Real Analysis ten-minute test questions

Which of the following are true for all real numbers x and y? [AA]

(a) $|x+y| \leq |x|+|y|$
(b) $|x-y| \leq |x|-|y|$
(c) $|x+y| = |x|+|y|$
(d) $|x+y| \geq |x|+|y|$
(e) $|x+y| < |x|+|y|$

Which of the following are true for all real numbers x and y? [AB]

(a) $|x-y| \leq |x|+|y|$
(b) $|x-y| \leq |x|-|y|$
(c) $|x-y| \geq |x|+|y|$
(d) $|x-y| = |x|+|y|$
(e) $|x-y| < |x|+|y|$

Which of the following are true for all real numbers x and y? [AC]

(a) $|x|-|y| \leq |x-y|$
(b) $|x|-y \leq |x-y|$
(c) $|x|-|y| \leq x-y$
(d) $|x|-|y| = |x-y|$
(e) $|x|-|y| < |x-y|$

Which of the following are true for all real numbers x and y? [AD]

(a) $|x| \leq |y|+|x-y|$
(b) $|x| \leq y+|x-y|$
(c) $|x| \leq |y|-|x-y|$
(d) $|x| \leq |y|+x-y$
(e) $|x| = |y|+|x-y|$

Which of the following are true for all real numbers x and y? [AE]

(a) $||x|-|y|| \leq |x-y|$
(b) $||x|+|y|| \leq |x-y|$
(c) $|x|-|y| \geq x-y$
(d) $||x|-y| \leq |x+y|$
(e) $||x|-|y|| = |x-y|$

Which of the following are true for all real numbers x, y, a and b? [AF]

(a) $|(x+y)-(a+b)| \leq |x-a|+|y-b|$
(b) $|(x+y)+(a-b)| \leq |x-a|+|y-b|$
(c) $|(x+y)+(a+b)| \leq |x-a|+|y-b|$
(d) $|(x+y)-(a+b)| \leq |x+a|+|y+b|$
(e) $|(x+y)-(a+b)| = |x-a|+|y-b|$

Which of the following are true for all real numbers x and y? [AG]

(a) $|xy| = |x||y|$
(b) $|xy| \leq |x||y|$
(c) $|xy| \geq |x||y|$
(d) $|xy| < |x||y|$
(e) $|xy| > |x||y|$

Typeset by $\mathcal{AMS}$-TEX

Which of the following are true for all real numbers x and all non-zero real numbers y? [AH]

(a) $|\frac{x}{y}| = \frac{|x|}{|y|}$

(b) $|\frac{x}{y}| \leq \frac{|x|}{|y|}$

(c) $|\frac{x}{y}| \geq \frac{|x|}{|y|}$

(d) $|\frac{x}{y}| < \frac{|x|}{|y|}$

(e) $|\frac{x}{y}| > \frac{|x|}{|y|}$

Given that x, y, a and b are real numbers, which of the following are sufficient to ensure that $x = y$? [AHD]

(a) $a \leq x$ and $x \leq y$ and $y \leq b$ and $b \leq a$
(b) $x \leq a$ and $x \leq y$ and $y \leq b$ and $b \leq a$
(c) $a \leq x$ and $y \leq x$ and $y \leq b$ and $b \leq a$
(d) $a \leq x$ and $x \leq y$ and $b \leq y$ and $b \leq a$
(e) $a \leq x$ and $x \leq y$ and $y \leq b$ and $a \leq b$

Which of the following are true for all non-zero real numbers x and a? [AK]

(a) $|\frac{1}{x} - \frac{1}{a}| = \frac{|x-a|}{|a||x|}$

(b) $|\frac{1}{x} - \frac{1}{a}| \leq \frac{|x-a|}{|a||x|}$

(c) $|\frac{1}{x} - \frac{1}{a}| \geq \frac{|x-a|}{|a||x|}$

(d) $|\frac{1}{x} - \frac{1}{a}| = \frac{x-a}{ax}$

(e) $|\frac{1}{x} - \frac{1}{a}| = \frac{a-x}{ax}$

Which of the following are true for all real numbers x and y? [AL]

(a) $|x| \geq |y| - |y - x|$
(b) $|x| \leq |y| - |y - x|$
(c) $|x| \geq |y| + |y - x|$
(d) $|x| \geq |y| - |y + x|$
(e) $|x| > |y| - |y - x|$

Which of the following are true for all real numbers x, y and z? [AM]

(a) $|x - y| \leq |x - z| + |y - z|$
(b) $|x - y| \geq |x - z| + |y - z|$
(c) $|x - y| \leq |x - z| - |y - z|$
(d) $|x - y| \leq |x + z| + |y - z|$
(e) $|x + y| \leq |x - z| + |y - z|$

Which of the following is the definition of '$\lim_{n\to\infty} x_n = b$'? [BA]

(a) $\forall\, \epsilon > 0\ \exists\, n_0 \in \mathbb{N}\ \forall\, n \geq n_0,\ |x_n - b| \leq \epsilon$
(b) $\forall\, \epsilon > 0\ \exists\, n_0 \in \mathbb{N}\ \forall\, n \geq n_0,\ |x_n - b| \geq \epsilon$
(c) $\forall\, \epsilon \geq 0\ \exists\, n_0 \in \mathbb{N}\ \forall\, n \geq n_0,\ |x_n - b| \leq \epsilon$
(d) $\exists\, \epsilon > 0\ \forall\, n_0 \in \mathbb{N}\ \exists\, n \geq n_0,\ |x_n - b| \leq \epsilon$
(e) $\forall\, \epsilon > 0\ \exists\, n_0 \in \mathbb{N}\ \forall\, n \leq n_0,\ |x_n - b| \leq \epsilon$

In the Analysis Game, the first player says '$\lim_{n\to\infty} 2^{-n} = 0$'. Which of the following are legal moves for the second player? [BB]

(a) $\epsilon = 1$
(b) $\epsilon = 0$
(c) $n_0 = 10^6$
(d) $n = 10^6$
(e) $\epsilon = -10^{24}$

In the Analysis Game, the first player says '$\lim_{n\to\infty} \sin\frac{n\pi}{2} = 1$' and the second player answers '$\epsilon = \frac{1}{4}$'. Which of the following are now legal moves for the first player? [BC]

(a) $n_0 = 57$
(b) $\epsilon = 2$
(c) $n_0 = \infty$
(d) $n = 5$
(e) π is irrational

In the Analysis Game, the first player says '$\lim_{n\to\infty} \frac{\ln n}{n} = 0$', the second player says '$\epsilon = 0{\cdot}00691$', and the first player says '$n_0 = 1000$'. Which of the following are now legal moves for the second player? [BD]

(a) $n = 10^{1000}$
(b) $n = 999$
(c) $\epsilon = 0{\cdot}001$
(d) $n_0 = 900$
(e) $n = \infty$

Which of the following is the definition of '$\lim_{x\to a} f(x) = b$'? [CA]

(a) $\forall\,\epsilon > 0\ \exists\,\delta > 0,\ f(x)$ is defined and $|f(x) - b| \le \epsilon$ whenever $0 < |x - a| \le \delta$
(b) $\forall\,\epsilon > 0\ \exists\,\delta > 0,\ |f(x) - b| \le \epsilon$ whenever $0 < |x - a| \le \delta$
(c) $\forall\,\epsilon > 0\ \exists\,\delta > 0,\ f(x)$ is defined and $|f(x) - b| \le \epsilon$ whenever $0 \le |x - a| \le \delta$
(d) $\forall\,\epsilon \ge 0\ \exists\,\delta > 0,\ f(x)$ is defined and $|f(x) - b| \le \epsilon$ whenever $0 < |x - a| \le \delta$
(e) $\forall\,\epsilon > 0\ \exists\,\delta \ge 0,\ f(x)$ is defined and $|f(x) - b| \le \epsilon$ whenever $0 < |x - a| \le \delta$

