From Essex and Thacher's Poll Tax to Walrasian Equilibrium in Twenty Four Steps

Peter J. Hammond (Dept. of Economics, U. Warwick, UK)

June 13, 2015

Essex Before 1971

- Founding professor: Richard Lipsey.
  His *Positive Economics* was my homework
  during the summer vacation of 1967,
  while working for a software startup.

  Three more star professors:
  Chris Archibald, David Laidler, Michael Parkin

  All had left by 1970
- Keynes Visiting Professors: Michio Morishima, Rex Bergstrom
  Bill Brainard (??), plus others
- Lecturers etc: David Horwell, Mike Martin, Alastair McAuley,
  PN (Raja) Junankar, David Mayston, Jim Richmond, etc.
- Some distinguished students:
  Chris Pissarides, Graham Loomes, Roy Bailey, Alec Chrystal

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1970: The Urban Legend

1. By 1970 Essex had no permanent professors of economics, but there were 2 (or 3) vacancies to fill urgently.
2. Vice Chancellor Albert Sloman gets on a train to Cambridge to talk to Christopher Bliss, then a university lecturer.
3. Christopher is invited to take up an appointment as Professor and Department Chair
4. He is also allowed to select one colleague to join him as another Professor
5. He selects Tony Atkinson, then aged 26.
6. Rex Bergstrom also stays on as a permanent professor.

1970–71: The Transition

- Doug Fisher is acting department chair at the time.
- Christopher Bliss, my ex-supervisor and "guru" at Cambridge, telephones me at Nuffield College, Oxford and invites me to apply for a lectureship.
- I am "flown out" (presumably by train) for a "job talk" during which I am grilled by David Mayston.
- Some legitimate doubts are expressed over whether I am "really an economist".

  But I am offered, and accept, a lectureship starting in Autumn 1971.

1972–79: An Accumulation of Talent

- Mike Wickens as Reader
- As Lecturers:
  Ken Burdett, Monojit Chatterji, Leslie Hannah,
  William Kennedy, James Macintosh, Peter Phillips.
- The best year of the 1970s:
  1974–75, when Essex "borrows" Oliver Hart for a year,
  and I am away on leave at the Australian National University.

Footnote: Joe Stiglitz had told me about IMSSS
(Institute of Mathematical Studies in the Social Sciences).

Founded during the 1950s
by Patrick Suppes, "scientific philosopher".

Economics section founded by Mordecai Kurz in 1969,
as a device to bring Ken Arrow back to Stanford every summer.

On my way back to the UK, I visited IMSSS in summer 1975.

1976–79: Some Decumulation of Talent

- 1976: Tony Atkinson leaves for UCL
  — and I am promoted as an inadequate replacement.
- Peter Phillips takes a chair at Birmingham
- Jim Mirrlees remembers that Christopher Bliss
  used to teach international trade in Cambridge,
  and recruits him to the Nuffield Readership in Oxford.
- Mike Wickens moves to a chair at York.

There is one thing worse than a department
that loses its personnel to promotions elsewhere:
it is a department that never loses anyone!

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1978–79: Massive New Recruitment

- New professors (or readers?) starting in 1979:
  Graciela Chichilnisky, Geoffrey Heal, Norman Schofield
  – the Essex acyclic triple?
- 6 lectureships. From an enormous field, we shortlisted 12, and then got down to 6 we appointed, including:

  Roy Bailey and Tim Hatton (right year?)
  Sajal Lahiri, Henri Lorie, Patricia Rice, Tony Venables

1979: Voting with My Feet

- 1970–74: In Edward Heath's government, Mrs. Thatcher was Secretary of State for Education and Science.
- Universities gradually starved.
- The library could not afford a subscription to the *International Journal of Game Theory*.
- By February 1979, in the "winter of discontent", it is obvious that Mrs. Thatcher will win the forthcoming general election.
- 1978: Stanford entices Kenneth Arrow back from Harvard.
- Stanford creates another post in theory to start in 1979.
- When I am snowed in at my home in Leavenheath, I accept an offer over the phone from Bert Hickman, chair of Economics at Stanford.

Postgraduate Students I Remember from 1971–79

Doctoral students:

- Paul Madden (Manchester);
- Paul Weller — my first PhD student,
  who went first to Warwick, then on to Iowa
- John Yeabsley, Jack Mintz
  (for whose Ph.D. theses I was the internal examiner)

  Plus econometricians Terence Agbeyegbe, Noxy Dastoor.

Masters students:

- Michael Riordan (briefly an assistant Professor at Stanford, eventually the Laurans A. and Arlene Mendleson Professor of Economics at Columbia University);
- Luigi Campiglio
  (Catholic University of the Sacred Heart in Milan)

Notable Undergraduates, I: Kevin Roberts

- Kevin Roberts graduated in Math Econ in 1973.
- Went on to obtain MPhil and then DPhil degrees at Nuffield College, Oxford; supervised by Jim Mirrlees.
- 1982–87: Professor of Economic Theory, University of Warwick
- 1987–99: Professor of Economics, London School of Economics
- 1999–date Sir John Hicks Professor of Economics, Nuffield College, Oxford.

Notable Undergraduates, II: Jean Drèze

- Son of Jacques Drèze, whom I have known since 1971.
- Recruited by Mike Martin in 1976 after Jean had left it too late to apply to most other universities. Essex was first after getting off the boat from Belgium!
- Graduated in Math Econ in 1979.
- PhD 1984, Indian Statistical Institute, New Delhi "On the Choice of Shadow Prices for Project Evaluation" Two volumes mailed to his external examiner at Stanford.
- Conceptualized and drafted the first version of the National Rural Employment Guarantee Act 2005, later renamed as the "Mahatma Gandhi National Rural Employment Guarantee Act" (or, MGNREGA).

