“Black-Scholes-Merton Approach To Modelling Financial Derivatives’ Prices”

Tomas Sinkariovas 0802869

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1. Introduction

In this paper I present Black, Scholes (1973) and Merton (1973) (BSM) general equilibrium option pricing model. As any financial model it depends on a number of 'strong' assumptions. The model reveals that the price of a financial option contract can be computed by only knowing a price of an underlying asset, strike price of underlying asset specified in the option contract, risk-free interest rate, maturity of an option (time to the expiration of an option) and volatility of the price of the underlying asset. The model's strength lies in its simplicity as only the volatility of the price of the underlying asset must be estimated and is unknown prior to the calculation of an option price. Other four parameters are observed prior to the computation.

The same option-pricing approach can also be applied to Contingent Claim Analysis (CCA), which employs option-pricing methodology for evaluation of companies' capital structures (liability structures), 'real'[1] options and other financial assets. This paper also explores application of option-pricing analysis in valuing company's liability structure, insurance of deposit and loan agreements and revolving credit agreements. In order to proceed, we first must define what is meant by a financial option contract.

Option is a financial contract, a derivative, whose value non-linearly depends on the value of the underlying asset and gives its holder a right to purchase or sell the underlying asset. Options differ in two ways: by their execution and purpose. There are two types of options, which differ by purpose: call and put options. The owner of the call option has a right to purchase the underlying asset at some specified price and

[1] Where by 'real' options I mean companies' future growth opportunities (options) through capital and other investment, acquisitions, etc.

date and the owner of the put option has a right to sell an asset at some specified price and date. The writer of the option is obliged by a contract to buy or sell the underlying asset at some specified price and date. Furthermore, options differentiate in their time of execution. Two types of options are presented: American and European. American style options can be exercised before the maturity day of an option and European style options are exercised only at the maturity day of the option. Black, Scholes (1973) analysis considers only European options[2]. We now proceed with our analysis.

The rest of the paper is constructed as follows: Section 2 presents the assumptions of the model and its implications, Section 3 constructs a critique of the model, Section 4 describes the application of option pricing methodology to contingent claim analysis and Section 5 concludes the paper.

2. Assumptions of general equilibrium option pricing model

As any financial model, the Black-Scholes option-pricing model is dependent on a number of assumptions. Some of them are 'standard' assumptions employed in financial models such as Sharpe (1964), Lintner (1965) and Mossin (1966) capital asset pricing model (CAPM) and a couple of them are distinctive from (CAPM). The following assumptions must be satisfied for the Black-Scholes model to be valid:

1. Security markets are frictionless – there are no transaction costs, taxes and restrictions on security trading. Moreover, all securities are perfectly divisible (any amount of security can be bought or sold) and short selling allowed.
2. There are no additional payments from the underlying asset (dividends, in case of common stock) during the lifetime of the option.

[2] However, with some modifications Merton (1973) shows that the same analysis can be applied to American options on non-dividend paying common stocks.

3. There are no riskless arbitrage opportunities.
4. Investors can borrow and lend at the same risk-free interest rate, which is constant for the lifetime of the option.
5. Trading in asset markets is continuous through time.
6. Price of the underlying asset has a lognormal distribution and evolves according to Brownian motion process with continuous sample paths. (There are no sudden jumps in underlying asset's price).
7. Investors prefer more to less and agree on the function of underlying asset's variance o^2. This variance is considered to be constant.

If the preceding assumptions are satisfied, when the Black and Scholes formula[3] is:

$$c = SN(d_1) - Xe^{-rT}N(d_2) \quad (1)$$

$$p = Xe^{-rT}N(-d_2) - SN(-d_1) \quad (2)$$

where

$$d_1 = \frac{\ln(S/X) + (r + o^2/2)T}{o\sqrt{T}} \quad (3)$$

$$d_2 = \frac{\ln(S/X) + (r + o^2/2)T}{o\sqrt{T}} = d_1 - o\sqrt{T} \quad (4)$$

where N(X) is a cumulative probability function for a variable with a standard normal distribution N(0,1), where mean =0 and variance =1, and N(X) illustrates the probability of any random variable with a standard normal distribution being less than x. Consequently, c and p are prices of European put and call options, X is the strike price embedded in the option contract , r - the risk-free interest rate, T - time to

[3] Formulas are taken from Hull (Introduction to Futures and Options Markets, 1997, third edition, Prentice-Hall International).

maturity (time to expiration of the option) of an option and o - volatility (standard deviation) of the underlying asset's price.