In the proof that 'if $\lim_{n\to\infty} x_n = b$ and $\lim_{n\to\infty} y_n = c$, then $\lim_{n\to\infty} x_n + y_n = b + c$', the last line reads

$|(x_n + y_n) - (b + c)| \le$ [redacted] $\le \frac{\epsilon}{2} + \frac{\epsilon}{2} \le \epsilon.$

Which of the following could be put in place of [redacted]? [CB]

(a) $|x_n - b| + |y_n - c|$
(b) $|x_n + b| + |y_n - c|$
(c) $|x_n - b| - |y_n - c|$
(d) $|x_n - b| + |y_n + c|$
(e) $|x_n + b| + |y_n + c|$

In the proof that 'if $\lim_{x\to a} f(x) = b$ and $\lim_{x\to a} g(x) = c$ then $\lim_{x\to a} f(x)g(x) = bc$', which of the following would you expect to see? [CC]

(a) Set $\delta = \min(\delta_1, \delta_2)$
(b) Set $\delta = \max(\delta_1, \delta_2)$
(c) Set $\delta = \delta_1 + \delta_2$
(d) Set $\delta = \delta_1\delta_2$
(e) Set $\delta = \epsilon$

In the proof that 'if $\lim_{n\to\infty} x_n = b$ and $\lim_{n\to\infty} y_n = c$, then $\lim_{n\to\infty} x_n + y_n = b + c$', which of the following would you expect to see? [CD]

(a) Set $n_0 = \max(n_1, n_2)$
(b) Set $n_0 = \min(n_1, n_2)$
(c) Set $n_0 = 10^{24}$
(d) Set $n_0 = |n_1 - n_2|$
(e) Set $n_0 = \infty$

In the proof that 'if $\lim_{n\to\infty} x_n = b$ and $\lim_{n\to\infty} y_n = c$ then $\lim_{n\to\infty} x_n + y_n = b + c$', three of the lines are of the form

There is an $n_1 \in \mathbb{N}$ such that $|x_n - b| \leq \blacksquare$ whenever $n \geq n_1$,
there is an $n_2 \in \mathbb{N}$ such that $|y_n - c| \leq \blacksquare$ whenever $n \geq n_2$,
$|(x_n + y_n) - (b + c)| \leq \ldots \leq \epsilon$.

Which of the following will serve in place of $\blacksquare$? [CE]

(a) $\frac{1}{2}\epsilon$
(b) ϵ
(c) 0
(d) ϵ^2
(e) $\sqrt{\epsilon}$

Which of the following are true for all real numbers x, y, a and b? [DA]

(a) $xy - ab = (x - a)(y - b) + a(y - b) + (x - a)b$
(b) $xy - ab = (x - a)(y - b) - a(y - b) + (x - a)b$
(c) $xy - ab = (x - a)(y - b) + a(y - b) - (x - a)b$
(d) $xy - ab = (x - a)(y - b) + a(y + b) + (x - a)b$
(e) $xy - ab = (x - a)(y - b) + a(y - b) + (x + a)b$

Which of the following is the definition of '$\lim_{x\to\infty} f(x) = b$'? [DB]

(a) $\forall\, \epsilon > 0\ \exists\, M \in \mathbb{R}\ \forall\, x \geq M$, $f(x)$ is defined and $|f(x) - b| \leq \epsilon$
(b) $\forall\, \epsilon > 0\ \exists\, M \in \mathbb{R}\ \forall\, x \geq M$, $|f(x) - b| \leq \epsilon$
(c) $\forall\, \epsilon \geq 0\ \exists\, M \in \mathbb{R}\ \forall\, x \geq M$, $f(x)$ is defined and $|f(x) - b| \leq \epsilon$
(d) $\forall\, \epsilon > 0\ \exists\, M \in \mathbb{R}\ \forall\, x \geq M$, $f(x)$ is defined and $|f(x) - b| \geq \epsilon$
(e) $\forall\, \epsilon > 0\ \exists\, M \in \mathbb{R}\ \forall\, x \leq M$, $f(x)$ is defined and $|f(x) - b| \leq \epsilon$

In the Analysis Game, the first player says '$\lim_{x\to\infty} \sin x = 1$' and the second player answers '$\epsilon = \frac{1}{4}$'. Which of the following are now legal moves for the first player? [DC]

(a) $M = e^{10}$
(b) $\epsilon = 2$
(c) $M = \infty$
(d) $x = 5$
(e) $M \geq 0$

In the proof that 'if $\lim_{n\to\infty} x_n = b$ and $\lim_{n\to\infty} y_n = c$, then $\lim_{n\to\infty} x_n y_n = bc$', the last line reads

$$|x_n y_n - bc| \leq \blacksquare \leq \eta^2 + \eta|c| + |b|\eta \leq \epsilon.$$

Which of the following could be put in place of $\blacksquare$? [DD]

(a) $|x_n - b||y_n - c| + |x_n - b||c| + |b||y_n - c|$
(b) $|x_n - b||y_n - c| + |x_n - b||c| + |b||y_n + c|$
(c) $|x_n - b||y_n - c| + |x_n - b||c| - |b||y_n - c|$
(d) $|x_n + b||y_n + c| + |x_n - b||c| + |b||y_n - c|$
(e) $|x_n + b||y_n + c| + |x_n + b||c| + |b||y_n + c|$

In the proof that 'if $\lim_{x\to\infty} f(x) = b$ and $\lim_{x\to\infty} g(x) = c$ then $\lim_{x\to\infty} f(x) + g(x) = b + c$', which of the following would you expect to see? [DE]

(a) Set $M = \max(M_1, M_2)$
(b) Set $M = \min(M_1, M_2)$
(c) Set $\delta = \min(\delta_1, \delta_2)$
(d) Set $M = M_1 + M_2$
(e) Set $M = \frac{1}{\epsilon}\max(M_1, M_2)$

In the proof that 'if $\lim_{n\to\infty} x_n = b$ and $\lim_{n\to\infty} y_n = c$ then $\lim_{n\to\infty} x_n y_n = bc$', three of the lines are of the form

There is an $n_1 \in \mathbb{N}$ such that $|x_n - b| \leq \blacksquare$ whenever $n \geq n_1$,
there is an $n_2 \in \mathbb{N}$ such that $|y_n - c| \leq \blacksquare$ whenever $n \geq n_2$,
$|x_n y_n - bc| \leq \ldots \leq \epsilon$.

Which of the following will serve in place of $\blacksquare$? [DF]

(a) $\min(1, \frac{\epsilon}{1+|b|+|c|})$
(b) $\frac{\epsilon}{1+|b|+|c|}$
(c) ϵ
(d) $\min(10^{-10}, \frac{1}{4}\epsilon)$
(e) 0

In the proof that 'if $\lim_{n\to\infty} x_n = b \neq 0$ then $\lim_{n\to\infty} \frac{1}{x_n} = \frac{1}{b}$', two of the lines are of the form

There is an $n_0 \in \mathbb{N}$ such that $|x_n - b| \leq \blacksquare$ whenever $n \geq n_0$,
$|\frac{1}{x_n} - \frac{1}{b}| \leq \ldots \leq \epsilon$.