  In its World Development Report 2014, the World Bank termed it a "stellar example of rural development".

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Academic Recollection Involving Rex Bergstrom

Dastoor, Naorayex K. (1983) "Some Aspects of Testing Non-nested Hypotheses" *Journal of Econometrics* 21: 213–228.

What I suspect was an early version of this was presented to a PhD student workshop.

Both Rex Bergstrom and I attended.

Noxy's example of non-nested hypotheses:

1. profit maximization by a firm;
2. cost minimization by a firm.

Rex and I both pointed out that 1 implies 2.
But not if you are an econometrician!

Econometric Equations

1. With profit maximization, you have:
   - input and output prices as exogenous variables;
   - input demands and output supplies as endogenous variables.

   So

$$\text{input demands} = f_1(\text{input prices, output prices}) + \text{error}$$
$$\text{output supplies} = f_2(\text{input prices, output prices}) + \text{error}$$

2. With cost minimization, you have:
   - input prices and output supplies as exogenous variables;
   - only input demands as endogenous variables.

   So

$$\text{input demands} = f(\text{input prices}, \text{output quantities}) + \text{error}$$

Those errors make all the difference!

Personal Productivity

According to Google scholar,
the papers ranked 1, 2, and 4–9 in my list of citations
were all produced at Essex.

Many thanks to Pam Hepworth for typing nearly all this work,
as well as editorial correspondence
for the *Review of Economic Studies*.

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Identification problem: how much is due to:

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RStud Symposium Article, 1979

"Straightforward Individual Incentive Compatibility
in Large Economies" *Review of Economic Studies* 46: 263–282.

Results:

1. Incentive compatibility of Walrasian (competitive) equilibrium without lump-sum transfers.

   This formalizes a remark due to Leo Hurwicz.

   Subject of the main part of the talk.

2. Incentive incompatibility of lump-sum transfers, thus formalizing an observation due to Paul Samuelson in his *Foundations of Economic Analysis*.

Thatcher's Downfall

In the late 80s, Mrs. Thatcher introduced a "community charge", generally labelled the " poll tax" (on people who have heads).

It was first imposed on the Scots!

Features prominently in Anthony King and Ivor Crewe (2013) *The Blunders of Our Governments*.

Third result:
The only incentive compatible and Pareto efficient method for financing publicly provided goods is a poll tax.

My article did not use the term "poll tax", or even "community charge".

The result came with a "health warning":
the poll tax was likely to be difficult to collect if consumer's true endowments were not publicly known.

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A Model

Continuum of consumers,
with tastes θ varying in a metric space Θ.

The taste distribution is ν,
a probability measure on the Borel measurable sets of Θ.

Each θ is a parameter which determines
that consumer's utility function $(x, z) \mapsto u(x, z; \theta)$
for private goods $x \in \mathbb{R}^G$ and public goods $z \in \mathbb{R}^H$.

Production possibiliities for public goods: $(-x^P, z) \in Y^P$

Sufficient Conditions

A sufficient condition for constrained Pareto efficiency, given the public goods vector z:

For each taste distribution ν, choose:

1. a Walrasian price vector $p(\nu) \in \mathbb{R}^G$ for private goods;
2. a public good vector $z(\nu) \in \mathbb{R}^H$;
3. for producing public goods, a cost-minimizing input vector
$$x^P(\nu) \in \arg\min_{x^P} \{p(\nu)\, x^P \mid (-x^P, z(\nu)) \in Y^P\}$$
4. a poll tax $w(\nu) = p(\nu)\, x^P(\nu) \in \mathbb{R}$;
5. for each taste parameter θ, a private good allocation vector
$$x(\nu, \theta) \in \arg\max_{x} \{u(x, z(\nu); \theta) \mid p(\nu)\, x \leq -w(\nu)\}$$

Together, all these must satisfy the market clearing condition

$$x^P(\nu) + \int_\Theta x(\nu, \theta) d\nu = 0$$

Constrained Pareto Efficiency

The linear pricing scheme brings about
a constrained Walrasian equilibrium, taking as given
the public sector production plan $(-x^P(\nu), z(\nu)) \in Y^P$.

By a suitable application of the standard
first efficiency theorem of welfare economics,
this is constrained Pareto efficient
given the public sector production plan $(-x^P(\nu), z(\nu)) \in Y^P$.

Strategyproofness

Given any true taste parameter $\theta \in \Theta$
and any false taste parameter $\eta \in \Theta$,
there is an incentive constraint

$$u(x(\nu, \theta), z(\nu); \theta) \geq u(x(\nu, \eta), z(\nu); \theta)$$

But there is a continuum of consumers,
so no individual can affect $p(\nu)$, $z(\nu)$, or $w(\nu)$.

Because $x(\nu, \theta)$ is optimal, and $x(\nu, \eta)$ also satisfies
the budget constraint $p(\nu)\, x \leq -w(\nu)$,
this ensures strategyproofness
(or "straightforward" individual incentive compatibility).

This property came to be known
as the taxation or decentralization principle.

Roger Guesnerie (1998) Econometric Society Monograph
A Contribution to the Pure Theory of Taxation.

Main Result Updated

- Not only is this scheme of markets and poll taxation sufficient for a constrained Pareto efficient allocation given the public sector production plan $(-x^P(\nu), z(\nu)) \in Y^P$;
- it is also necessary given:
  1. for each fixed public vector $z \in \mathbb{R}^H$, a sufficiently rich domain of smooth preferences corresponding to the utility functions $x \mapsto u(x, z; \theta)$;
  2. no further information such as age (the basis of any state pension system), or residence (the basis of the "community charge");
  3. the restriction that all consumers with the same tastes receive the same allocation.

