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The Capital Asset Pricing Model: A
Review of Theory and Empirical Evidence

1. Introduction

Almost fifty years after its development by Sharpe(1964) and Lintner (1965) the Capital Asset Pricing Model (CAPM) is still widely used. The attractiveness of the model is based on its simplicity in linking risk with expected returns. Despite its popularity the model has never really been an empirical success. The purpose of this paper is to examine whether the empirical failure of the early version of the CAPM is due to its theoretical shortcomings or to its empirical implementation. The paper is organised as follows: Section 2 reviews the CAPM. Section 3 examines the empirical record of the model and section 4 we assess some of the theoretical extensions proposed in order to improve the empirical applicability of the model. The paper ends with a conclusion.

2. The CAPM

Sharpe (1964) and Lintner (1965) extend the Markowittz (1952) mean-variance model to capture the whole market[1]. For the implementation of the extension a number of assumptions are necessary: a) investor behaviour follows a mean-variance objective and all investors have identical beliefs regarding the joint distribution of asset returns; b) there is one risk-free interest rate available to all investors both for borrowing and lending and it is independent from the volume of transactions; c) there are no restrictions on borrowing (short-selling) of any assets and there are not any taxes; d)there is perfect liquidity and divisibility of assets;

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[1] Markowitz (1952) relied on the notion that risks across assets are correlated proceed to work the basic principles of portfolio construction. Under the assumption that returns follow a normal distribution the risk-averse investor cares only about the mean and the variance of their investment returns. Investors choose from "mean-variance efficient" portfolios those which either maximize their expected returns at t given the level of risk or otherwise minimize the level of risk given their expected return.

e) there is fixed quantity of assets and that all investors are price takers[2]. By using these assumptions Sharpe and Lintner built on the mean variance analysis and developed a general equilibrium theory in the capital markets known as the Capital Asset Pricing Model (CAPM). The main message of the model is that the return of an asset (or a portfolio) has as linear relationship to its riskiness. To understand the basic intuition of the model we turn our attention to the following diagram:
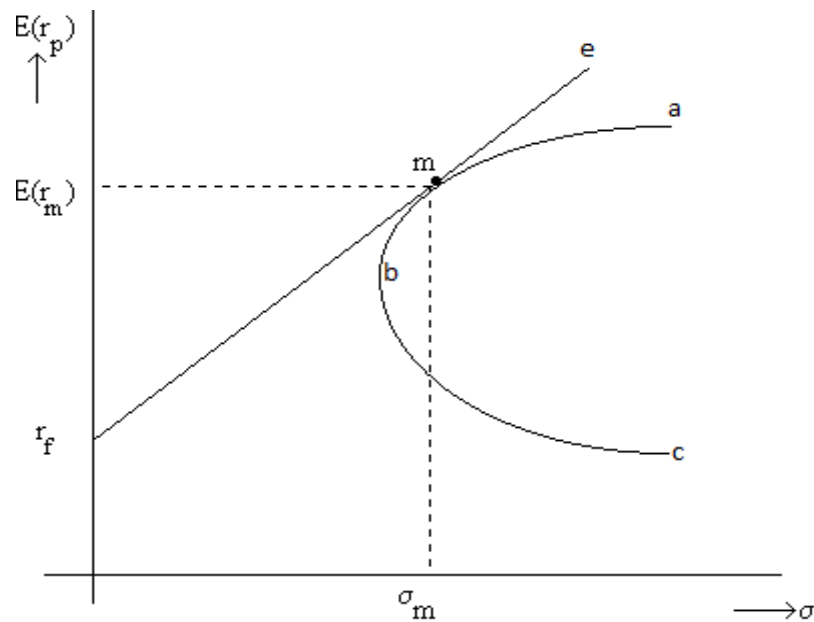