Two assumptions, which are different from the capital asset pricing model, are concerned with the distributions of underlying asset's price and continuous trading. The only unobservable variable in equations (1) and (2) prior to the computation of an option price is the volatility of the underlying asset's price. In addition, it should be noted that the price of an option does not depend on the expected return of the underlying asset, aggregate supply of assets and risk preferences of investors, as neither of these variables appear in equations (1), (2), (3) and (4). It is also imperative to emphasize that the preceding formulas are derived for European options only as American options can be exercised before the expiration (maturity) date of the option.

The derived option pricing formulas are also robust to relaxation of some of their assumptions. Merton (1973) shows that, if there are no additional payments during the lifetime of the option such as dividends in common stock case, then it is not rational for an investor to exercise an American option before the maturity day, therefore, the Black and Scholes (1973) analysis can also be applied to evaluating American options on non-dividend paying common stocks. Moreover, Merton (1973) modifies the equations (1) and (2) to account for both American and European style options and stochastic interest rate. Finally, Merton (1976) and Cox and Ross (1976) modify the model for discontinuous stock (underlying asset) price movements.

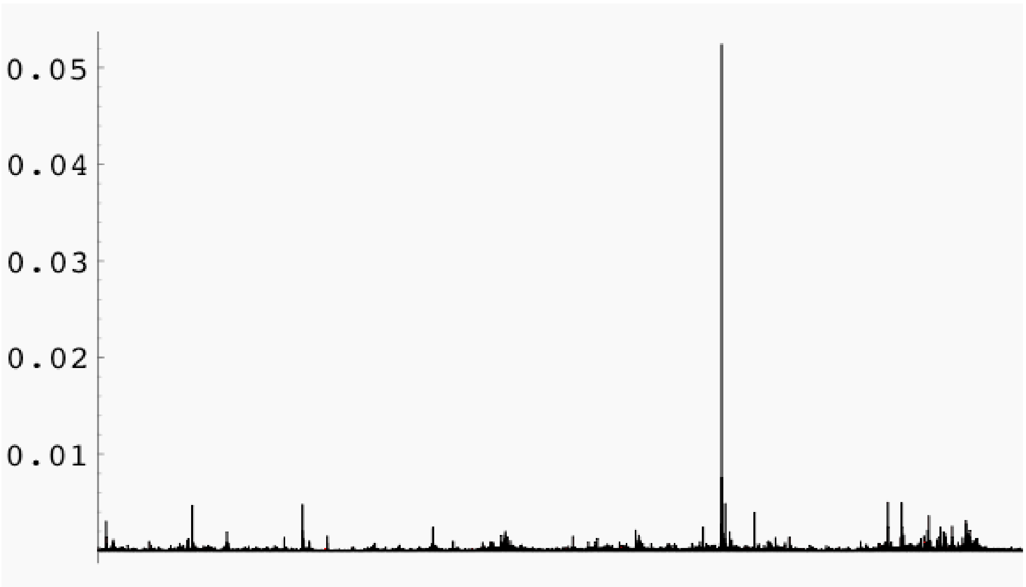
In order to proceed with the critique and application of a general equilibrium option-pricing model, it is imperative to understand the basis of the derivation of formulas

(1) and (2). Lets consider a financial intermediary, which issues (sells) a call option[4] to a client. It is obvious that the issuance of options is a risky activity; therefore, the financial intermediary would ideally want to minimize or entirely eliminate the risks associated with the issued option during the maturity of the contract. In order to do that, the financial intermediary constructs a hedging portfolio with a purpose to exactly replicate the payoffs of an option. The hedging portfolio consists of some risky and risk-free assets. When the payoffs of an option are accurately replicated this portfolio in Merton (1998) is called a 'replicating' portfolio and replication is attained using dynamic hedging strategies (continuous adjustment of weights in the hedging portfolio of risky and risk-free assets). If perfect replication is possible, then it is showed that the hedging portfolio consists only of the underlying asset on which a call option was sold. However, such dynamic hedging strategies are possible only theoretically as Black and Scholes (1973) assumptions do not hold in a real world. If dynamic hedging is not possible, there are risks involved in hedging the payoffs of a call option. Knowing the predictions of the capital asset pricing model, the only risk, which cannot be hedged – market risk. Therefore, if the hedging portfolio is well diversified, which implies elimination of non-systematic risk (company-specific risk in case of common stock) when the return of a hedging portfolio is dependent on systematic risk and the return – market risk-free return. Hence, the payoffs of the call option are hedged and only market risk affects the hedging portfolio return.

3. Critique of Black-Scholes model

[4] The same argument can be constructed, if the intermediary buys a put option from some investor or intermediary and lends securities (takes a short position) to some investor or intermediary. The analysis is taken from Merton (1998).

