA]
(a) S_4
(b) A_4
(c) D_6, the dihedral group of order 12
(d) D_{12}, the dihedral group of order 24
(e) S_3

Which of the following is isomorphic to the rotation group of the regular tetrahedron? [NB]
(a) S_4
(b) A_4
(c) D_6, the dihedral group of order 12
(d) D_{12}, the dihedral group of order 24
(e) S_3

Regarding the rotation group of the cube as acting on its four long diagonals, what kind of permutation of the diagonals corresponds to a 90° rotation about an axis through the midpoints of opposite faces? [NC]
(a) a 4-cycle
(b) a 3-cycle
(c) a transposition
(d) a double-transposition
(e) the identity permutation

Regarding the rotation group of the cube as acting on its four long diagonals, what kind of permutation of the diagonals corresponds to a 120° rotation about a long diagonal? [ND]
(a) a 4-cycle
(b) a 3-cycle
(c) a transposition
(d) a double-transposition
(e) the identity permutation

Regarding the rotation group of the cube as acting on its four long diagonals, what kind of permutation of the diagonals corresponds to a 180° rotation about an axis through the midpoints of opposite edges? [NE]
(a) a 4-cycle
(b) a 3-cycle
(c) a transposition
(d) a double-transposition
(e) the identity permutation

Regarding the rotation group of the cube as acting on its four long diagonals, what kind of permutation of the diagonals corresponds to a 180° rotation about an axis through the midpoints of opposite faces? [NF]
(a) a 4-cycle
(b) a 3-cycle
(c) a transposition
(d) a double-transposition
(e) the identity permutation

Which of the following is isomorphic to the rotation group of the cube? [OA]
(a) S_4
(b) A_4
(c) D_6, the dihedral group of order 12
(d) D_{12}, the dihedral group of order 24
(e) S_3

Which of the following are isomorphic to the rotation group of the regular octahedron? [OB]
(a) S_4
(b) the rotation group of the cube
(c) the isometry group of the regular tetrahedron
(d) A_4
(e) D_{12}, the dihedral group of order 24

How many vertices does a regular dodecahedron have? [OC]
(a) 20
(b) 12
(c) 30
(d) 60
(e) 15

How many edges does a regular dodecahedron have? [OD]
(a) 20
(b) 12
(c) 30
(d) 60
(e) 15

How many isometries does a regular dodecahedron have? [OE]
(a) 20
(b) 120
(c) 30
(d) 60
(e) 15

How many rotations does a regular dodecahedron have? [OF]
(a) 20
(b) 120
(c) 30
(d) 60
(e) 15

Consider a rotation of a dodecahedron through 120° about a long diagonal. What is the corresponding permutation of the five inscribed cubes? [PA]
(a) a 3-cycle
(b) a transposition
(c) a double-transposition
(d) a 5-cycle
(e) a 3-cycle multiplied by a disjoint transposition

Consider a rotation of a dodecahedron through 72° about an axis through the midpoints of opposite faces. What is the corresponding permutation of the five inscribed cubes? [PB]
(a) a 3-cycle
(b) a 4-cycle
(c) a double-transposition
(d) a 5-cycle
(e) a 3-cycle multiplied by a disjoint transposition

Consider a rotation of a dodecahedron through 180° about an axis through the midpoints of opposite edges. What is the corresponding permutation of the five inscribed cubes? [PC]
(a) a 3-cycle
(b) a 4-cycle
(c) a double-transposition
(d) a 5-cycle
(e) a transposition

How many long diagonals does a regular dodecahedron have? [PD]
(a) 20
(b) 10
(c) 30
(d) 60
(e) 15

Which of the following are true, where D is a regular dodecahedron? [PE]
(a) $\mathrm{Rot}(D) \cong A_5$
(b) $\mathrm{Isom}(D) \cong S_5$
(c) S_5 has a normal subgroup with 2 elements
(d) $\mathrm{Isom}(D)$ has a normal subgroup with 2 elements
(e) $\mathrm{Rot}(D) \lhd \mathrm{Isom}(D)$

(Recall that two elements x, y of a group G are *conjugate* in G if there is an $a \in G$ such that $axa^{-1} = y$.)
Which of the following are true? [QA]
(a) any two 5-cycles in S_5 are conjugate in S_5
(b) any two 3-cycles in S_5 are conjugate in S_5
(c) any two 5-cycles in A_5 are conjugate in A_5
(d) any two 3-cycles in A_5 are conjugate in A_5
(e) any two double-transpositions in A_5 are conjugate in A_5