Fig. A: The static Sharpe-Lintner CAPM

From the assumption of risk-free lending and borrowing we deduce that an investor is restricted in choice between two classes of the assets the risky, and the risk–free. The homogeneous assumption implies that a selection of a risky portfolio by an investor would be the same for all investors and therefore it is the market portfolio. All investors will hold combinations of the risky portfolio and the risk-free asset. The frontier abc depicts the combinations of the risky assets with the maximum expected return at different levels of variance in the absence of any risk-free asset. The set of efficient combinations of the risky assets however are depicted only along the ab part of the frontier since only here the expected return maximised given the level of variance. Therefore point b is the set of risky assets with the lower level of variance and thus the lower risk. With the presence of a risk-free asset the efficient set is the tangent line $r_f e$ usually referred to as the capital market line. The point m is the tangent point between the risky assets frontier and the risk free line and is

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[2] The CAPM model includes additional assumptions, among others being that all assets are marketable.

the point where the proportion of risk-free asset is zero. With all investors having identical beliefs the efficient portfolio consisting of only risky assets is the same for every investor (point m in Fig. A). Yet in the selected portfolio the fraction of risk-free assets differs among investors according to each investor's attitude to risk.

The model can also be expressed in mathematical terms. The more rigorous derivation of the model will be useful later on when we relax different assumptions of the models.

The basic equation of the portfolio equilibrium is as follow:

$$E(R_i) = R_f + [\ E(R_M) - R_f]\ \beta_{iM} \qquad i = 1,\ldots,n. \qquad\qquad (1)$$

where:

$E(R_i)$ = is the expected return on asset i $\qquad$ i=1,2....n

R_f = the risk free rate

$\beta_{iM} = \sigma_{iM} / \sigma^2_M$ $\qquad$ M denotes the efficient market portfolio comprising only risky assets[3]

This says that for an individual asset the risk premium required is equal to its beta times the market risk premium.

3. Empirical testing of CAPM

The CAPM model has been subject to numerous empirical studies. In practice, equation 1 is modified, with the addition of a constant, so as to be in a form amenable to empirical testing:

$$E(R_i - R_f) = a_0 + a_1\ \beta_{iM} \qquad\qquad (2)$$

where $\quad a_1 = [\ E(R_M) - R_f]$

For the CAPM hypothesis to hold the constant term a_0 should not be significantly different to zero otherwise it would imply that a significant variable has been left out. In addition, since

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[3] To get the result that market β of an asset i is the ratio of the covariance to the variance of the market return, we should minimize the error of equation 1. An error is defined as the difference between actual and predicted returns.

expected returns should only be explained by β and this relationship must be linear. Finally, a_1 should be equal to $[E(R_M) - R_f]$.

Most of the empirical tests on the CAPM use either cross-section or time-series regressions[4]. The simpler cross-section tests were based on the intercept and slope of the regressed standard Sharpe-Lintner model. As mentioned above, theoretically the model implies that the intercept is zero and the slope is the difference between the expected return of the market portfolio minus the risk free-rate $E(R_M) - R_f$. However, the relevant empirical tests have revealed two problems. First the estimations of individual betas and alphas, though statistically significant, seem to violate the CAPM. Second, in the resulting regressions the residuals are positively correlated. This positive correlation of the residuals points to a missing variables bias and therefore both estimators are biased[5].

To confront the problem of imprecise individual beta estimations Black, Jensen and Scholes (1972) and Blume (1970) use portfolios rather than individual assets in their tests. The reasoning for this approach is that estimations in diversified portfolios are more accurate than for individual assets. In this way they manage to reduce measurement errors in variables. However, amassing securities into groups reduces the available volume of betas and the power of statistical estimations. Therefore in order to avoid this problem they rank the stocks into deciles according to the size of their betas from the lowest to the highest. So the first portfolio comprises assets with low betas and assets with high betas. Their findings seem to conform to the previous findings that when b is less (greater) than one then alpha should be positive (negative)

Jensen (1968) applied the first in-depth time-series analysis. He utilized the extended (with α_i) CAPM econometric equation 2

where α_i named as " Jensen's alpha" is the intercept of the equation.

