“Black-Scholes-Merton approach – merits and shortcomings”

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1. Introduction

The Black-Scholes and Merton method of modelling derivatives prices was first introduced in 1973, by the Nobel Prize winners Black, Scholes (1973) and Merton (1973), after which the model is named. Essentially, the Black-Scholes-Merton (BSM) approach shows how the price of an option contract can be determined by using a simple formula of the underlying asset's price and its volatility, the exercise price – price of the underlying asset that the contract stipulates – time to maturity of the contract and the risk-free interest rate prevailed in the market. Its discovery has highly influenced the pricing and hedging of options since the 1970's, offering also the possibility of extending the approach to other derivative instruments that have similar characteristics to options. The present paper is intended to offer an assessment of this approach, by discussing its economic and empirical validity.

An option is a financial contract, the value of which is derived from the price of an underlying asset, hence the title of "derivatives" attributable to these types of contracts. The holder of such a contract purchases the right, but not the obligation, to buy or sell the asset specified in the option contract – the underlying. The classification of options varies according to their scope and flexibility in the terms of their fulfilment. Hence, a call/ put option owner has the possibility of buying/ selling the underlying asset at the specified contract price and date, for a specified fee – the option premium. This is the price modelled using the BSM approach, which is straightforward in implementation for European-style options. American options can be exercised any time prior to or at maturity, whereas European options can be executed only at the maturity date, hence the simplicity in modelling European option prices. The second participating party to the option contract is the option writer, who must satisfy the terms of the agreement to buy or sell the asset, should the option holder decide to exercise his right.

The paper is organised as follows. Section 2 will explain the role of the assumptions underlying the BSM model. Section 3 will provide an overview of the meaning of volatility and how it is modelled and will discuss the main critiques of the BSM approach. The following section will summarize other general, yet important applications of this model. Section 5 concludes on the main aspects of the assessment of the BSM method.

2. Main assumptions of BSM approach

The BSM approach offers a model of general equilibrium of options' prices, which is valid only under a certain set of assumptions. Some of the assumptions are necessary for every model of option price determination, whereas the BSM model requires two additional assumptions regarding the trading of options and the underlying asset's price. The BSM model is based on the following assumptions:

1. Frictionless markets, which implies that the cost of trading is zero, and there are no legal restrictions on trading in the options and in the underlying asset, or on short-selling the asset. Moreover, perfect divisibility of the asset ensures that one unit can be readily fragmented into tradable parts.

2. The underlying asset offers no additional payments during the life of the option (for example, stock that pays no dividends and is protected against stock splits)[1].
3. Unlimited borrowing and lending is possible at a constant risk-free interest rate.
4. The arbitrage principle is at work – no profits can be made from an investment with zero initial outlay and zero risk.
5. Investor preference assumption – given a constant variance of the underlying asset's price, investors will always choose more wealth to less.
6. Continuous trading in the asset markets.
7. The logarithm of the price of the underlying asset has a normal distribution and the changes in this price are described by a geometric Brownian motion (a sample-continuous process).

Given these assumptions, Black and Scholes (1973) arrived at the following formulas[2] for a European call option premium, c, and a European put option premium, p:

$$c = S_0 N(d_1) - Xe^{-rT} N(d_2) \qquad (1)$$

$$p = Xe^{-rT} N(-d_2) - S_0 N(-d_1) \qquad (2)$$

with $$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad (3)$$

and $$d_2 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \qquad (4)$$

[1] Black and Scholes (1973) use this assumption because option prices change when there is a change in the payout policy of a company; however, Merton (1973) proposes an adjustment in the reasoning of the model to account for dividend payments during the life of the option.
[2] Source: Hull (Introduction to Futures and Options Markets, 2011, Seventh Edition, Pearson Education).

The determinants of the prices for the European call and put options are: the stock price - S_0, the strike price - X, time until option expires (time to maturity) - T, the risk-free interest rate - r and the volatility of the stock price (measured by its standard deviation) - σ. The N(x) function represents the cumulative probability function for a variable with zero mean and constant variance of one, following a standard normal distribution N(0,1). N(x) essentially gives the probability that a variable with these mean and variance properties will be less than x.

The first five assumptions are also required for the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965), being viewed as "standard" assumptions of asset price derivation. The innovation of the BSM comes from assumptions 6 and 7, which are essential to the model's predictions. Moreover, it is important to note that the price of a European option as derived in the BSM model does not depend on the expected return and total supply of the underlying asset, and investors' attitudes to risk. Hence, investors may have different expectations on the rate of return on the asset and the model will still holds. However, investors must agree that the geometric Brownian motion process best describes price fluctuations in the asset. In order for the predictions of the model to hold, the volatility term is valued the same by each investor. The factors that determine the option price, as they appear in the formulas (1), (2), (3) and (4), are computed with ease and a high degree of accuracy, except for the standard deviation term. The measurement difficulties of this variable stem from the fact that volatility is not an observed value, rather it is necessary to estimate it using information on the asset price movements.