Where to Find Details

Disagreement over how to treat null sets:

Paul Champsaur and Guy Laroque (1982)
"A Note on Incentives in Large Economies"
Review of Economic Studies 49: 627–635.

For some important details,
and an attempt to meet arguments like Champsaur and Laroque's
(at least with only private goods),
see especially Sections 2 and 14 of:

"Competitive Market Mechanisms as Social Choice Procedures"
in Kenneth J. Arrow, Amartya Sen and Kotaro Suzumura (eds.)
Handbook of Social Choice and Welfare, Vol. II
(Amsterdam: North-Holland, 2011), ch. 15, pp. 47–151.

Lindahl Pricing

The 1979 paper used Lindahl pricing to achieve
an overall first-best Pareto efficient allocation
of both private and public goods.

But it ignored non-convexities in producing public goods
— or the public environment in general.

My last seminar at Essex (when Karl Shell was visiting):

(with Antonio Villar) "Efficiency with Non-Convexities:
Extending the 'Scandinavian Consensus' Approaches"
Scandinavian Journal of Economics 100 (1998), 11–32.

Motivation

The REStud 1979 paper had many practical limitations,
even for implementing a Walrasian equilibrium in spot markets
for an economy with only private goods.

Problems to take care of:

1. Typically traders are privately informed,
   not only of their tastes,
   but also of their endowments and production possibilities.
2. Traders may not deliver the supplies they agreed to make.
3. Traders may not even understand what preferences are;
   at best, we expect them to announce
   demand correspondences
   — perhaps choosing from a menu of computer programs.
   Compare double auctions, where the demand correspondence
   is a step "function" with a closed graph.
4. Equilibrium may not exist without restrictions on demand.

Arrow's Exceptional Case, I

Arrow's (1951) original example involved
an Edgeworth box economy with two traders and two goods
in which one trader has non-monotone preferences.

Koopmans (1957, p. 34) appears to have been the first
to present a version in which preferences are monotone.

The agent's feasible set is the non-negative quadrant $\mathbb{R}^2_+$.

Preferences are assumed to be represented by the "quasi-linear"
utility function $u(x_1, x_2) = x_1 + \sqrt{x_2}$ restricted to the domain $\mathbb{R}^2_+$.

So all the indifference curves are given by $x_2 = (u - x_1)^2$
for $0 \leq x_1 \leq u$.

Arrow's Exceptional Case, II

The trader's indifference curves must therefore be parts of parabolae which touch the x_1-axis, as indicated by the dotted curves.

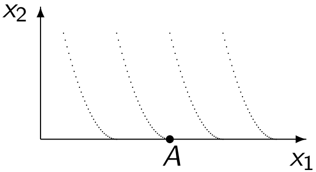

Figure: Arrow's Exceptional Case

Arrow's Exceptional Case, III

The marginal willingness to pay for the second good becomes infinite as $x_2 \to 0$.

Consider an economy with one consumer whose endowment vector is $(x_1, 0)$ with x_1 positive, such as A in the figure.

To make this an equilibrium requires a budget line touching the indifference curve at A.

So the only possible price vector is $(0, p_2)$ where $p_2 > 0$.

The corresponding budget constraint would have to be $p_2 \, x_2 \leq 0$, which is equivalent to $x_2 \leq 0$.

But then the agent could always move to preferred points by increasing x_1 indefinitely while keeping $x_2 = 0$.

There is no Walrasian equilibrium in this economy.

Agents' Preference Types and Utilities

Over the space of consumption vectors $c^i \in \mathbb{R}^G_+$,
assume that each agent $i \in I$ has:

- a continuous and strictly monotone preference ordering $\succsim^i$;
- which is represented by a continuous and strictly increasing (but not necessarily quasi-concave) utility function $u^i : \mathbb{R}^G_+ \to \mathbb{R}$.

Following Harsanyi (1967–8), each agent i's utility function is assumed to take the form $u^i(c^i) \equiv u(c^i; \theta^i)$, where:

- θ^i is a taste parameter in a compact Polish type space Θ;
- the mapping $\mathbb{R}^G_+ \times \Theta \ni (c, \theta) \mapsto u(c; \theta) \in \mathbb{R}$ is:
  1. independent of i;
  2. jointly continuous in c and θ;
  3. strictly increasing in c for each fixed θ.

Agents' Types and Joint Distributions

Each agent's endowment vector is assumed to lie in the set

$$E := \{e \in \mathbb{R}^G_+ \mid e \leqq \bar{e}\}$$

for a suitable common upper bound $\bar{e} \geqq 0$.

Then the agents' type space is taken to be the set $T := \Theta \times E$ of preference–endowment pairs $t = (\theta, e)$.

We consider many spaces such as $\mathcal{M}_\lambda(L; T)$.

It is defined as the set of probability measures τ
on the product measurable space $(L \times T, \mathcal{B}(L \times T))$
satisfying the requirement that $\text{marg}_L \, \tau = \lambda$
— i.e., for every Borel set $K \subseteq L$, one has $\lambda(K) = \tau(K \times T)$.

A Continuum Economy or Game of Incomplete Information

Each agent $i \in \mathbb{N}$ is given a random label $\ell^i \in L := [0, 1]$,
which the auctioneer/principal can observe.

