Present a critique of the Capital Asset Pricing Model, and hence discuss the claim that 'beta is dead' in the context of empirical models of assets' returns.

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Word count: 2997

# 1. Introduction

The early work of the mean-variance portfolio theory by Harry Markowitz (1959) provided the foundation upon which the Capital asset pricing Model will be built upon by William Sharpe (1964) and John Linter (1965). Under its three key assumptions the model attempts to explain the expected risk premium of an assets return, this is achieved by an analysis on the entire market rather than at an individual level.

The beta term in the CAPM symbolizes the non-diversifiable (systematic) risk (variance) of an assets return, often referred to as an assets "exposure to the market". This demonstrates the correlation of the assets excess return to the excess return of the market portfolio, which in reality is determined by a proxy. The claim "beta is dead" refers to the models failure. If beta is incapable of predicting the risk premium of an asset i.e. not statistically different from 0, it is seen as insignificant and thus dead. This paper will investigate this claim and give a critique of the model.

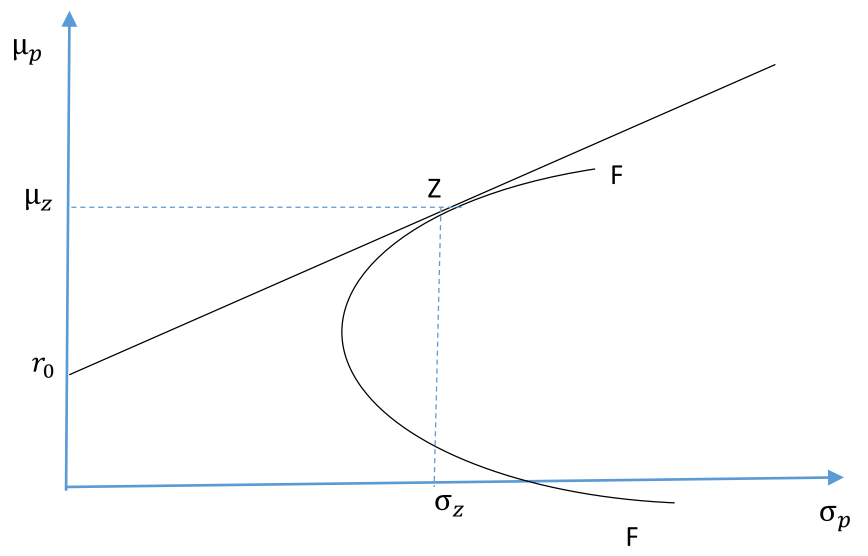
This paper will be organised as follows;

- Section one: Introductory section.
- Section two: An overview of the capital asset pricing model and a critique.
- Section three: Empirical results.
- Section four: Conclusion.

## 2. CAPM overview

I will initially give a brief overview of the portfolio theory developed by Markowitz to give the foundations of the CAPM. Then an overview of the CAPM, including its derivation. Lastly the implications/predictions and a critique will be given.

The portfolio frontier (FF) depicts all the combinations of assets in a portfolio. Asset diversification provides a shifting of the portfolio frontier to the left and therefore a reduction in the risk for a given expected return. The part of the portfolio frontier with an infinite (minimum risk portfolio) or positive gradient indicates all efficient portfolios. Investors are assumed to want to maximise an assets expected return relative to its variance, the optimal portfolio is therefore on the positive slope. After the introduction of a risk free asset ($R_0$), the tangency line from the risk free asset to the frontier indicates the new set of efficient portfolios. Investors will therefore choose a combination of the risk free asset and the optimal portfolio to satisfy their indifference curve.



*Capital Asset Pricing Model*

If we assume all investors have homogenous mean-variance beliefs, then all investors hold the tangency portfolio. If this is true the entire wealth of the market can be expressed by this portfolio and hence is the market portfolio. The tangency line is now described as the capital market line (CML). All points on this line provide a more efficient investment than investing in only risky assets. The assumptions required for this model are as follows;

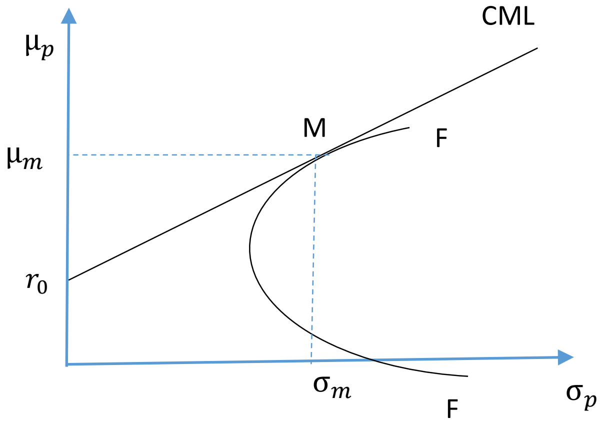
1. Markets are in equilibrium;
    a. Frictionless markets
    b. Unlimited risk-free borrowing and lending
    c. Divisible assets
    d. Complete markets
    e. Price-taking investors
    f. Neutral taxes
2. Mean-Variance investors
3. Homogenous beliefs about the mean-variances (e.g. based on historic data).