As mentioned in the previous section the Black-Scholes-Merton equilibrium option-pricing model is a convenient and straightforward way for computing prices of European call and put options. However, the model has its limitations due to unrealistic assumptions. Firstly, Haug and Talleb (2009) argue that violation of the assumptions associated with continuous trading, dynamic hedging and lognormal distribution of underlying assets' price exposes model to risks of tail events, events, which are considered to be highly improbable and have dear consequences. Graph 1[5] shows how risk is managed in dynamically hedged portfolio, according to Black-Scholes formula:



Graph 1 shows the variation of the dynamically hedged portfolio according to Black and Scholes argument.

It is obvious that the variance of a dynamically hedged portfolio is modest, however, occasionally the variance might be very high and losses - substantial. Thus, some modification must be applied to Black and Scholes formulae to account for these tail events.

[5] Graph 1 is obtained from Haug and Talleb (2009).

Moreover, another problem with Black and Scholes analysis is that dynamic hedging is not possible in practice due to discontinuous trading and high transaction costs; thus, the dynamic portfolio hedging is not riskless and actually involves substantial risks, which are not properly adjusted in the model. Moreover, exchange-traded options are usually traded in blocks, so it is not possible to take any position in the option trade, thus, the assumption of asset indivisibility is clearly violated. Consequently, Garleanu, Pedersen and Poteshman (2009) demonstrate that the demand for options affect the price of option contracts. Other studies document that liquidity in option markets is important and might have effect on option prices. Also, it is plausible to assume that option prices are affected by the market structures and exchange rules in which they are traded. For instance, a sudden increase/decrease on the margin account requirements by the exchange operators can have effect on option prices, which are not accounted for in Black and Scholes framework. Furthermore, even if we consider that continuous trading is well approximated by the trading during the 'common' trading day, still, there are time lags in execution of the trading orders, which violate the assumption of continuous trading. The emergence of new 'high-frequency' traders only strengthens the importance of continuous trading assumption. Finally, one of the most important limitations of the model is the assumption of a constant volatility of the underlying asset's price. People involved in trading of options contracts commonly use the Black and Scholes (1973) analysis for calculating 'implied' volatilities from the formulas (1) and (2) when the prices of options are observed. They find that volatility of an underlying asset's price is not constant and forms a 'smile', 'smirk', 'reverse skew', 'forward skew' and many more configurations. In addition, Black and Scholes (1972) empirically test their option-pricing formula and find that the formula tends to underestimate the price of options on stocks with lower price variance and overestimate the option prices on stocks with higher price variance. Hence, the variance of underlying asset's price is not constant.

These preceding examples are the clear violations of Black and Scholes option-pricing model's assumptions, although professionals in financial markets still widely use the model[6].

4. Contingent Claim Analysis

Black and Scholes (1973) present a method to employ option-pricing analysis for pricing of other financial claims[7]. It was also realized that the liabilities' side of companies' balance sheets and other assets with option like structures can be evaluated using Black and Scholes (1973) approach. This approach is called Contingent Claim Analysis (CCA). It should be noted that the following analysis is applied to European style options. This section presents three cases of different contingent claim analysis (CCA): in terms of put, call and both put and call options.

4.1 View of company's capital structure

Modigliani and Miller (1958) argued that, if a particular set of assumptions is satisfied[8], when the way in which a company raises its capital does not affect its value and it was an influential view before the establishment of a general equilibrium option-pricing theory and its various applications. It was called a 'capital structure irrelevance' hypothesis. However, Black and Scholes (1973) argued and demonstrated that company's capital structure can be viewed from a different perspective. If one

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[6] However, Haug and Talleb (2009) argue that professionals do not use the precise formula of Black and Scholes (1973), only various extensions of it.

[7] This approach can be used in pricing warrants, preferred stocks, bonds etc.

[8] There are no taxes, bankruptcy and transaction costs, securities are issued in perfect capital markets, individual and corporations can borrow at the same rates.

assumes that company issues pure discount bonds, which do not pay coupon payments, and has physical assets or financial assets such as common stock of another company and intends to issue common stock at the maturity of its discount bonds in order to repay its debt, then this situation can be viewed from the perspective of option-pricing analysis. It is thought that company's shareholders buy a call option on company's stock (assets) when issuing discount bonds because bond investors have seniority in claiming company's assets, therefore, current shareholders take a 'short' position in company's assets and 'long' position in a call option. It is believed that the option will be exercised only, if company's net asset position (total assets – total value of discount bonds) is larger than zero at the maturity date of bonds. If this is the case, when company's stockholders will be willing to issue new common stock (exercise a call option) in order to repay the holders of discount bonds.