Which of the following will serve in place of $\blacksquare$? [EA]

(a) $\min(\frac{|b|}{2}, \frac{1}{2}b^2\epsilon)$
(b) $\frac{1}{2}b^2\epsilon$
(c) $\min(\frac{|b|}{2}, \frac{1}{2}b\epsilon^2)$
(d) $\min(\frac{b}{2}, \frac{1}{2}b^2\epsilon)$
(e) $\max(\frac{|b|}{2}, \frac{1}{2}b^2\epsilon)$

Which of the following is the definition of '$\lim_{x\to-\infty} f(x) = b$'? [EB]

(a) $\forall\, \epsilon > 0\ \exists\, M \in \mathbb{R}\ \forall\, x \leq M$, $f(x)$ is defined and $|f(x) - b| \leq \epsilon$
(b) $\forall\, \epsilon > 0\ \exists\, M \in \mathbb{R}\ \forall\, x \leq M$, $|f(x) - b| \leq \epsilon$
(c) $\forall\, \epsilon \geq 0\ \exists\, M \in \mathbb{R}\ \forall\, x \leq M$, $f(x)$ is defined and $|f(x) - b| \leq \epsilon$
(d) $\forall\, \epsilon > 0\ \exists\, M \in \mathbb{R}\ \forall\, x \leq M$, $f(x)$ is defined and $|f(x) - b| \geq \epsilon$
(e) $\forall\, \epsilon > 0\ \exists\, M \in \mathbb{R}\ \forall\, x \geq M$, $f(x)$ is defined and $|f(x) - b| \leq \epsilon$

Which of the following is the definition of 'f is continuous at x_0'? [EC]

(a) $\forall\, \epsilon > 0\ \exists\, \delta > 0\ \forall\, x \in [x_0 - \delta, x_0 + \delta] \cap \mathrm{dom}(f)$, $|f(x) - f(x_0)| \leq \epsilon$
(b) $\forall\, \epsilon > 0\ \exists\, \delta > 0\ \forall\, x \in [x_0 - \delta, x_0 + \delta]$, $f(x)$ is defined and $|f(x) - f(x_0)| \leq \epsilon$
(c) $\forall\, \epsilon > 0\ \exists\, \delta > 0\ \forall\, x \in [x_0 - \delta, x_0 + \delta]$, $|f(x) - f(x_0)| \leq \epsilon$
(d) $\forall\, \epsilon > 0\ \exists\, \delta > 0\ \forall\, x \in [x_0 + \delta, x_0 - \delta] \cap \mathrm{dom}(f)$, $|f(x) - f(x_0)| \leq \epsilon$
(e) $\forall\, \epsilon > 0\ \exists\, \delta \geq 0\ \forall\, x \in [x_0 - \delta, x_0 + \delta] \cap \mathrm{dom}(f)$, $|f(x) - f(x_0)| \leq \epsilon$

In the Analysis Game, the first player says 'ln is continuous'. (It is understood that $\operatorname{dom}\ln =]0,\infty[$). Which of the following is a complete legal move for the second player? [ED]

(a) $x_0 = 10^{10}$, $\epsilon = \frac{1}{x_0}$
(b) $x_0 = 1$, $\epsilon = 0$
(c) $x_0 = 1$
(d) $\epsilon = 10^{-10}$
(e) $x_0 = 0$, $\epsilon = 10^{-10}$

In the proof that 'if f and g are continuous, so is $f \times g$', which of the following would you expect to see? [FA]

(a) Set $\delta = \min(\delta_1, \delta_2)$
(b) Set $\delta = \max(\delta_1, \delta_2)$
(c) Set $M = \max(M_1, M_2)$
(d) Set $\delta = \delta_1\delta_2$
(e) Set $\delta = \frac{1}{2}\epsilon$

In the Analysis Game, the first player says 'the function $x \mapsto x^2 : \mathbb{R} \to \mathbb{R}$ is continuous' and the second player says '$x_0 = -10$, $\epsilon = \frac{1}{10}$'. Which of the following are legal moves for the first player? [FB]

(a) $\delta = 10^{-10}$
(b) $\delta = \epsilon$
(c) $\delta = \epsilon^2/x_0$
(d) $\delta = \epsilon/x_0^2$
(e) $\delta = 0$

In the proof that 'if f and g are continuous, so is $f + g$', three of the lines are of the form

if $x \in [x_0 - \delta_1, x_0 + \delta_1] \cap \operatorname{dom}(f)$, $|f(x) - f(x_0)| \leq \blacksquare$,
if $x \in [x_0 - \delta_2, x_0 + \delta_2] \cap \operatorname{dom}(g)$, $|g(x) - g(x_0)| \leq \blacksquare$,
$|(f+g)(x) - (f+g)(x_0)| \leq \ldots \leq \epsilon$.

Which of the following will serve in place of $\blacksquare$? [FC]

(a) $\frac{1}{2}\epsilon$
(b) ϵ
(c) ϵ^2
(d) $\frac{\epsilon}{1+|x_0|}$
(e) 0

In the proof that 'if f and g are continuous, so is $f \times g$', three of the lines are of the form

if $x \in [x_0 - \delta_1, x_0 + \delta_1] \cap \operatorname{dom}(f)$, $|f(x) - f(x_0)| \leq \blacksquare$,
if $x \in [x_0 - \delta_2, x_0 + \delta_2] \cap \operatorname{dom}(g)$, $|g(x) - g(x_0)| \leq \blacksquare$,
$|(f \times g)(x) - (f \times g)(x_0)| \leq \ldots \leq \epsilon$.

Which of the following will serve in place of $\blacksquare$? [FD]

(a) $\min(1, \frac{\epsilon}{1+|f(x_0)|+|g(x_0)|})$
(b) $\frac{\epsilon}{1+|f(x_0)|+|g(x_0)|}$
(c) $\min(1, \frac{\epsilon}{1+|f(x_0)+g(x_0)|})$
(d) $\min(1, \epsilon^2)$
(e) 0

In the proof that ‘if f is continuous, so is $\frac{1}{f}$’, two of the lines are

There is a $\delta > 0$ such that $|f(x) - f(x_0)| \leq \blacksquare$ whenever $x \in \operatorname{dom} f$ and $|x - x_0| \leq \delta$,

$|\frac{1}{f(x)} - \frac{1}{f(x_0)}| \leq \ldots \leq \epsilon$.