(Recall that two elements x, y of a group G are *conjugate* in G if there is an $a \in G$ such that $axa^{-1} = y$.)
Which of the following are true? [QB]
(a) $(\overleftarrow{12345})$ and $(\overleftarrow{54321})$ are conjugate in A_5
(b) $(\overleftarrow{12345})$ and $(\overleftarrow{21345})$ are conjugate in A_5
(c) $(\overleftarrow{123})$ and $(\overleftarrow{213})$ are conjugate in A_5
(d) $(\overleftarrow{123})$ and $(\overleftarrow{213})$ are conjugate in A_4
(e) $(\overleftarrow{123})$ and $(\overleftarrow{213})$ are conjugate in A_3

How many transpositions are there in S_5? [QC]
(a) 8
(b) 6
(c) 10
(d) 12
(e) 15

How many members of S_5 are of the form $(\overleftarrow{i_1\, i_2\, i_3})(\overleftarrow{i_4\, i_5})$ where $i_1, \ldots, i_5$ are distinct? [QD]
(a) 30
(b) 24
(c) 12
(d) 20
(e) 15

How many 4-cycles are there in S_5? [QE]
(a) 30
(b) 24
(c) 60
(d) 20
(e) 15

Let G be a group. Which of the following define actions of G on itself? [RA]
(a) $a \bullet x = ax$
(b) $a \bullet x = a^{-1}x$
(c) $a \bullet x = xa$
(d) $a \bullet x = xa^{-1}$
(e) $a \bullet x = axa^{-1}$

In S_4, let $K = \{e, (\overleftarrow{12})(\overleftarrow{34}), (\overleftarrow{13})(\overleftarrow{24}), (\overleftarrow{14})(\overleftarrow{23})\}$. Which of the following are true? [RB]
(a) $S_4/K \cong S_3$
(b) $S_4/K \cong \mathbb{Z}_6$
(c) $A_4/K \cong S_3$
(d) $S_4/A_4 \cong S_3$
(e) $S_4/A_4 \cong \mathbb{Z}_2$

,
Let G be a group and $\bullet$ the left action of G on itself, so that $a \bullet x = ax$ for a, $x \in G$. For arbitrary $x \in G$, which of the following are necessarily true? [SA]
(a) $\mathrm{Stab}(x) = \{e\}$
(b) $\mathrm{Orb}(x) = X$
(c) $\mathrm{Stab}(x) = G$
(d) $\mathrm{Orb}(x) = \{x\}$
(e) if G is finite, $\#(\mathrm{Orb}(x)) \cdot \#(\mathrm{Stab}(x)) = \#(G)$

Let G be the rotation group of the regular tetrahedron, and X the set of six edges of the tetrahedron; let $\bullet$ be the natural action of G on X. For arbitrary $x \in X$, which of the following must be true? [SB]
(a) $\mathrm{Stab}(x) = \{e\}$
(b) $\mathrm{Orb}(x) = X$
(c) $\mathrm{Stab}(x) = G$
(d) $\mathrm{Orb}(x) = \{x\}$
(e) $\#(\mathrm{Stab}(x)) = 2$

Let G be the rotation group of the cube, and X the set of six axes through midpoints of opposite edges; let $\bullet$ be the natural action of G on X. For arbitrary $x \in X$, which of the following must be true? [SC]
(a) $\mathrm{Stab}(x) = \{e\}$
(b) $\mathrm{Orb}(x) = X$
(c) $\#(\mathrm{Stab}(x)) = 4$
(d) $\mathrm{Orb}(x) = \{x\}$
(e) $\#(\mathrm{Stab}(x)) = 2$

Let G be a group and C its centre. Which of the following are true? [TA]
(a) $C = \{a : ax = xa \; \forall \, x \in G\}$
(b) $C = \{a : axa^{-1} = x \; \forall \, x \in G\}$
(c) $C \lhd G$
(d) G/C is cyclic
(e) $C = \{a : axa^{-1} = a \; \forall \, x \in G\}$

Let G be a finite group. Which of the following are true? [TB]
(a) if $\#(G)$ is prime, G is cyclic
(b) if G is cyclic, $\#(G)$ is prime
(c) if $\#(G)$ is the square of a prime, G is abelian
(d) if $\#(G)$ is the square of a prime, G is cyclic
(e) if $\#(G)$ is the cube of a prime, G is abelian