It is assumed that a completely informed observer could estimate:

1. a distribution of agents' labels $\ell^i \in L$
   which is described by Lebesgue measure λ on $[0, 1]$;
2. a type distribution measure τ
   in the set $\mathcal{M}_\lambda(L; T)$ of Borel probability measures on $L \times T$,
   the space of agents' label–type pairs (ℓ^i, t^i),
   whose marginal on L is equal to λ.

Assume, however, that the auctioneer/principal
is unable to observe any agent's type,
and does not know the type distribution τ either.

Commodities, Prices, Agents, and Endowments

To exclude Arrow's exceptional case, we only allow trade in an appropriate subset K of a fixed commodity set G.

Let $p = (p_g)_{g \in K} \in \mathbb{R}^K$ denote a typical price vector.

Let $P_K := \left\{ p \in \mathbb{R}^K_+ \mid \sum_{g \in K} p_g = 1 \right\}$
denote the unit simplex of non-negative normalized price vectors.

Let $P^0_K := \left\{ p \in \mathbb{R}^K_{++} \mid \sum_{g \in K} p_g = 1 \right\}$
denote the relative interior of P,
with relative boundary $\mathrm{bd}\, P_K := P_K \setminus P^0_K$.

Consider a pure exchange economy
with a countably infinite set $I = \mathbb{N} := \{1, 2, \ldots\}$ of traders.

Each trader $i \in I$ has a fixed endowment vector $e^i \in \mathbb{R}^K_+$.

Net Trades

A typical net trade vector for agent i
will be denoted by $z^i \in \mathbb{R}^K$,
with components z^i_g for $g \in K$.

It can be written uniquely as $z^i = x^i - y^i$
where $x^i \in \mathbb{R}^K_+$ and $y^i \in \mathbb{R}^K_+$
are separate non-negative demand and supply vectors
with $x^i_g = \max\{z^i_g, 0\}$
and $y^i_g = \max\{-z^i_g, 0\} = -\min\{z^i_g, 0\}$ for all $g \in K$.

In lattice notation, one can then write

$$x^i = z^i \vee 0 \quad \text{and} \quad y^i = (-z^i) \vee 0 = -(z^i \wedge 0)$$

Feasible Sets

Assume each agent $i \in I$ can:

1. supply any vector $y^i \in Y^i := \{y^i \in \mathbb{R}^K_+ \mid y^i \leqq e^i\}$;
2. demand any vector $x^i \in \mathbb{R}^K_+$.

This leads to the closed feasible set $Z^i := \mathbb{R}^K_+ - \{e^i\}$
of net trade vectors $z^i = x^i - y^i$ with $x^i \in \mathbb{R}^K_+$ and $y^i \leqq e^i$.

A Walrasian Budget Correspondence

Consider the Walrasian budget correspondence

$$P_K^0 \ni p \mapsto\!\!\!\to B^K(p) := \{z \in \mathbb{R}^K \mid p\, z \leq 0 \text{ and } z \geqq -\bar{e}\}$$

Its value is the Walrasian budget set $B^K(p)$
of all net trade vectors satisfying both the budget constraint,
as well as what the feasibility constraint $z + \bar{e} \geqq 0$
would be for any agent whose endowment
happened to equal the upper bound $\bar{e}$
of the range E of all possible endowment vectors in $\mathbb{R}^K$.

A Net Demand Correspondence

A possible net demand correspondence will be a mapping

$$P_K^0 \ni p \mapsto\!\!\!\rightarrow Z(p; \mathfrak{Z}) \subset B^K(p)$$

also denoted by $\mathfrak{Z}$, that for each price vector $p \in P_K^0$
selects a non-empty subset $Z(p; \mathfrak{Z})$ of the budget set $B^K(p)$.

Notation

Recall that $\bar{e} \in \mathbb{R}^K_{++}$ denotes
the uniform upper bound on traders' endowment vectors.

Given any effective endowment set $J \subseteq K$, define:

- $\bar{e}^J := (\bar{e})_{g \in J} \in \mathbb{R}^J_+$ as the subvector of $\bar{e}$ that results from considering only goods $g \in J$;
- $(\bar{e}^J, 0^{K \setminus J}) \in \mathbb{R}^K_+$ as the vector that results after replacing by zero all the components $(\bar{e}_g)_{g \in K \setminus J}$ corresponding to the other goods.

Similarly, for each price vector $p \in P_K$,
define the vector p^J of associated components.

Regular Net Demand Correspondences

The net demand correspondence $P^0_K \ni p \mapsto\!\!\!\to Z(p; \mathfrak{Z}) \subset B^K(p)$ is *J*-regular just in case it satisfies four conditions.

Of these, the first three conditions are:

- budget exhaustion: $p\, z = 0$ for all $p \in P^0_K$ and all $z \in Z(p; \mathfrak{Z})$;
- continuity: the correspondence $\mathfrak{Z}$ has a graph

$$\Gamma_{\mathfrak{Z}} := \{(p, z) \in P^0_K \times \mathbb{R}^K \mid z \in Z(p; \mathfrak{Z})\}$$

  which is a relatively closed subset of $P^0_K \times \mathbb{R}^K$;
- feasibility: $z \geqq -(\bar{e}^J, 0^{K \setminus J})$ for all $p \in P^0_K$ and $z \in Z(p; \mathfrak{Z})$;

The feasibility condition prohibits the trader
from offering to supply any commodity that has not been included in the effective endowment set J.

The Fourth Boundary Condition

Suppose that $(p^{\mathbb{N}}, z^{\mathbb{N}}) = (p^n, z^n)_{n \in \mathbb{N}} \in P_K^0 \times \mathbb{R}^K$
is any infinite sequence of points in the graph $\Gamma_{\mathfrak{Z}}$ of $\mathfrak{Z}$
with the property that the price sequence $p^{\mathbb{N}}$ converges
to a point $\bar{p}$ on the boundary of P_K satisfying $\bar{p}^J \neq 0^J$
— i.e., $\bar{p}_j > 0$ for at least one $j \in J$.