Which of the following will serve in place of $\blacksquare$? [GA]

(a) $\min(\frac{1}{2}|f(x_0)|, \frac{1}{2}\epsilon f(x_0)^2)$

(b) $\min(\frac{1}{2}f(x_0), \frac{1}{2}\epsilon f(x_0)^2)$

(c) $\min(\frac{1}{2}|f(x_0)|, \epsilon f(x_0)^2)$

(d) $\frac{1}{2}\epsilon f(x_0)^2$

(e) $\min(|f(x_0)|, \frac{1}{2}\epsilon f(x_0)^2)$

In the proof that ‘$x \mapsto \frac{1}{x} : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ is continuous’, which of the following would you expect to see? [GA]

(a) $\delta = \min(\frac{1}{2}|x_0|, \frac{1}{2}\epsilon x_0^2)$

(b) $\delta = \min(\frac{1}{2}x_0, \frac{1}{2}\epsilon x_0^2)$

(c) $\delta = \min(\frac{1}{2}|x_0|, \epsilon x_0^2)$

(d) $\delta = \frac{1}{2}\epsilon x_0^2$

(e) $\delta = \min(|x_0|, \frac{1}{2}\epsilon x_0^2)$

In the proof that ‘if f and g are continuous, so is $f \circ g$’, which of the following would you expect to see, following ‘Let $x_0 \in \operatorname{dom}(f \circ g)$, $\epsilon > 0$’? [GB]

(a) Let $\eta > 0$ be such that $|f(y) - f(g(x_0))| \leq \epsilon$ whenever $y \in \operatorname{dom} f$ and $|y - g(x_0)| \leq \eta$
(b) Let $\eta > 0$ be such that $|g(x) - g(x_0)| \leq \epsilon$ whenever $x \in \operatorname{dom} g$ and $|x - x_0| \leq \eta$
(c) Let $\eta \geq 0$ be such that $|f(y) - f(g(x_0))| \leq \epsilon$ whenever $y \in \operatorname{dom} f$ and $|y - g(x_0)| \leq \eta$
(d) Let $\eta > 0$ be such that $|f(x) - f(x_0)| \leq \epsilon$ whenever $x \in \operatorname{dom} f$ and $|x - x_0| \leq \eta$
(e) Let $\eta > 0$ be such that $|f(y) - f(g(x_0))| \leq \epsilon$ whenever $|y - g(x_0)| \leq \eta$

In the proof that ‘the function $x \mapsto x : \mathbb{R} \to \mathbb{R}$ is continuous’, which of the following would you expect to see? [GC]

(a) Set $\delta = \epsilon$
(b) Set $\delta = \epsilon^2$
(c) Set $\delta = \min(\delta_1, \delta_2)$
(d) Set $\epsilon = \delta$
(e) Set $\delta = \epsilon / x_0^2$

Which of the following are true, for arbitrary real functions f and g? [GD]

(a) $\operatorname{dom}(f \times g) = \operatorname{dom}(f + g)$
(b) $\operatorname{dom}(1/f) = \operatorname{dom} f$
(c) $\operatorname{dom}(f \times g) \subseteq \operatorname{dom} f$
(d) $\operatorname{dom}(f \circ g) \subseteq \operatorname{dom} g$
(e) $\operatorname{dom}(f \circ g) \subseteq \operatorname{dom} f$

Which of the following would you expect to see in the proof that $| \ |$ is continuous? [HA]
(a) Set $\delta = \epsilon$
(b) Set $\delta = \epsilon^2$
(c) Set $\epsilon = \delta$
(d) Set $\delta = \epsilon/x_0^2$
(e) Set $\delta = \epsilon/x_0$

If $A =]-1, 1[$, which of the following are true? [HB]
(a) A has a least upper bound, which does not belong to A
(b) A has a least upper bound, which belongs to A
(c) A has an upper bound, but no least upper bound
(d) A has no upper bounds
(e) A has no lower bounds

If $A = \mathbb{Z}$, which of the following are true? [HC]
(a) A has no upper bounds
(b) A has a least upper bound, which does not belong to A
(c) A has a least upper bound, which belongs to A
(d) A has an upper bound, but no least upper bound
(e) A has no lower bounds

If $A = \{x : x \in \mathbb{R},\ x^2 \leq 1\}$, which of the following are true? [HD]
(a) A has a least upper bound, which belongs to A
(b) A has a least upper bound, which does not belong to A
(c) A has an upper bound, but no least upper bound
(d) A has no upper bounds
(e) A has no lower bounds

If $A = \{x : x \in \mathbb{R},\ x^2 < 0\}$, which of the following are true? [HE]
(a) A has an upper bound, but no least upper bound
(b) A has a least upper bound, which belongs to A
(c) A has a least upper bound, which does not belong to A
(d) A has no upper bounds
(e) A has no lower bounds

In the Analysis Game, the first player says 'every bounded non-decreasing sequence is convergent'. Which of the following is a legal move for the second player? [IA]
(a) Set $x_n = \frac{n}{n+1}$ for $n \in \mathbb{N}$
(b) Set $x_n = 2^n$ for $n \in \mathbb{N}$
(c) Set $b = 1$
(d) Set $x_n = (-1)^n$ for $n \in \mathbb{N}$
(e) Set $\epsilon = \frac{1}{100}$

In the Analysis Game, the first player says 'every bounded non-decreasing sequence is convergent' and the second player says 'Set $x_n = \sum_{i=0}^{n} \frac{1}{i!}$ for $n \in \mathbb{N}$'. Which of the following is a winning legal move for the first player? [IB]
(a) $b = e$
(b) $b = \lim_{n\to\infty} x_n$
(c) $n_0 = 10^{10}$
(d) $b = 2{\cdot}7$
(e) $\epsilon = 10^{-10}$

In the proof that 'every bounded non-decreasing sequence is convergent', which of the following would you expect to see? [IC]

(a) Set $b = \sup\{x_n : n \in \mathbb{N}\}$
(b) Set $b = \inf\{x_n : n \in \mathbb{N}\}$
(c) Set $b = \sum_{n=0}^{\infty} x_n$
(d) Set $b = x_0$
(e) Set $b = 0$

In the proof that 'every bounded non-decreasing sequence is convergent', which of the following would you expect to see? [ID]

(a) Let $n_0 \in \mathbb{N}$ be such that $x_{n_0} > b - \epsilon$
(b) Let $n_0 \in \mathbb{N}$ be such that $x_{n_0} < b + \epsilon$
(c) Let $n_0 \in \mathbb{N}$ be such that $x_{n_0} \leq b$
(d) Let $n_0 \in \mathbb{N}$ be such that $x_{n_0} \geq 0$
(e) Let $n_0 \in \mathbb{N}$ be such that $x_{n_0} = b$

In the proof that 'every Cauchy sequence is bounded', which of the following would you expect to see? [JA]

(a) Set $M = \max(|x_0|, \ldots, |x_{n_0}|, |x_{n_0}| + 1)$
(b) Set $M = \max(|x_0|, \ldots, |x_{n_0}|)$
(c) Set $M = |x_{n_0}| + 1$
(d) Set $M = \max(x_0, \ldots, x_{n_0}) + 1$
(e) Set $M = \sup\{|x_n| : n \in \mathbb{N}\}$

Which of the following is the definition of '$\langle x_n \rangle_{n\in\mathbb{N}}$ is a Cauchy sequence'? [JB]

(a) $\forall\, \epsilon > 0\; \exists\, n_0 \in \mathbb{N}\; \forall\, m, n \geq n_0,\; |x_m - x_n| \leq \epsilon$
(b) $\forall\, \epsilon > 0\; \exists\, n_0 \in \mathbb{N}\; \forall\, n \geq n_0,\; |x_m - x_n| \leq \epsilon$
(c) $\forall\, \epsilon \geq 0\; \exists\, n_0 \in \mathbb{N}\; \forall\, m, n \geq n_0,\; |x_m - x_n| \leq \epsilon$
(d) $\forall\, \epsilon > 0\; \exists\, n_0 \in \mathbb{N}\; \forall\, m, n \leq n_0,\; |x_m - x_n| \leq \epsilon$
(e) $\forall\, \epsilon > 0\; \exists\, n_0 \in \mathbb{N}\; \forall\, m, n \geq n_0,\; |x_m - x_n| = \epsilon$

Suppose that $\langle x_n \rangle_{n\in\mathbb{N}}$ is a bounded sequence, and $y_n = \inf\{x_i : i \geq n\}$ for each $n \in \mathbb{N}$. Which of the following must be true for every $n \in \mathbb{N}$? [JC]