4.2 Deposit insurance and loan guarantees

Merton (1977) argued that option-pricing methodology might be applied to evaluation of loan guarantees and deposit insurance. The business model of a financial intermediary, in its simplest case, is to gather short-term deposits and to lend these deposits in the form of long-term loans to businesses, which are in need of capital. Thus, there is a maturity mismatch on financial intermediaries balance sheet. In addition, financial intermediaries usually do not have enough assets to cover their current liabilities, so they are prone to bank runs, if all creditors desire to withdraw their funds at the same time, therefore, if there is no deposit insurance supplied, then 'small' depositors are advised to diversify and keep small amounts of their wealth in different financial intermediaries. However, this process involves gathering valuable information about the intermediaries' risks, which is costly and time consuming for ‘small’ investors. Furthermore, it is not clear, whether the diversification of depositors wealth through deposits in various financial intermediaries would really protect their wealth as the financial crisis of 2007-2009 demonstrates - financial intermediaries are interconnected and a run on one intermediary might have dear consequences for the whole financial system, as a result there is a need for a third party to provide deposit insurance. In general, deposit insurance is provided by governments were financial intermediaries are domiciled. However, it is not clear, who has to pay for this deposit insurance. Merton (1977)[9] shows how to apply Black and Scholes option-pricing methodology in calculating the cost of deposit or loan insurance. The guarantee of deposit or loan insurance implies selling of the put option to investors by a third party. Put options give the holder of the option the right to sell the underlying assets at some specified price and date. European put options are usually exercised when the value of the underlying asset declines below the strike price on the option at maturity, therefore, option is ‘in money’ and exercised. Hence, government or an insurance company writes (sells) a put option on the underlying asset (bank liabilities to depositors) and banks together with depositors buy a put option. Put options are also usually used as insurance tool for investment portfolio and lays the ‘floor’ under portfolio value, therefore, investment portfolio declines only as much as specified in the strike price of the option contract. In this case, government’s and insurance company’s ‘issuance’ of the put option works as an insurance policy in the portfolio insurance case.

4.3 Revolving credit agreements

[9] The mathematical derivation of deposit/loan insurance cost can be found in original Merton (1977) paper. We do not include it here.

Hawkins (1982) demonstrates that revolving credit agreements can be viewed from the prospective of both call and put options, depending on the needs of the borrower. Revolving credit agreements are such agreements, where company, which is in need of new financing, agrees with another party to access additional capital on request. However, company is not obliged to access new capital and has an option to choose, whether to use a credit facility, therefore, this analysis can be viewed from the perspective of options. The logic behind the revolving credit agreements is the following: company has an option to borrow from the financial intermediary; however, it is not obliged to do so. There are fixed costs, which bank faces, when arranging such a financing facility, thus, company, which want to access such a financing option, pays for it (acquires an option)[10]. Due to the fact, that a company is not obliged to use the financing facility, company has an opportunity to look for a less expensive financing alternative. If the current market interest rates are higher than provided by the facility, then a company, which have already acquired a call option, uses the arranged facility. At the same time financial intermediary is 'short' a call option and 'long' debt securities (credit line). On the other hand, if there is a liquidity shortage in financial markets and companies cannot borrow at any feasible interest rate, then availability of such a facility is as if a company has acquired a put option, because it is hedged against such an adverse conditions in financial markets. Hence, we can observe that revolving credit agreements exhibit characteristics of both put and call options and option-pricing methodology can be applied in order to determine the cost of such a financing arrangement.

5. Conclusions

[10] The derivation of the mathematical cost formula can be obtained from the original paper Hawkins (1982).

The Black, Scholes (1973) and Merton (1973b) general equilibrium option-pricing model provides a convenient and quick way for computing prices of exchange-traded options. The main strength of the model is its simplicity, as option prices are by the specifications of the option contract and risk-free interest rate – observables and underlying asset's price volatility – unobservable. However, the model has its limitations. The estimation of the volatility is a complex problem and the assumption that volatility is constant is violated on a number of occasions. Other 'idealistic' assumptions are violated and the core of the model – dynamic hedging is not attainable in practice. Hence, Black, Scholes (1973) and Merton (1973) analysis of option pricing must be approached with a great caution. Nevertheless, if one knows the limitations of the model, one can successfully apply the general formulae for analysis and calculation of option prices.

The analysed approach of option pricing is also beneficial when applied to Contingent Claim Analysis (CCA). In section 4 of this paper we demonstrated that option-pricing analysis is applicable to evaluating costs of companies' liabilities structures, deposit and loan insurance and revolving credit agreements. Moreover, this approach can be applied more generally to evaluation of any asset with option-like characteristics. Hence, the general equilibrium option-pricing model does not have to be narrowly viewed as a model of pricing financial option contracts; it should be viewed as a new way in evaluating any asset with option-like characteristics.

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