Then the sum $\sum_{g \in K} z_g^n \to +\infty$.

The boundary condition requires that,
as one or more prices p_g $(g \in K)$ tend to zero,
so total net demand over all commodities $g \in K$ tends to infinity
whenever the "cheaper point" property holds
— i.e., whenever the effective announced endowment set J
contains at least one commodity g with $\bar{p}_g > 0$ in the limit,
thus ensuring that the trader has positive wealth there.

Regularity imposes no restriction of rationality;
all the revealed preference axioms may be violated.

A Statistical Demand Revelation Mechanism

The auctioneer/principal chooses a statistical mechanism which, for each possible demand distribution $\zeta \in \mathcal{M}_\lambda(L; \mathcal{Z}^K)$, specifies both:

1. a price vector $p(\zeta) \in P$;
2. a measurable deterministic allocation function

$$L \times \mathcal{Z}^K \ni (\ell, \mathfrak{Z}) \mapsto z^\ell(\mathfrak{Z}, \zeta) \in \mathbb{R}^K$$

specifying the net trade vector of any agent labelled $\ell \in L$, whose announced net demand correspondence is $\mathfrak{Z}$.

Moreover, for each possible demand distribution $\zeta \in \mathcal{M}_\lambda(L; \mathcal{Z}^K)$, the function $(\ell, \mathfrak{Z}) \mapsto z^\ell(\mathfrak{Z}, \zeta)$ satisfies:

- demand selection: $z^\ell(\mathfrak{Z}, \zeta) \in Z(p(\zeta))$ throughout $L \times \mathcal{Z}^K$;
- market clearing: $0 = \int_{L \times \mathcal{Z}^K} z^\ell(\mathfrak{Z}, \zeta)\, \zeta(d\ell \times d\mathfrak{Z})$.

An Extensive Form Game in 24 Steps

We will define a 24-step extensive form game whose players are:

- the countably infinite set of traders $k \in \mathbb{N}$
  whose respective label–type pairs $(\ell^k, t^k) \in L \times T$
  are independently drawn
  from the probability measure $\tau \in \mathcal{M}_\lambda(L; T)$;
- player 0, who manages the market
  by combining all three roles
  of auctioneer, principal, and warehouse manager.

Labels and Reported Endowment Sets: Steps 1–3

Step 1 Player 0 announces a finite set G
of all possible goods that are candidates for trade.

Step 2 Each trader $k \in \mathbb{N}$ who participates in the market:

- is assigned a random label ℓ^k
  from the probability measure λ on L;
- then reports to player 0
  an endowment set $H^k \subseteq G$.

Player 0 records the countably infinite list
of all reported pairs $(\ell^k, H^k) \in L \times 2^G$.

Step 3 Player 0 estimates
the joint endowment distribution $\gamma \in \mathcal{M}_\lambda(L; 2^G)$
of all the pairs $(\ell, H) \in L \times 2^G$
that were recorded at step 2.

Determining the Trading Space: Step 4

Step 4 Given the distribution γ estimated at step 3, player 0 publicly announces the trading set K of goods $g \in G$ in the reported endowment set H of a positive measure of agents — i.e.,

$$K := G(\gamma) := \{g \in G \mid \gamma(L \times \{H \in 2^G \mid g \in H\}) > 0\}$$

First-Stage Demand Revelation: Step 5

Step 5 Each trader with recorded pair $(\ell, H) \in L \times 2^G$ faces the relevant trading set $J := H \cap K$.

For the first-stage demand revelation mechanism, player 0 requires this trader to report a J-regular correspondence $\mathfrak{Z} \in \mathcal{Z}_J^K$.

Any trader with recorded pair (ℓ, H) who refuses to report a J-regular correspondence is penalized by being recorded as having the autarkic net demand correspondence $\mathfrak{Z}^{\mathbb{A}}$ that satisfies $Z(p, \mathfrak{Z}^{\mathbb{A}}) = \{0^K\}$ for all $p \in P_K^0$.

First Stage of Demand Revelation: Steps 6–7

Step 6 Player 0 updates each trader's pair $(\ell, H) \in L \times 2^G$ that was recorded at step 2 by:

- first replacing H with $J := H \cap K$;
- then appending the same trader's net demand correspondence $\mathfrak{z} \in \mathcal{Z}_J^K$.

The result is a new record taking the form of a triple $(\ell, J, \mathfrak{z}) \in L \times 2^K \times \mathcal{Z}^K$.

Step 7 Player 0 estimates the joint distribution $\zeta \in \mathcal{M}_\gamma^*$ of all the triples $(\ell, J, \mathfrak{z}) \in L \times 2^K \times \mathcal{Z}^K$ that were recorded at step 6.