(a) $y_n \leq x_n$
(b) $y_n \leq y_{n+1}$
(c) $y_{n+1} \leq x_n$
(d) $x_n \leq y_n$
(e) $y_{n+1} \leq y_n$

In the proof that every convergent sequence is Cauchy, which of the following would you expect to see? [KA]

(a) $|x_m - x_n| \leq |x_m - b| + |x_n - b|$
(b) $|x_m - x_n| \leq |x_m - b|$
(c) $|x_m - x_n| \geq |x_m - b| + |x_n - b|$
(d) $|x_m - x_n| \leq |x_m + b| + |x_n + b|$
(e) $|x_m - x_n| \leq |x_m - b| - |x_n - b|$

Which of the following is the definition of '$\sum_{n=0}^{\infty} x_n = b$'? [KB]

(a) $\lim_{n\to\infty} \sum_{k=0}^{n} x_k = b$
(b) $\lim_{n\to\infty} x_n = b$
(c) $\lim_{n\to\infty} \sum_{k=1}^{n} x_k = b$
(d) $\lim_{k\to\infty} \sum_{k=0}^{n} x_k = b$
(e) $\lim_{y\to\infty} \int_{t=0}^{y} x_t = b$

In the proof that every absolutely summable series is summable, which of the following would you expect to see, somewhere after '$s_n = \sum_{k=0}^{n} x_k$, $t_n = \sum_{k=0}^{n} |x_k|$'? [KC]
(a) $|s_m - s_n| \leq |t_m - t_n|$
(b) $|t_m - t_n| \leq |s_m - s_n|$
(c) $s_m - s_n \leq t_m - t_n$
(d) $|s_m - s_n| \leq t_m - t_n$
(e) $s_m - s_n \leq |t_m - t_n|$

Given that $s_n = \sum_{k=0}^{n} x_k$ and $t_n = \sum_{k=0}^{n} |x_k|$ for each n, which of the following must be true, for all m, $n \in \mathbb{N}$? [KCA]
(a) $|s_m - s_n| \leq |t_m - t_n|$
(b) $|t_m - t_n| \leq |s_m - s_n|$
(c) $s_m - s_n \leq t_m - t_n$
(d) $|s_m - s_n| \leq t_m - t_n$
(e) $s_m - s_n \leq |t_m - t_n|$

In the proof that any continuous function on a closed bounded interval is bounded, which of the following would you expect to see? [LA]
(a) Set $A = \{x : a \leq x \leq b,\ f \text{ is bounded on } [a, x]\}$
(b) Set $A = \{x : a \leq x \leq b,\ f \text{ is bounded on } [x, a]\}$
(c) Set $A = \{x : a \leq x \leq b,\ f \text{ is constant on } [a, x]\}$
(d) Set $A = \{x : a \leq x \leq b,\ f \text{ is not constant on } [a, x]\}$
(e) Set $A = \{x : b \leq x \leq a,\ f \text{ is bounded on } [a, x]\}$

Which of the following is the definition of 'f is bounded on D'? (You may take it for granted that $D \subseteq \operatorname{dom} f$.) [LB]
(a) $\exists\, M \in \mathbb{R}\ \forall\, x \in D,\ |f(x)| \leq M$
(b) $\forall\, x \in D\ \exists\, M \in \mathbb{R},\ |f(x)| \leq M$
(c) $\exists\, M \in \mathbb{R}\ \forall\, x \in D,\ |f(x)| \geq M$
(d) $\exists\, M \in \mathbb{R}\ \forall\, x \in D,\ f(x) \leq M$
(e) $\exists\, x \in D\ \forall\, M \in \mathbb{R},\ |f(x)| \leq M$

If $a \leq b \leq c$, which of the following must be true? [LC]
(a) $[a, c] = [a, b] \cup [b, c]$
(b) $]a, c[=]a, b[\cup]b, c[$
(c) $]a, c] =]a, b] \cup]b, c]$
(d) $[a, c[= [a, b[\cup [b, c[$
(e) $[a, c] = [a, b[\cup [b, c]$

If we know that $A \subseteq \mathbb{R}$ and that $c = \sup A$, which of the following must be true? [LD]
(a) $\forall\, \delta > 0\ \exists\, x \in A,\ x \geq c - \delta$
(b) $\forall\, \delta > 0\ \exists\, x \in A,\ c \geq x \geq c - \delta$
(c) $\forall\, \delta > 0\ \exists\, x \in A,\ c > x > c - \delta$
(d) $\forall\, \delta > 0\ \exists\, x \in A,\ x > c - \delta$
(e) $\forall\, \delta > 0\ \exists\, x \in A,\ |x - c| \leq \delta$

Which of the following is the definition of '$\lim_{n\to\infty} x_n = b$'? [BE]
(a) $\forall\, \epsilon > 0\ \exists\, n_0 \in \mathbb{N}\ \forall\, n \geq n_0,\ |x_n - b| \leq \epsilon$
(b) $\exists\, \epsilon > 0\ \forall\, n_0 \in \mathbb{N}\ \exists\, n \geq n_0,\ |x_n - b| \leq \epsilon$
(c) $\exists\, n_0 \in \mathbb{N}\ \forall\, \epsilon > 0\ \forall\, n \geq n_0,\ |x_n - b| \leq \epsilon$
(d) $\forall\, n \geq n_0\ \exists\, n_0 \in \mathbb{N}\ \forall\, \epsilon > 0,\ |x_n - b| \leq \epsilon$
(e) $\exists\, n_0 \in \mathbb{N}\ \forall\, n \geq n_0\ \forall\, \epsilon > 0,\ |x_n - b| \leq \epsilon$

You have to prove that if $f : [a, b] \to \mathbb{R}$ is continuous and $f(a) < c < f(b)$ then there is a $z \in [a, b]$ such that $f(z) = c$. Which of the following works? [NA]

(a) $z = \sup\{x : a \le x \le b,\ f(x) \le c\}$
(b) $z = \inf\{x : a \le x \le b,\ f(x) \le c\}$
(c) $z = \sup\{x : a \le x \le b,\ f(x) \ge c\}$
(d) $z = \sup\{x : a \le x \le b,\ f(x) \le 0\}$
(e) $z = \sup\{x : a \le x \le b,\ f$ is bounded on $[a, x]\}$

In the proof of the Intermediate Value Theorem, three of the lines are

> if $f(z) > c$, let $\delta > 0$ be such that $|f(x) - f(z)| \le \blacksquare$ whenever $x \in [a, b]$ and $|x - z| \le \delta$
> there is an x such that $f(x) \le c$ and $x \ge \max(a, z - \delta)$
> and this is impossible.

Which of the following can be put in place of $\blacksquare$? [NB]

(a) $\frac{1}{2}(f(z) - c)$
(b) $f(z) - c$
(c) 0
(d) $c - f(z)$
(e) ϵ

In the proof of the Intermediate Value Theorem, two of the lines are

> if $f(z) < c$, let $\delta > 0$ be such that $|f(x) - f(z)| \le \blacksquare$ whenever $x \in [a, b]$ and $|x - z| \le \delta$
> setting $x = \min(b, z + \delta)$, $f(x) \le c$.