The arbitrage principle is a core element in the economic validity of the model. As Merton (1998) describes the reasoning, in order to minimise the risk associated with issuing an option, it is possible to construct a “replicating” portfolio consisting of both risk-free and risky assets and whose payoffs perfectly reproduce the payoffs of the option. This is obtained through dynamic hedging which means that the weights in this portfolio are constantly amended until it consists of only the underlying asset of the option contract. Such perfect hedging is possible in the BSM model through the continuous trading and frictionless markets assumptions. Continuous trading implies that hedging is possible by trading in the option and in the underlying asset simultaneously. Otherwise, arbitrage opportunities arise and the option price is no longer determined by the formula derived in the model. The unrealistic nature of the model’s assumptions would make dynamic hedging strategies impossible in practice. This implies that hedging options’ payoffs is risky. The capital asset pricing model shows that firm-specific risk can be eliminated through a well-diversified portfolio. Hence, by diversification, the return on the “replicating” portfolio yields market risk only and the option payoffs are successfully hedged.

3. Main critiques

Whereas the main merit of the BSM approach is its simplicity in application, the BSM model was developed purely theoretical, leaving most of the literature on option pricing theory to question its assumptions: their unrealistic nature may prevent the model from producing accurate predictions when tested empirically. Another approach used in testing the theory is to compare the model’s predictions on the option prices with the observed market prices. If the predictions are constantly accurate, the BSM model would be the best model, regardless of its assumptions. The literature on these two approaches shows mixed results, of which the most prominent ones are discussed in the following.

One of the most noted shortcomings of the model is related to the way the volatility term is computed. Measurement errors in the option price determinants, such as volatility, cause misleading predictions. In order to clearly show how this affects the model's predictions, a distinction between "explicit" and "implicit" volatility[3] is necessary. Explicit volatility is measured by the standard deviation of realised asset prices. In contrast, implicit volatility is that measure of volatility which satisfies the Black-Scholes formula. To be obtained, one would substitute the observed option price in the relevant formula (either for a call or for a put option), and compute the value of σ. The estimated volatility is forward-looking, based on expectations on the future and is used in evaluating the risk of holding an asset. More importantly to the analysis of the BSM model is the equality of the estimated volatilities from different option contracts written on the same underlying asset. If these equalities do not hold, there is evidence that volatility of the asset's price is not constant. It is found that implicit volatility patterns take many graphical forms, most observed being "volatility smiles", "smirks" and "skews". A study by Macbeth and Merville (1979) shows that the implied volatility varies whether the option considered is out or in the money and that the longer the time to maturity the less volatile the underlying asset price is. Black and Scholes (1972) themselves admit the drawbacks of their formula, more precisely the inaccurate measure of explicit volatility which overestimates the value of an option written on an asset with a high volatility of its return.

[3] Source: Bailey, R. E. (The Economics of Financial Markets, 2005, Cambridge University Press)

Given that volatility varies over time, whether the asset price follows a geometric Brownian motion is a good approximation of reality is treated with scepticism in the literature. Bollerslev and Zhou (2007) use a "model-free" approach to account for time-varying volatility and find better predictive power than for the Black-Scholes model. Their argument is built on the difference between explicit and implicit volatilities of asset prices, which would be accounted by a risk premium.

Asset indivisibility assumption is violated because very often option contracts are traded in blocks on organised exchanges, hence the impossibility of trading in a single option from that block. Furthermore, in practice, dynamic hedging is not feasible due to market frictions such as excessive transaction costs, and discontinuous trading. Hence the resulting portfolio is only an approximation of a "replicating" portfolio. Haug and Taleb (2010) argue the weakness of the Black-Scholes formula to jumps in the underlying asset's price and also to tail events. These cause the variance of the portfolio constructed using dynamic hedging to increase abruptly, thus experiencing enormous losses on this portfolio. In the light of these findings, the Black-Scholes approach can have high associated risks, which need to be accounted for by a modification in the analysis.

The above mentioned difficulties in applying the model can result in discrepancies between the observed market prices of options and the prices predicted by the Black-Scholes formula. As a result, the literature on derivatives pricing has found additional relevant factors that determine the prices of option contracts. Garleanu, Pedersen and Poteshman (2009) study the implications of option demand on option prices and find that demand is a relevant predictor.

The BSM model has its shortcomings when it comes to accurately predicting option prices. However, the intuition used to construct it is relevant for the theory of option pricing as it provides a benchmark for assessing competing model. Moreover, it can be easily modified to account for different relaxations of assumptions, while in other aspects it can prove difficult to amend[4]. Black and Scholes (1973) restricted their analysis to European style options, but the assumptions imposed can be relaxed so as to extend this theory, for example to the American-style options. Merton (1973) shows that the Black-Scholes approach can be applied to American call options, provided that the underlying asset pays no dividends during the life of the option. He demonstrates the it is not profitable for an investor to exercise a call option on such assets prior to maturity, hence his argument also allows for varying interest rates over time; however, the crucial assumption of homogeneous beliefs of investors must still hold in order to derive the Black-Scholes formula. Moreover, Merton (1973) also demonstrates the Cox and Ross (1976) allow for discontinuous asset price movements, hence eliminating the geometric Brownian motion unrealistic assumption.