By definition of $\mathcal{M}_\gamma^*$, this distribution must have a marginal of γ on $L \times 2^K$, as well as satisfying

$$\zeta(\{(\ell, J, \mathfrak{z}) \in L \times 2^K \times \mathcal{Z}^K \mid \mathfrak{z} \in \mathcal{Z}_J^K\}) = 1$$

First Equilibrium Prices: Step 8

Step 8 Player 0 determines,
as a function $\mathcal{M}^*_\gamma \ni \zeta \mapsto \bar{p}(\zeta) \in P^0_K$
of the demand distribution ζ
that was estimated at step 7,
a first-stage price vector $\bar{p}(\zeta) \in P^0_K$
which clears markets for goods $g \in K$
in the sense that

$$0^K \in \int_{L \times 2^K \times \mathcal{Z}^K} Z(\bar{p}(\zeta); \mathfrak{z})\, \zeta(d\ell \times dJ \times d\mathfrak{z})$$

First Equilibrium Allocation: Step 9

Step 9 Player 0 specifies, as a function
of each trader's triple $(\ell, J, \mathfrak{z}) \in L \times 2^K \times \mathcal{Z}^K$
that was recorded at step 6,
as well as of the demand distribution ζ
that was estimated at step 7,
a market-clearing allocation rule

$$L \times 2^K \times \mathcal{Z}^K \times \mathcal{M}^*_\gamma \ni (\ell, J, \mathfrak{z}, \zeta) \mapsto \bar{z}^\ell(J, \mathfrak{z}; \zeta) \in \mathbb{R}^G_+$$

Its values are net trade vectors that,
for each fixed $\zeta \in \mathcal{M}^*_\gamma$,
are described by the measurable selection

$$L \times 2^K \times \mathcal{Z}^K \ni (\ell, J, \mathfrak{z}) \mapsto \bar{z}^\ell(J, \mathfrak{z}; \zeta) \in Z(\bar{p}(\zeta); \mathfrak{z}) \subset \mathbb{R}^K$$

which also satisfies the market clearing condition

$$0^K = \int_{L \times 2^K \times \mathcal{Z}^K} \bar{z}^\ell(J, \mathfrak{z}; \zeta)\, \zeta(d\ell \times dJ \times d\mathfrak{z})$$

Mandated Warehouse Deposits: Step 10

Step 10 Player 0 uses the allocation rule found at step 9 in order to calculate each trader's mandated warehouse supply vector

$$\bar{y}^{\ell}(J, \mathfrak{z}; \zeta) := -[\bar{z}^{\ell}(J, \mathfrak{z}; \zeta) \wedge 0] = [-\bar{z}^{\ell}(J, \mathfrak{z}; \zeta)] \vee 0 \in \mathbb{R}^G_+$$

as a function of:

- each trader's triple $(\ell, J, \mathfrak{z}) \in L \times 2^K \times \mathcal{Z}^K$ that was recorded at step 6;
- the first-stage demand distribution ζ that was estimated at step 7.

Chosen Warehouse Deposits: Steps 11–12

Step 11 Player 0 opens up the warehouse
to accept traders' deposits.

Each trader described by the recorded triple $(\ell, J, \mathfrak{Z})$
is informed, as they arrive at the warehouse,
that their mandated supply vector is $\bar{y}^\ell(J, \mathfrak{Z}; \zeta)$.

Step 12 Each trader with endowment vector e
offers to the warehouse
any supply vector y satisfying $0 \leqq y \leqq e$.

The Warehouse Deposit Distribution: Steps 13–14

Step 13 Player 0, acting as warehouse manager, when confronted by a trader who is described by the recorded triple $(\ell, J, \mathfrak{Z})$:

- accepts any supply vector $y \geqq 0$ satisfying $y \leqq \bar{y}^\ell(J, \mathfrak{Z}; \zeta)$;
- turns away the excess supply $[y - \bar{y}^\ell(J, \mathfrak{Z}; \zeta)] \vee 0$ that is offered — if any.

A record is made of the resultant list of all traders' quadruples

$$(\ell, J, \mathfrak{Z}, y) \in L \times 2^K \times \mathcal{Z}^K \times \mathbb{R}^G_+$$

Step 14 Player 0 uses the list of all the quadruples $(\ell, J, \mathfrak{Z}, y)$ that were recorded at step 13 to estimate the supply distribution

$$\eta \in \mathcal{M}_\zeta(L \times 2^K \times \mathcal{Z}^K; \mathbb{R}^G_+)$$

Testing for Compliance: Step 15

Step 15 Player 0 constructs the set

$$C(\zeta) := \{(\ell, J, \mathfrak{Z}, y) \in L \times 2^K \times \mathcal{Z}^K \times \mathbb{R}^G_+ \mid y = \bar{y}^\ell(J, \mathfrak{Z}; \zeta)\}$$

of all compliant quadruples $(\ell, J, \mathfrak{Z}, y)$,
and estimates its measure $\eta(C(\zeta))$.

There are now two cases:

compliance when $\eta(C(\zeta)) = 1$;

non-compliance when $\eta(C(\zeta)) < 1$.

The Compliant Case: Step 16

Step 16 Consider first the compliant case when $\eta(C(\zeta)) = 1$.

Here, player 0 allows each trader with quadruple $(\ell, J, \mathfrak{Z}, y)$ recorded at step 13 to withdraw any demand vector $x \geqq 0$ satisfying:

1. in case $(\ell, J, \mathfrak{Z}, y)$ is a compliant quadruple in $C(\zeta)$, the quota restriction

$$x \leqq \bar{x}^\ell(J, \mathfrak{Z}; \zeta) := [\bar{z}^\ell(J, \mathfrak{Z}; \zeta) \vee 0] \in \mathbb{R}^K_+$$

   where $\bar{z}^\ell(J, \mathfrak{Z}; \zeta)$ is the net demand vector that was specified at step 9;
2. in case $(\ell, J, \mathfrak{Z}, y)$ is non-compliant, the budget constraint $\bar{p}(\zeta)\, x \leq \bar{p}(\zeta)\, y$, where $\bar{p}(\zeta)$ is the market-clearing price in P^0_K that was chosen at step 8.

In the compliant case when $\eta(C(\zeta)) = 1$, player 0 follows this step by jumping directly to the concluding step 24 of the game.