Which of the following can be put in place of $\blacksquare$? [NC]

(a) $c - f(z)$
(b) $\frac{1}{2}(f(z) - c)$
(c) 0
(d) $f(z) - c$
(e) ϵ

You are seeking to prove that, for any $c \in \mathbb{R}$, there is a $z \in \mathbb{R}$ such that $z - \sin z = c$. Which of the following is a suitable interval $[a, b]$ on which to apply the Intermediate Value Theorem? (You may assume that $\sin : \mathbb{R} \to [-1, 1]$ is continuous.) [ND]

(a) $a = c - 1$, $b = c + 1$
(b) $a = 1 - c$, $b = c + 1$
(c) $a = c + 1$, $b = c - 1$
(d) $a = c$, $b = c + 1$
(e) $a = 0$, $b = c + 2$

You are given a continuous function $f : [a, b] \to \mathbb{R}$ and you wish to prove that there is a $z_0 \in [a, b]$ such that $f(x) \ge f(z_0)$ for every $x \in [a, b]$. Which of the following approaches work? [OA]

(a) Set $c = \inf\{f(x) : x \in [a, b]\}$, consider $g(x) = \frac{1}{f(x)-c}$
(b) Set $c = \sup\{f(x) : x \in [a, b]\}$, consider $g(x) = \frac{1}{f(x)-c}$
(c) Set $c = \inf\{f(x) : x \in [a, b]\}$, consider $g(x) = \frac{1}{f(x)+c}$
(d) Set $c = \min(f(a), f(b))$, consider $g(x) = \frac{1}{f(x)-c}$
(e) Set $c = \inf\{f(x) : x \in [a, b]\}$, consider $g(x) = f(x) - c$

Suppose that f is a real function and $x_0 \in \operatorname{dom} f$. Which of the following are true? [OB]

(a) If $\lim_{x\to x_0} f(x) = f(x_0)$ then f is continuous at x_0
(b) If f is continuous at x_0 then $\lim_{x\to x_0} f(x) = f(x_0)$
(c) If f is differentiable at x_0 then f is continuous at x_0
(d) If $\lim_{x\to x_0} f(x) = f(x_0)$ then f is differentiable at x_0
(e) If f is continuous at x_0 then f is differentiable at x_0

Which of the following are equivalent to '$|\frac{f(x)-f(x_0)}{x-x_0} - b| \le \epsilon$ whenever $0 < |x - x_0| \le \delta$'? [PA]
(a) $|f(x) - f(x_0) - b(x - x_0)| \le \epsilon|x - x_0|$ whenever $|x - x_0| \le \delta$
(b) $|f(x) - f(x_0) + b(x - x_0)| \le \epsilon|x - x_0|$ whenever $|x - x_0| \le \delta$
(c) $|f(x) - f(x_0) - b(x - x_0)| \le \epsilon|x + x_0|$ whenever $|x + x_0| \le \delta$
(d) $|f(x) - f(x_0) - b(x + x_0)| \le \epsilon|x + x_0|$ whenever $|x - x_0| \le \delta$
(e) $|f(x) - f(x_0) - b(x - x_0)| < \epsilon|x - x_0|$ whenever $|x - x_0| < \delta$

Given that $|g(x) - g(x_0) - b(x - x_0)| \le \eta|x - x_0|$, which of the following must be true? [PB]
(a) $|g(x) - g(x_0)| \le (\eta + |b|)|x - x_0|$
(b) $|g(x) - g(x_0)| \le (\eta + b)|x - x_0|$
(c) $|g(x) + g(x_0)| \le (\eta + |b|)|x + x_0|$
(d) $|g(x) - g(x_0)| \ge (|b| - \eta)|x - x_0|$
(e) $g(x) - g(x_0) \le (\eta + |b|)(x - x_0)$

In the proof that if $\lim_{x\uparrow a} f(x) = \lim_{x\downarrow a} f(x) = b$ then $\lim_{x\to a} f(x) = b$, which of the following would you expect to see? [RA]
(a) Set $\delta = \min(\delta_1, \delta_2)$
(b) Set $\delta = \max(\delta_1, \delta_2)$
(c) Set $M = \max(M_1, M_2)$
(d) Set $M = \min(M_1, M_2)$
(e) Set $\delta = \epsilon$

In the proof that if $f(x) \le c$ for $z < x \le b$ and $\lim_{x\downarrow z} f(x) = d$ then $d \le c$, three of the lines are

Suppose, if possible, that $d > c$.
There is a $\delta > 0$ such that $|f(x) - d| \le \blacksquare$ whenever $z < x \le z + \delta$.
Setting $x = \blacksquare\blacksquare\blacksquare$, $c \ge f(x) \ge d - \blacksquare$, which is impossible.

Which pair of expressions can you use in place of $\blacksquare$, $\blacksquare\blacksquare\blacksquare$? [RBA]
(a) $\frac{1}{2}(d - c)$, $\min(b, z + \delta)$
(b) $d - c$, $\min(b, z + \delta)$
(c) $\frac{1}{2}(d - c)$, $\max(b, z + \delta)$
(d) $d - c$, $\max(b, z + \delta)$
(e) $\frac{1}{2}(c - d)$, $\min(b, z + \delta)$

In the proof that if $f(x) \ge c$ for $a \le x < z$ and $\lim_{x\uparrow z} f(x) = d$ then $d \ge c$, three of the lines are

Suppose, if possible, that $d < c$.
There is a $\delta > 0$ such that $|f(x) - d| \le \blacksquare$ whenever $z - \delta \le x < z$.
Setting $x = \blacksquare\blacksquare\blacksquare$, $c \le f(x) \le d + \blacksquare$, which is impossible.

Which pair of expressions can you use in place of $\blacksquare$, $\blacksquare\blacksquare\blacksquare$? [RBB]
(a) $\frac{1}{2}(c - d)$, $\max(a, z - \delta)$
(b) $c - d$, $\max(a, z - \delta)$
(c) $\frac{1}{2}(c - d)$, $\min(a, z - \delta)$
(d) $c - d$, $\min(a, z - \delta)$
(e) $\frac{1}{2}(d - c)$, $\max(a, z - \delta)$

Given that f is a real function and that $[a, b] \subseteq \operatorname{dom} f$, where $a < b$, which of the following is Rolle's theorem? [SA]

(a) if f is continuous on $[a, b]$ and differentiable on $]a, b[$, and $f(a) = f(b)$, then there is an $x \in]a, b[$ such that $f'(x) = 0$

(b) if f is differentiable on $[a, b]$ and continuous on $]a, b[$, and $f(a) = f(b)$, then there is an $x \in]a, b[$ such that $f'(x) = 0$

(c) if f is continuous on $[a, b]$ and differentiable on $]a, b[$, and $f(a) \neq f(b)$, then there is an $x \in]a, b[$ such that $f'(x) \neq 0$

(d) if f is continuous on $[a, b]$ and differentiable on $]a, b[$, then there is an $x \in]a, b[$ such that $f'(x) = 0$

(e) if f is differentiable on $]a, b[$, and $f(a) = f(b)$, then there is an $x \in]a, b[$ such that $f'(x) = 0$

Given that $x_0 \neq 0$ and $\epsilon > 0$, which of the following is enough to ensure that $|\frac{x_0 - x}{x x_0^2}| \leq \epsilon$? [SB]

(a) $|x - x_0| \leq \min(\frac{1}{2}|x_0|, \frac{1}{2}\epsilon|x_0|^3)$

(b) $|x - x_0| \leq \max(\frac{1}{2}|x_0|, \frac{1}{2}\epsilon|x_0|^3)$

(c) $|x - x_0| \leq \frac{1}{2}\epsilon|x_0|^3$

(d) $|x - x_0| \leq \frac{1}{2}|x_0|$

(e) $|x - x_0| \leq \min(\frac{1}{2}|x_0|, \frac{1}{2}\epsilon|x_0|^2)$

Which of the following theorems are needed in the proof of Rolle's theorem? [SC]

(a) A continuous function on a closed bounded interval is bounded and attains its bounds.