Let G be a finite group. For $x \in G$, write C_x for the centralizer of x and E_x for the set of elements conjugate to x. Which of the following are necessarily true? [TC]
(a) $\#(G) = \#(C_x) \cdot \#(E_x)$ for every $x \in G$
(b) $x^{-1} \in C_x$ for every $x \in G$
(c) $C_x \lhd G$ for every $x \in G$
(d) $x^{-1} \in E_x$ for every $x \in G$
(e) if $x \in E_y$ then $y \in E_x$

Suppose that ρ, σ are the generators of the dihedral group D_n, where $n \geq 3$, so that $\rho^n = \sigma^2 = e$ and $\sigma\rho\sigma^{-1} = \rho^{-1}$. What is the centralizer of ρ? [UA]
(a) $< \rho >$
(b) $\{\rho\}$
(c) D_n
(d) $\{\rho, \rho^{-1}\}$
(e) $\{e, \sigma, \rho, \sigma\rho^{-1}\}$

Suppose that ρ, σ are the generators of the dihedral group D_n, where $n \geq 3$, so that $\rho^n = \sigma^2 = e$ and $\sigma\rho\sigma^{-1} = \rho^{-1}$. What is the centralizer of σ? [UB]
(a) $< \sigma >$
(b) $\{\sigma\}$
(c) D_n
(d) $< \sigma >$ if n is odd, $\{e, \sigma, \rho^{n/2}, \sigma\rho^{n/2}\}$ if n is even
(e) $\{e, \sigma, \rho^{n/2}, \sigma\rho^{n/2}\}$

Suppose that ρ, σ are the generators of the dihedral group D_n, where $n \geq 3$, so that $\rho^n = \sigma^2 = e$ and $\sigma\rho\sigma^{-1} = \rho^{-1}$. What is the conjugacy class of ρ? [UC]
(a) $< \rho >$
(b) $\{\rho\}$
(c) D_n
(d) $\{\rho, \rho^{-1}\}$
(e) $\{e, \sigma, \rho, \sigma\rho^{-1}\}$

Suppose that ρ, σ are the generators of the dihedral group D_n, where $n \geq 3$, so that $\rho^n = \sigma^2 = e$ and $\sigma\rho\sigma^{-1} = \rho^{-1}$. What is the conjugacy class of σ? [UD]
(a) $D_n \setminus < \rho >$
(b) $\{\sigma\rho^{2i} : i \in \mathbb{Z}\}$
(c) D_n
(d) $\{\sigma\}$
(e) $< \rho >$

Let G be a group. For $a, x \in G$ set $\hat{a}(x) = axa^{-1}$. If $a \in G$ and $\psi \in \operatorname{Aut} G$, for which $b \in G$ is $\psi\hat{a}\psi^{-1} = \hat{b}$? [VA]
(a) $b = \psi(a)$
(b) $b = \psi^{-1}(a)$
(c) $b = a$
(d) $b = a\psi(a)$
(e) $b = \psi(a^{-1})$

Let G be a group, S_G the group of bijections from G to itself, and J the set of inner automorphisms of G. Which of the following are necessarily true? [VB]
(a) J is a subgroup of S_G
(b) $J \lhd S_G$
(c) $J = S_G$
(d) $J = \operatorname{Aut} G$
(e) $J \lhd \operatorname{Aut} G$

Let G be a group, J the set of inner automorphisms of G and C the centre of G. Which of the following are necessarily true? [VC]

(a) $J \subseteq \operatorname{Aut} G$
(b) J is a subgroup of $\operatorname{Aut} G$
(c) $J \cong G/C$
(d) $J \cong G$
(e) $\operatorname{Aut} G \subseteq J$

If K_4 is the Klein group (the non-cyclic group with four elements), which of the following are isomorphic to $\operatorname{Aut} K_4$? [WA]

(a) S_3
(b) D_3
(c) A_3
(d) $\mathbb{Z}_6$
(e) the isometry group of an equilateral triangle

If G is a cyclic group with four elements, which of the following are isomorphic to $\operatorname{Aut} G$? [WB]

(a) $\{1, 3\}$ with multiplication mod 4
(b) $\mathbb{Z}_2$
(c) G
(d) a group with one element
(e) S_2

If G is a cyclic group with five elements, which of the following are isomorphic to $\operatorname{Aut} G$? [WC]

(a) $\{1, 2, 3, 4\}$ with multiplication mod 5
(b) $\mathbb{Z}_4$
(c) the Klein group
(d) G
(e) $\mathbb{Z}_2$