The Non-Compliant Case: Beginning Step 17

Step 17 Alternatively, in case $\eta(C(\zeta)) < 1$,
player 0 constructs the set

$$G^+(\eta) := \{g \in G \mid \mathbb{E}_\eta[y_g] > 0\}$$

of goods $g \in K$ in positive mean supply per trader,
thus allowing good g to be traded
in the second-stage market.

If the number of goods to be traded
satisfies $\#G^+(\eta) < 2$,
then player 0 returns to the depositors
all deposits of every good,
and declares universal autarky.

The Non-Compliant Case: Concluding Step 17

Step 17, continued Otherwise, if $\#G^+(\eta) \geq 2$, then player 0:

- announces the set $G^+(\eta)$;
- returns any deposits of goods $g \in G \setminus G^+(\eta)$ to the traders who made those deposits;
- starting at step 18, institutes a second-stage demand revelation mechanism somewhat like that used in the first stage.

Second Stage of Demand Revelation: Preliminaries

Consider the commodity space $\mathbb{R}^{G^+(\eta)}$, as well as the new version

$$Q := \left\{ q \in \mathbb{R}_+^{G^+(\eta)} \mid \sum_{g \in G} q_g = 1 \right\}$$

of the unit simplex $P \subset \mathbb{R}^G$
of semi-positive normalized price vectors,
together with the new version

$$Q^0 := \left\{ q \in \mathbb{R}_{++}^{G^+(\eta)} \mid \sum_{g \in G} q_g = 1 \right\}$$

of the relative interior $P^0 \subset P$.

For each supply vector $y = \langle y_g \rangle_{g \in G} \in \mathbb{R}_+^G$
we introduce the notation

$$y^+ := \langle y_g \rangle_{g \in G^+(\eta)} \in \mathbb{R}_+^{G^+(\eta)}$$

for the subvector whose components correspond
to goods in positive mean supply.

Second Stage of Demand Revelation: Step 18

Step 18 Each trader, given the already recorded deposit vector $y^+ \in \mathbb{R}_+^{G^+(\eta)}$, is required to report to player 0 a $J(y^+)$-regular second-stage net demand correspondence $Q^0 \ni q \mapsto\!\!\!\rightarrow Z^+(q) \in \mathbb{R}^{G^+(\eta)}$, also denoted by $\mathfrak{Z}^+ \in \mathcal{Z}^+$.

Any trader with recorded quadruple $(\ell, J, \mathfrak{Z}, y^+)$ who fails to report a $J(y^+)$-regular second-stage net demand correspondence $\mathfrak{Z}^+$ is recorded as having the autarkic net demand correspondence $(\mathfrak{Z}^+)^{\mathbb{A}}$ that satisfies

$$Z^+(q, (\mathfrak{Z}^+)^{\mathbb{A}}) = \{0^{G^+(\eta)}\} \text{ for all } q \in Q^0$$

The Non-Compliant Case: Steps 19–20

Step 19 Player 0 updates each trader's quadruple $(\ell, J, \mathfrak{z}, y) \in L \times 2^K \times \mathcal{Z}^K \times \mathbb{R}^G_+$ that was recorded at step 13 by appending the same trader's second-stage net demand correspondence $\mathfrak{z}^+ \in \mathcal{Z}^+$ that was recorded at step 18.
The resulting new record for the trader is the quintuple

$$(\ell, J, \mathfrak{z}, y, \mathfrak{z}^+) \in L \times 2^K \times \mathcal{Z}^K \times \mathbb{R}^G \times \mathcal{Z}^+$$

Step 20 Given all the quintuples

$$(\ell, J, \mathfrak{z}, y, \mathfrak{z}^+) \in L \times 2^K \times \mathcal{Z}^K \times \mathbb{R}^G_+ \times \mathcal{Z}^+$$

that were recorded at step 19,
player 0 estimates the demand distribution

$$\zeta^+ \in \mathcal{M}_\eta(L \times 2^K \times \mathcal{Z}^K \times \mathbb{R}^G_+; \mathcal{Z}^+)$$

Second-Stage Equilibrium Prices: Step 21

Step 21 Player 0 determines, as a function

$$\mathcal{M}_\eta(L \times 2^K \times \mathcal{Z}^K \times \mathbb{R}^G_+; \mathcal{Z}^+) \ni \zeta^+ \mapsto \tilde{q}(\zeta^+) \in Q^0$$

of the distribution ζ^+ that was estimated at step 20,
a second-stage price vector $\tilde{q}(\zeta^+)$
that satisfies the market clearing condition

$$0 \in \int_{L \times \mathcal{Z} \times \mathbb{R}^G_+ \times \mathcal{Z}^+} Z^+(\bar{q}(\zeta^+); \mathfrak{Z}^+)\, \zeta^+(d\ell \times dZ \times dy \times d\mathfrak{Z}^+)$$

Second-Stage Equilibrium Allocation: Step 22

Step 22 Player 0 specifies a market-clearing allocation rule

$$(\ell, J, \mathfrak{z}, y, \mathfrak{z}^+, \zeta^+) \mapsto \tilde{z}^\ell(\mathfrak{z}, \zeta, y, \mathfrak{z}^+; \zeta^+) \in \mathbb{R}^{G^+(\eta)}$$

as a function of:

- each trader's quintuple $(\ell, J, \mathfrak{z}, y, \mathfrak{z}^+)$ that was recorded at step 19;
- the demand distribution

$$\zeta^+ \in \mathcal{M}_\eta(L \times 2^K \times \mathcal{Z}^K \times \mathbb{R}^G_+; \mathcal{Z}^+)$$

of relevant observable trader histories that was estimated at step 20.