4. Alternative applications of the BSM approach

The Black-Scholes-Merton analysis of option prices can be generally applied to contingent claims – securities that have returns dependent on the returns of other assets. The contingent claims analysis is thus concerned with finding such a relationship and using it to determine the price of the contingent claim. This is the same reasoning applied in the BSM model.

[4] This is the case of time-varying volatility, as argued by Bollerslev and Zhou (2011).

Merton (1998) outlines the numerous applications of the Black-Scholes formula that have evolved together with financial innovations. One of the significant applications of the BSM approach to “option-like” securities involved the pricing of insurance contracts such as loan guarantees and deposit insurance policies and is attributable to Merton (1977). In essence, the strategy of acquiring both a put option on an asset and the asset as well can be viewed as an insurance policy against losses that would result from a decline in the asset price. Similarly, loan guarantees and deposit insurance policies protect the owner from losses incurred in the event of default. Loan guarantees insure banks from losing the payments on the loans they issue should a borrower default. Deposit insurance, in a similar fashion, are guarantees that depositors will receive the full amount or some percentage of it in the case of a bank run. To make the similarity of deposit insurance and loan guarantees with a put option evident, the insurance company (or the government), writes a put option that gives its holder the right but not the obligation to sell the underlying asset: the deposit made with a bank –where the holder is a depositor who wishes to protect himself should the bank default – and the loan issued – where a bank holds the option on the loan and will exercise it in the case that the borrower defaults on his payments. Merton (1977) uses the Black-Scholes formula to price the insurance contracts described above.

Another application of the option pricing theory of BSM is to revolving credit agreements (or line of credit agreements, as they represent one of the ways in which banks offer credit), which are contracts similar to options in the sense that a company in need of finance for its projects signs an agreement with another company who is obliged to lend to it the amount needed should this be requested. The company which owns this call option makes a decision according to the cost of borrowing that other banks impose, the aim of the company being to borrow at the lowest rate possible. Hawkins (1982) uses the theoretical set-up of BSM model to calculate the prices of revolving credit agreements.

5. Conclusion

The option-pricing model developed by Black, Scholes and Merton in 1973 provides a straightforward way of computing the prices of option contracts, being widely used by traders since its publication. The main strengths discussed in are the following. By using the information provided in the option contract (which are observed values) and the estimated volatility of the underlying asset price (unobserved value) one can easily determine the option price through the Black-Scholes formulas. The study of the model's assumptions and predictions have enabled Merton (1977) along with other researchers to broaden its applicability to other contingent claims such as line-credit agreements and insurance contracts.

As with any model of asset pricing, the weaknesses emerge from an unrealistic set of assumptions, which cause difficulties in estimation and evaluating the model's predictions. Given these shortcomings, the BSM approach is still one of the most accessible and relied-upon methods. Researchers have worked on amending the model to incorporate assumptions closer to reality and have discovered that by knowing the model's weaknesses, its application can still prove useful in analysing option prices.

Bibliography

Bailey, R. E. (2005). *The Economics of Financial Markets.* United Kingdom: Cambridge University Press.

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy , 81* (3), 637-654.

Black, F., & Scholes, M. (1972). The Valuation of Option Contracts and a Test of Market Efficiency. *The Journal of Finance , 27* (2), 399-417.

Bollerslev, T., & Zhou, H. (2007-11). *Expected Stock Returns and Variance Risk Premia.* (unpublished) Retrieved April 2013, from http://www.federalreserve.gov/Pubs/feds/2007/200711/200711pap.pdf

Cox, J. C., & Ross, S. A. (1976). The Valuation of Options For Alternative Stochastic Processes. *Journal of Financial Economics , 3* (1), 145-166.

Garleanu, N., Pedersen, L. H., & Poteshman, A. M. (2009). Demand-Based Option Pricing. *The Review of Financial Studies , 22* (10), 4260-4299.

Haug, E. G., & Taleb, N. N. (2010). *Option traders use (very) sophisticated heuristics, never the Black–Scholes formula.* (unpublished) Retrieved April 2013, from http://ssrn.com/abstract=1012075

Hawkins, G. D. (1982). An Analysis of Revolving Credit Agreements. *Journal of Financial Economics , 10* (1), 59-81.

Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics , 47* (1), 13-37.

Macbeth, J. D., & Merville, L. J. (1979). An Empirical Examination of the Black-Scholes Call Option Pricing Model. *Journal of Finance , 34* (5), 1173-1186.

Merton, R. C. (1977). An Analytical Derivation of the Cost of Deposit Insurance and Loan Guarantees. *Journal of Banking and Finance , 1* (1), 3-11.

Merton, R. C. (1998). Applications of Option-Pricing Theory: Twenty-Five Years Later. *The American Economic Review , 88* (3), 323-349.

Merton, R. C. (1973). Theory of Rational Option Pricing. *The Bell Journal of Economics and Management Science , 4* (1), 141-183.

Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Financial Economicsv , 10* (3), 425-442.

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