Jensen advocates that if the CAPM risk premium $[\beta_{iM}(R_{Mt} - R_{ft})]$ fully explains the expected value of a security's excess return $(R_{it} - R_{ft})$ then the intercept α must be equal to zero for every security. His econometric test failed to verify the validity of the CAPM. He found a

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[4] The CAPM model measurements are expressed in terms of future values. The explanation arises from the fact that the CAPM model is based on expectations. However due to lack of systematic observations on expectations nearly all the empirical studies have used ex-post or observed values for the variables.

[5] Miller and Scholes (1972) test for other sort of misspecification such as nonlinearities and the presence of heteroscedasticity

relation (positive) between expected returns and beta but was too "horizontal". These findings were confirmed by other studies like Friend and Blume (1970) Black Jensen and Scholes (1972) and Stambaugh (1982).

Fama and McBeth (1973) proposed a new methodology to examine the CAPM. Their method is one of the most influential on the CAPM empirical examination. They categorise securities in 20 portfolios to examine Betas for a time-series regression. They then perform a month-by-month cross-section regressions for the monthly returns on betas.

The form of the equation is:

$$\tilde{R}_{it} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t}\,\beta_i - \hat{\gamma}_{2t}\,\beta_i^2 + \hat{\gamma}_{3t}S_{ei} + \eta_{it} \qquad (3)$$

where S denotes the error term obtained from the time series regression

This type of CAPM allows three hypotheses to be tested. The first hypothesis is whether the expected values of the second and the third γ coefficient individually are equal to zero and if the expected value of the first γ is greater than zero. In the case that γ_3 does not equal to zero then residual risk affects return. For parameter γ_2 if the hypothesis does hold then no nonlinearities exist in the security market line. Finally, if $E(\gamma_{1t})$ is greater than zero then there is a positive price or risk in the capital markets.

Fama and McBeth's tests showed that γ_3 is not statistically different from zero so residual risk does not affect expected return on an asset. For the γ_2 coefficient the results do not differ from zero so they concluded that market beta does not affect average returns. Furthermore the γ_{1t} hypothesis tests have showed that there is positive and linear relationship between market beta and average return. The authors also found that $\hat{\gamma}_{0t}$ is noticeably higher than the risk-free rate and that γ_{1t} is less than the difference between average return on market portfolio and the risk free rate.

4. Theoretical Considerations

Although the Fama and McBeth (1973) paper adequately explains expected returns and the positivity of risk premium of beta it fails to address the Sharpe-Lintner CAPM argument that beta premium is the expected market return minus the risk-free rate. The empirical failure of the original version of the CAPM model led many academics to seek modifications that could

improve the empirical performance of the model. In particular, it was believed that the failure of the CAPM model might be due its strong assumptions that seem to contradict reality. Several authors look into how the relaxations of the assumptions affect the theoretical and empirical validity of the model. In this paper we focus on of just two of the assumptions, namely the riskless borrowing/lending and the mean-variance objective in a single period.

4.1 Black CAPM

Black (1972) extends the principal model by removing risk-free borrowing and lending. Instead, Black's model introduces the assumption of unrestricted short-selling (borrowing) of risky assets. In the absence of the opportunity of a risk free rate the CAPM can be treated only with the risky assets as it is depicted in the figure B by the frontier abc. The efficient set is the upward part of the frontier given by ab since with the same level of variance you can have a higher expected return. The market portfolio then in Black's CAPM is the weighed of all the efficient portfolios chosen by investors when market prices clear. Consequently with no restrictions on the short-selling of risky assets the efficient market portfolio lies at any point on the ab line.
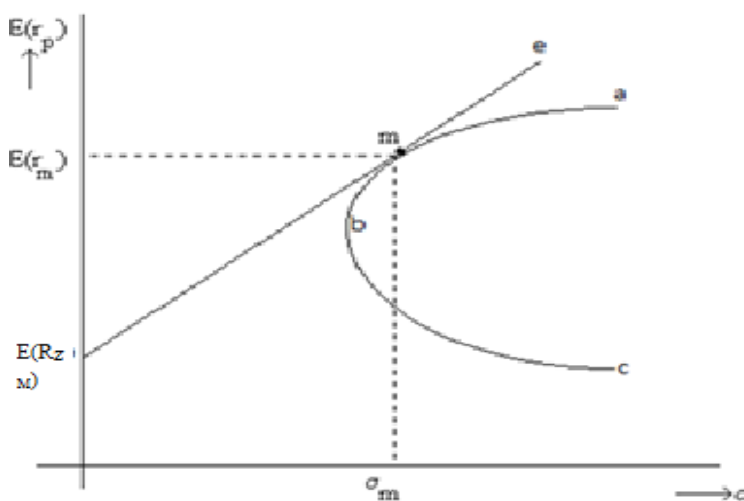