More on Step 22

Each value $\tilde{z}^\ell(\mathfrak{Z}, \zeta, y, \mathfrak{Z}^+; \zeta^+)$ of this function
is a net demand vector that,
for each fixed distribution ζ^+, satisfies both:

- the measurable selection condition

$$\tilde{z}^\ell(J, \mathfrak{Z}, y, \mathfrak{Z}^+; \zeta^+) \in Z^+(q(\zeta^+); \mathfrak{Z}^+) \subseteq \mathbb{R}^{G^+(\eta)}$$

  for ζ^+-a.e. $(\ell, J, \mathfrak{Z}, y, \mathfrak{Z}^+) \in L \times 2^K \times \mathcal{Z} \times \mathbb{R}^G_+ \times \mathcal{Z}^+$;

- the market clearing condition

$$0 = \int_{L \times 2^K \times \mathcal{Z} \times \mathbb{R}^G_+ \times \mathcal{Z}^+} \tilde{z}^\ell(J, \mathfrak{Z}, y, \mathfrak{Z}^+; \zeta^+) \, \zeta^+(d\ell \times dJ \times dZ \times dy \times d\mathfrak{Z}^+)$$

Wrapping Up: Steps 23–24

Step 23 Consider any trader with the quintuple $(\ell, J, \mathfrak{Z}, y, \mathfrak{Z}^+)$
that was recorded at step 19,
and so with the second-stage
net trade vector $\tilde{z}^\ell(\mathfrak{Z}, \zeta, y, \mathfrak{Z}^+; \zeta^+) \in \mathbb{R}_+^{G^+(\eta)}$
that was specified in step 22.

This trader is allowed
any warehouse withdrawal $x^+ \in \mathbb{R}_+^{G^+(\eta)}$ satisfying

$$x^+ \leqq x^\ell(J, \mathfrak{Z}, y, \mathfrak{Z}^+; \zeta^+) := \tilde{z}^\ell(J, \mathfrak{Z}, y, \mathfrak{Z}^+; \zeta^+) + y^+$$

where y^+ is the unique vector in $\mathbb{R}_+^{G^+(\eta)}$
satisfying $y_g^+ = y_g$ for all $g \in G^+(\eta)$.

Step 24 When all traders have made their withdrawals
within their specified limit,
player 0 disposes of any remaining surplus.

The Principal's Commitments

Suppose that player 0, who combines the roles
of auctioneer, principal and warehouse manager,
is committed to the specified sequence of actions
at steps 1, 3–4, 6–11, 13–17, 19–22, and 24 of the 24-step
extensive form game.

Each Agent's Five-Part Strategy

Given these commitments by player 0,
each agent $i \in \mathbb{N}$, with label $\ell \in L$ and type $t = (\theta, e)$,
has a five-stage strategy:

1. at step 2, announce an endowment set $H \subseteq G$;
2. at step 5, after player 0 has chosen a commodity set $K \subseteq G$, announce a first-stage J-regular net demand correspondence $\mathfrak{Z} \in \mathcal{Z}_J^K$, where $J = H \cap K$;
3. at step 12, after player 0 has chosen first-period prices $\bar{p}(\zeta)$ and suggested a net trade vector $\bar{z}^\ell(\mathfrak{Z}, \zeta)$, choose a warehouse deposit vector $y \in \mathbb{R}_+^G$ with $y \leqq e$;
4. in case the second-stage market has to open, at step 18, announce a second-stage regular demand correspondence $\mathfrak{Z}^+$;
5. at step 23, given player 0's chosen second-stage prices $\tilde{q}(\zeta^+)$, choose a withdrawal vector $x \in \mathbb{R}_+^G$ with $x \leqq \tilde{x}^\ell(\mathfrak{Z}, y, \mathfrak{Z}^+; \zeta^+)$.

Perfect Bayesian Strategyproofness

For each trader described by the triple $(\ell, \theta, e) \in L \times T$:

- At step 23, a strictly dominant strategy for the agent is to withdraw the allowed amount $\tilde{x}^\ell(\mathfrak{Z}, y, \mathfrak{Z}^+; \zeta^+)$ in full.
- At step 18, a weakly dominant strategy is to announce the true Walrasian 2nd period demand correspondence $\mathfrak{Z}^+$, followed by the above full withdrawal strategy at step 23;
- At step 12, the unique dominant strategy is to deposit the mandated supply $\bar{y}^\ell(\mathfrak{Z}, \zeta) = [-\bar{z}^\ell(\mathfrak{Z}, \zeta)] \vee 0$, followed by the above strategy starting at step 18;
- At stage 5, a weakly dominant strategy is to announce the true Walrasian first period demand correspondence $\mathfrak{Z}$, followed by the above strategy starting at step 12.
- At stage 1, a weakly dominant strategy is to announce the true endowment set $H = \{g \in G \mid e_g > 0\}$, followed by the above strategy starting at step 5.

Summary of Main Results

The analysis has been limited to a static spot market system,
with many competing agents facing one omnipotent principal.

Demand revelation, in principle,
allows Walrasian equilibrium to be reached,
but without making any concessions to computational limitations.

When agents cannot trust each other to fulfil supply contracts,
one also needs a warehouse system,
along with a procedure for deterring defaulters.

Specifying the mechanism in full,
allowing for every possible deviation,
is surprisingly complicated.

Final Thoughts

Somebody had to try this!

Many thanks for the invitation,
and for letting me share the attempt!!

Many thanks to the Economics Department at Essex,
and to the University as whole,
for supporting me so well as I went through important stages
of both my academic career and life more generally.

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