(b) The Intermediate Value Theorem.

(c) If $\lim_{x \to z} f(x) = c$ then $\lim_{x \downarrow z} f(x) = c$.

(d) If $\lim_{x \uparrow z} f(x) = c$ and $f(x) \geq 0$ for $a \leq x < z$, where $a < z$, then $c \geq 0$.

(e) The General Principle of Convergence.

Given that f is a real function and that $[a, b] \subseteq \operatorname{dom} f$, where $a < b$, which of the following is the Mean Value Theorem? [TA]

(a) if f is continuous on $[a, b]$ and differentiable on $]a, b[$, then there is an $x \in]a, b[$ such that $f'(x) = \frac{f(b) - f(a)}{b - a}$

(b) if f is differentiable on $[a, b]$ and continuous on $]a, b[$, then there is an $x \in]a, b[$ such that $f'(x) = \frac{f(b) - f(a)}{b - a}$

(c) if f is continuous on $[a, b]$ and differentiable on $]a, b[$, then there is an $x \in]a, b[$ such that $f'(x) = 0$

(d) if f is continuous on $[a, b]$ and differentiable on $]a, b[$, and $f(a) = f(b)$, then there is an $x \in]a, b[$ such that $f'(x) = \frac{f(b) - f(a)}{b - a}$

(e) if f is continuous on $[a, b]$ and differentiable on $]a, b[$, and $f(a) \neq f(b)$, then there is an $x \in]a, b[$ such that $f'(x) \neq 0$

Given that f and g are real functions and that $[a, b] \subseteq \operatorname{dom} f \cap \operatorname{dom} g$, where $a < b$, which of the following hypotheses are used in Cauchy's Mean Value Theorem? [TB]

(a) f is continuous on $[a, b]$

(b) g is differentiable on $]a, b[$

(c) $g'(x) \neq 0$ for every $x \in]a, b[$

(d) $g(b) \neq g(a)$

(e) f is differentiable on $[a, b]$

Which of the following is the conclusion of Cauchy's Mean Value Theorem? [TC]

(a) $\exists\, x \in\,]a,b[,\ \frac{f'(x)}{g'(x)} = \frac{f(b)-f(a)}{g(b)-g(a)}$
(b) $\exists\, x \in [a,b],\ \frac{f'(x)}{g'(x)} = \frac{f(b)-f(a)}{g(b)-g(a)}$
(c) $\exists\, x \in\,]a,b[,\ \frac{f'(x)}{g'(x)} = \frac{g(b)-g(a)}{f(b)-f(a)}$
(d) $\exists\, x \in\,]a,b[,\ \frac{f'(x)}{g'(x)} = \frac{f(b)+f(a)}{g(b)+g(a)}$
(e) $\exists\, x \in\,]a,b[,\ \frac{f'(x)}{g'(x)} = \frac{f(b)-g(b)}{f(a)-g(a)}$

When proving Cauchy's Mean Value Theorem from Rolle's theorem, which of the following works? [TD]

(a) $h(x) = f(x)(g(b) - g(a)) - g(x)(f(b) - f(a))$
(b) $h(x) = f(x)(g(b) - g(a)) + g(x)(f(b) - f(a))$
(c) $h(x) = (f(x) - g(a))(g(x) - f(a))$
(d) $h(x) = f(x)(g(b) + g(a)) - g(x)(f(b) + f(a))$
(e) $h(x) = f(x)(b - a) - g(x)(b - a)$

Which of the following are true for all real numbers x and a such that $0 < |x - a| < |a|$? [xxAL]

(a) $\left|\frac{1}{x} - \frac{1}{a}\right| \leq \frac{|x-a|}{|a|(|a|-|x-a|)}$
(b) $\left|\frac{1}{x} - \frac{1}{a}\right| \leq \frac{|x-a|}{|a|(|a|+|x-a|)}$
(c) $\left|\frac{1}{x} - \frac{1}{a}\right| \leq \frac{|x+a|}{|a|(|a|+|x-a|)}$
(d) $\left|\frac{1}{x} - \frac{1}{a}\right| \leq \frac{|x+a|}{|a|(|a|-|x-a|)}$
(e) $\left|\frac{1}{x} - \frac{1}{a}\right| \leq \frac{|x-a|}{|x|(|x|-|a-x|)}$

Which of the following are true for all real numbers x, y, a and b? [xxAJ]

(a) $|xy - ab| \leq |x - a||y - b| + |a||y - b| + |b||x - a|$
(b) $|xy - ab| \leq |x + a||y - b| + |a||y - b| + |b||x - a|$
(c) $|xy - ab| \leq |x - a||y + b| + |a||y - b| + |b||x - a|$
(d) $|xy - ab| \leq |x - a||y - b| + |a||y + b| + |b||x - a|$
(e) $|xy - ab| \leq |x - a||y - b| + |a||y - b| + |b||x + a|$

Which of the following are negations of the sentence '$\forall\, x > 0,\ x \in A$'? [xxBA]

(a) $\exists\, x > 0,\ x \notin A$
(b) $\forall\, x > 0,\ x \notin A$
(c) $\exists\, x > 0,\ x \in A$
(d) $\forall\, x \leq 0,\ x \in A$
(e) $\exists\, x \leq 0,\ x \notin A$

Which of the following are negations of the sentence '$a > 0$ or $a \in B$'? [xxBB]

(a) $a \leq 0$ and $a \notin B$
(b) $a < 0$ and $a \notin B$
(c) $a \leq 0$ or $a \notin B$
(d) $a \leq 0$ and $a \in B$
(e) $a > 0$ and $a \notin B$

Which of the following are negations of the sentence '$x \leq 2$ and $x > a$'? [xxBC]
(a) $x > 2$ or $x \leq a$
(b) $x \leq 2$ or $x \leq a$
(c) $x > 2$ or $x > a$
(d) $x > 2$ and $x \leq a$
(e) $x \geq 2$ or $x \leq a$

Which of the following are negations of the sentence 'for every $x \in A$, if $x \leq 3$ then $x \leq 2$'? [xxBD]
(a) $\exists$, $x \in A$, $2 < x \leq 3$
(b) $\exists$, $x \in A$, $2 \leq x \leq 3$
(c) $\forall$, $x \in A$, $2 < x \leq 3$
(d) $\exists$, $x \in A$, $x > 2$ or $x \leq 3$
(e) $\exists$, $x \in A$, $x \leq 3$

Which of the following are negations of the sentence '$\exists\, x \in A,\, x > 0$'? [xxBE]
(a) $\forall\, x \in A,\, x \leq 0$
(b) $\exists\, x \in A,\, x \leq 0$
(c) $\forall\, x \in A,\, x > 0$
(d) $\forall\, x \in A,\, x < 0$
(e) $\forall\, x \notin A,\, x > 0$

Which of the following are negations of the sentence 'a is an upper bound of A'? (You may take it for granted that $a \in \mathbb{R}$ and $A \subseteq \mathbb{R}$.) [xxBF]
(a) $\exists\, x \in A,\, x > a$
(b) $\forall\, x \in A,\, x > a$
(c) $\exists\, x \in A,\, x \leq a$
(d) $\forall\, x \in A,\, x \leq a$
(e) $a \notin A$