Fig. B The Black CAPM

The equation for the Black's CAPM is similar as the original CAPM but now R_f is replaced with $E(R_{ZM})$ where $E(R_{ZM})$ represents the expected rate of return on the zero-beta portfolio. Thus, the equation in Black's model becomes:

$$E(R_i)= E(R_{ZM}) + [E(R_M)- E(R_{ZM})]\beta_{iM} \quad \text{for } i=1,2\ldots,\ldots,n \qquad (4)$$

Black's CAPM model overcomes the unrealistic assumption of unrestricted risk-free lending by accepting the assumption of no limits in short-selling of risky assets. In the case, however, that both the former and the latter assumptions are removed then the portfolios are not efficient and there is no link between expected return and beta. Therefore, the Black CAPM assumption of unlimited short-selling of risky assets is equally problematic and unrealistic as the assumption the risk-free lending and borrowing.

Early empirical tests point indicated that the Black CAPM was a successful model that fully explained expected returns. Black's CAPM had also been confirmed by Fama & McBeth (1973). However, later empirical studies, such as Basus (1977), Banz (1981) and Bhandari (1988), questioned the success of Black's model. Specifically, Basus (1977) and Banz (1981) demonstrated that in some cases expected returns on stocks are higher than those predicted by Black. Similarly, Bhandari (1988) showed that higher returns could also be found in debt-equity ratios. Therefore, we conclude that although Black CAPM eliminates some theoretical and empirical weaknesses of the original CAPM it still remains a model with many flaws.

4.2 ICAMP

The empirical problems associated with the previous described models led to the seeking of more complicated models. This meant dropping some unrealistic assumption of the classic CAPM model such as that investors are only interested about the mean and variance of one-period portfolio returns. It is reasonable to claim that apart from expected return and risk of portfolios investors care about how asset returns covaries with other economic variables, such as wages and future investment plans. Seen from this perspective the CAPM beta seems to be lack information and does not signify the actual level of risk. Furthermore, it is uncertain whether market beta captures all the differences in expected returns.

To correct for the aforementioned deficiencies Merton (1973) introduced the ICAPM (Intertemporal Capital Asset Pricing Model). The ICAPM is not a static model that describes how investors seek to maximise their wealth at the end of a given period but a dynamic model that also deals with upcoming investments. So when investors choose a portfolio at t-

1 they also care about how their terminal wealth at t might fluctuate by shifts in other variables like inflation, labour income, and other portfolio investment opportunities. In addition, an investor also takes into account expectations about future prices or returns of the variables mentioned above. As a result, the ICAPM model, in contrast to CAPM, expresses the higher expected payoff given the variances and the covariances of the related variables. Although the empirical support for this model is rather limited, one conclusion to be drawn from this model is that the CAPM is unlikely to hold in a multiperiod setting.