Which of the following are negations of the sentence 'a is not an upper bound of A'? (You may take it for granted that $a \in \mathbb{R}$ and $A \subseteq \mathbb{R}$.) [xxBG]
(a) $\forall\, x \in A,\, x \leq a$
(b) $\exists\, x \in A,\, x \leq a$
(c) $\forall\, x \in A,\, x > a$
(d) $\exists\, x \in A,\, x > a$
(e) $\forall\, x \in A,\, x < a$

Which of the following are negations of the sentence '$\forall\, \epsilon \in A\; \exists\, n \in \mathbb{N} \setminus \{0\},\, \frac{1}{n} \leq \epsilon$'? [xxCF]

(a) $\exists\, \epsilon \in A\; \forall\, n \in \mathbb{N} \setminus \{0\},\, \frac{1}{n} > \epsilon$

(b) $\forall\, \epsilon \in A\; \forall\, n \in \mathbb{N} \setminus \{0\},\, \frac{1}{n} > \epsilon$

(c) $\exists\, \epsilon \in A\; \exists\, n \in \mathbb{N} \setminus \{0\},\, \frac{1}{n} > \epsilon$

(d) $\exists\, \epsilon \in A\; \forall\, n \in \mathbb{N} \setminus \{0\},\, \frac{1}{n} \leq \epsilon$

(e) $\forall\, \epsilon \in A\; \exists\, n \in \mathbb{N} \setminus \{0\},\, \frac{1}{n} \leq \epsilon$

Which of the following are negations of the sentence '$\forall\, x \in A\; \exists\, y \in B,\, x < y$'? [xxCG]
(a) $\exists\, x \in A\; \forall\, y \in B,\, y \leq x$
(b) $\forall\, x \in A\; \forall\, y \in B,\, y \leq x$
(c) $\exists\, x \in A\; \exists\, y \in B,\, y \leq x$
(d) $\exists\, x \in A\; \forall\, y \in B,\, y > x$
(e) $\forall\, x \in A\; \exists\, y \in B,\, x < y$

Which of the following are negations of the sentence '$\forall\,\epsilon > 0\ \exists\ n_0 \in \mathbb{N}\ \forall\ n \geq n_0,\ |x_n - a| \leq \epsilon$'? [xxCH]

(a) $\exists\,\epsilon > 0\ \forall\,n_0 \in \mathbb{N}\ \exists\,n \geq n_0,\ |x_n - a| > \epsilon$
(b) $\forall\,\epsilon > 0\ \exists\,n_0 \in \mathbb{N}\ \forall\,n \geq n_0,\ |x_n - a| > \epsilon$
(c) $\exists\,\epsilon \leq 0\ \forall\,n_0 \in \mathbb{N}\ \exists\,n \geq n_0,\ |x_n - a| > \epsilon$
(d) $\exists\,\epsilon > 0\ \forall\,n_0 \in \mathbb{N}\ \exists\,n < n_0,\ |x_n - a| > \epsilon$
(e) $\exists\,\epsilon \geq 0\ \forall\,n_0 \in \mathbb{N}\ \exists\,n \geq n_0,\ |x_n - a| > \epsilon$

Which of the following are negations of the sentence '$\forall\,\epsilon > 0\ \exists\ \delta > 0\ \forall\ x \in \mathbb{R}$, if $0 < |x - x_0| \leq \delta$ then $|f(x) - a| \leq \epsilon$'? (You may take it for granted that f is a function from $\mathbb{R}$ to itself.) [xxCI]
(a) $\exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x \in \mathbb{R},\ 0 < |x - x_0| \leq \delta$ and $|f(x) - a| > \epsilon$
(b) $\exists\,\epsilon \geq 0\ \forall\,\delta > 0\ \exists\,x \in \mathbb{R},\ 0 < |x - x_0| \leq \delta$ and $|f(x) - a| > \epsilon$
(c) $\exists\,\epsilon > 0\ \forall\,\delta \geq 0\ \exists\,x \in \mathbb{R},\ 0 < |x - x_0| \leq \delta$ and $|f(x) - a| > \epsilon$
(d) $\exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x \in \mathbb{R},\ 0 < |x - x_0| \leq \delta$ or $|f(x) - a| > \epsilon$
(e) $\exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x \in \mathbb{R},\ 0 < |x - x_0| < \delta$ and $|f(x) - a| \geq \epsilon$

Which of the following are negations of the sentence '$\forall\,\epsilon > 0\ \exists\ \delta > 0\ \forall\ x \in [x_0 - \delta, x_0 + \delta] \cap \mathrm{dom}\, f,\ |f(x) - f(x_0)| \leq \epsilon$'? (You may take it for granted that f is a real function and that $x_0 \in \mathrm{dom}\, f$.) [xxCJ]
(a) $\exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x \in [x_0 - \delta, x_0 + \delta] \cap \mathrm{dom}\, f,\ |f(x) - f(x_0)| > \epsilon$
(b) $\exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x \in [x_0 - \delta, x_0 + \delta] \cap \mathrm{dom}\, f,\ |f(x) - f(x_0)| \geq \epsilon$
(c) $\exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x \in \mathrm{dom}\, f \setminus [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| > \epsilon$
(d) $\exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x \in [x_0 - \delta, x_0 + \delta] \cap \mathrm{dom}\, f,\ |f(x) - f(x_0)| \leq \epsilon$
(e) $\forall\,\epsilon > 0\ \exists\,\delta > 0\ \forall\,x \in [x_0 - \delta, x_0 + \delta] \cap \mathrm{dom}\, f,\ |f(x) - f(x_0)| > \epsilon$

Given that $f : \mathbb{R} \to \mathbb{R}$ is a function, which of the following are negations of the sentence '$\forall\,x_0 \in \mathbb{R}\ \forall\,\epsilon > 0\ \exists\,\delta > 0\ \forall\,x \in [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| \leq \epsilon$'? [xxCK]
(a) $\exists\,x \in \mathbb{R}\ \exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x \in [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| > \epsilon$
(b) $\exists\,x \in \mathbb{R}\ \forall\,\epsilon > 0\ \exists\,\delta > 0\ \forall\,x \in [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| > \epsilon$
(c) $\exists\,x \in \mathbb{R}\ \exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x \in [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| \leq \epsilon$
(d) $\exists\,\epsilon > 0\ \exists\,x_0 \in \mathbb{R}\ \forall\,\delta > 0\ \exists\,x \in [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| > \epsilon$
(e) $\exists\,x \in \mathbb{R}\ \exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x \in [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| \geq \epsilon$

Given that f is a real function with domain D, which of the following are negations of the sentence '$\forall\,\epsilon > 0\ \exists\,\delta > 0\ \forall\,x_0 \in D\ \forall\,x \in D \cap [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| \leq \epsilon$'? [xxCL]
(a) $\exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x_0 \in D\ \exists\,x \in D \cap [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| > \epsilon$'
(b) $\exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x_0 \in D\ \forall\,x \in D \cap [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| > \epsilon$
(c) $\exists\,\epsilon > 0\ \exists\,x_0 \in D\ \forall\,\delta > 0\ \exists\,x \in D \cap [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| > \epsilon$
(d) $\forall\,\delta > 0\ \exists\,\epsilon > 0\ \exists\,x_0 \in D\ \exists\,x \in D \cap [x_0 - \delta, x_0 + \delta],\ |f(x) - f(x_0)| > \epsilon$
(e) $\exists\,\epsilon > 0\ \forall\,\delta > 0\ \exists\,x_0 \in D\ \exists\,x \in D,\ |x - x_0| \leq \delta$ and $|f(x) - f(x_0)| > \epsilon$