4.3 CCAPM

Another extension of the CAPM model is the CCAPM that was first developed by Lucas(1978) and Breeden (1979). CCAPM is based on the assumption that most people invest with the aim of ensuring future consumption for them and their families. Therefore the CCAPM abandons the assumption that people solely pay attention to their future total wealth. In the consumption model wealth appears implicitly and is not perfectly correlated with market portfolio return. The portfolio returns on assets in the CCAPM is connected with future consumption. The difference between CAPM and CCAPM according to Chen (2003) is that in case of the CAPM it is the asset's covariance with the stock market that is used as a measurement of risk while for the CCAPM it is the covariance of the return with the capita consumption. The consumption model, however, has as a major drawback: the difficulty in measuring consumption. It is extremely hard for individuals to accurately measure their total consumption and hence to estimate the model. Many recent studies have formulated tests for the CCAPM model. Breeden, Gibbons & Litzenberg (1989) is one of the most noteworthy empirical studies of the CCAPM. For their empirical tests the following equation is used

$R_i = R_z + \gamma_1 \beta_i$

where

$\beta_i = Cov(R_i, C_t) / Var(C_t)$

C_t = the rate of growth in per capita consumption at time t

γ_1 = the market price of consumption risk (beta)

The equation for CCAPM is similar to Black's CAPM with the difference that market expected return is replaced by the growth in consumption. Breeden, Gibbons & Litzenberg faced four consumption measurement related problems during their empirical analysis. The first is the pure sampling error in consumption measures. The second problem arises from the fact that data are recorded for expenditures not for consumption. The third problem is that expenditures are recorded as the average for a time interval (a month or quarter) instead of a specific point in time. An additional problem is the frequency of the data, with data prior to 1958 being available only on a quarterly basis.

The measurement difficulties seem to weigh on the empirical results. Breeden, Gibbons & Litzenberg perform a number of tests on the CCAPM model to find that the Consumption CAPM predicts that as expected return increases risk increases. However, it fails to prove at the 5% significance level the linear relationship of expected returns and risk that is implied by the theory of CCAPM. According to the authors the failure to explain empirically the linearity of risk with rewards is due to the poor quality of consumption data. They argue that if the quality of data were better then the linear relationship would be proven. Other authors also provide little support for the CCAPM. Campbell (1996) and Cochrane (1996) suggest that in the case of cross-section regression tests then CAPM provides a better explanation for the asset returns rather than the Consumption CAPM. The main empirical finding of the CCAPM is that asset returns are closely correlated with the business cycle

5. Conclusions

The development of the standard CAPM by Sharpe (1964) and Lintner (1965) generalised for the whole market Markowittz's 1952 model, which assumes that investors choose portfolios according to a mean –variance – efficient analysis. The addition of multiple unrealistic assumptions ensures that market efficient portfolio M is the point of the tangency between the risk-free rate and the set of risky assets. Nevertheless, early and recent empirical analysis proved that many of the concepts of the CAPM model reflected a failure that invalidates its use in applications. For this purpose, several researchers (Jensen 1968, Fama & McBeth 1973 and Black, Jensen and Scholes 1972) applied extended econometric equations of the CAPM in an effort to improve the empirical valitity the Sharpe-Lintner model. Their proposed tests mitigated some of the inconsistencies of the model but were considered inadequate in providing valid empirical explanations consistent with every theoretical implication.

Aside from the weakness in the empirical application of the CAPM's its restricted assumptions had been questioned. Thus, researchers turned their attention to extended CAPM models by relaxing some of the unrealistic assumptions. Chronologically the first noteworthy model was the Black CAPM which removed the assumption of unlimited borrowing and lending. For many years this model was considered as providing a good description of expected returns. However, more recent studies have disputed the empirical success of the Black CAPM.

The empirical failure of the model was then attributed to its misspecification. Additional variables are needed to explain expected returns. To this end a new model, ICAPM, was first proposed by Merton (1973). This is a multiperiod model that assumes that investors are concerned not only about their wealth but also about future investments and consumption. Furthermore, it includes investor expectations about future investment plants. A variant of the ICAPM is the CCAPM which in measuring risk replaces the stock market with capita consumption. Once again these models fail to address all the problems faced by the CAPM.

In a nutshell, the CAPM still provides a sound theoretical base though its empirical validity has been extensively questioned. Competing models manage to provide additional useful insights into capital markets and they could be useful in certain cases. However, none of these models rectifies all the weakness of the CAPM. On the empirical front, comparing the performance of the various models does not seem to deliver a clear winner. (Lawrence, Geppert, Prakash, 2007)

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