The slope of the portfolios are also referred to as the Sharpe ratios. The Sharpe ratios for all portfolios expressed along the capital market line must equal one another. Setting the gradient of an efficient portfolio and the market portfolio equal results in the CAPM equation;

$$\frac{\mu_m - r_0}{\sigma_m} = \frac{\left((\mu_j - \mu_m)\,\sigma_{jm}\right)}{\left(\sigma_{jm} - \sigma^2{}_m\right)}$$

$$\mu_j = r_0 + \frac{\sigma_{jm}}{\sigma^2{}_m}(\mu_m - r_0)$$

$$\beta_{jm} = \beta_j = \frac{Cov(j,m)}{Var(m)} = \frac{\sigma_{jm}}{\sigma^2{}_m}$$



$\mu_j = r_0 + \beta_j(\mu_m - r_0)$

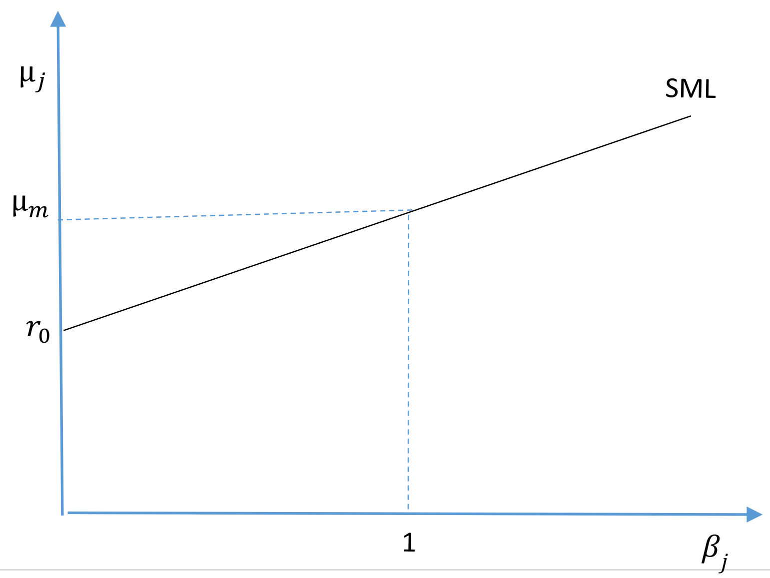
$\mu_j - ro = \beta_j(\mu_m - r_0)$ [This is the prediction of the assets expected risk premium]

The beta $(\beta_j)$ is the regression coefficient when the return of asset J is regressed onto the excess return of the market portfolio. It demonstrates the non-diversifiable (systematic) risk of the assets returns. A high absolute value indicates a strong exposure to the market.

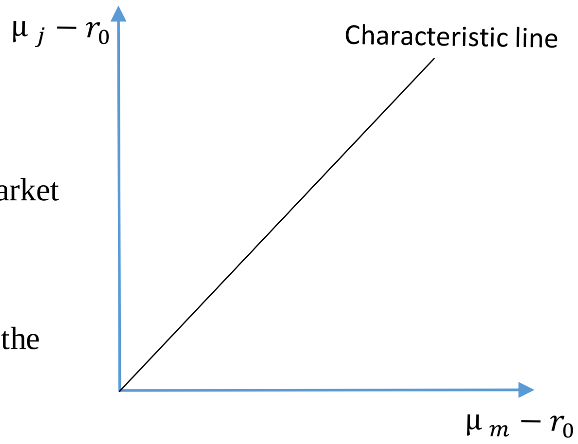
*Implication one: Security market line*

The security market line is the predicted linear line of best fit when the expected returns are plotted against the beta $(\beta_j)$ values. An asset is said to be in CAPM disequilibrium if the observed return is different to the expected return. This disequilibrium can be represented by the inclusion of an alpha term. If this term is different from zero it can be interpreted as evidence against the model. However, Bailey (2005) points out caution must be taken; a non-zero alpha term may be reflective of the assets in the sampling period being in disequilibrium and therefore not an indicator of the models failure. The SML is often modified to predict a zero intercept via making (uj-r0) the dependent term.

$\mu_j = \alpha_j + r_0 + \beta_j(\mu_m - r_0)$

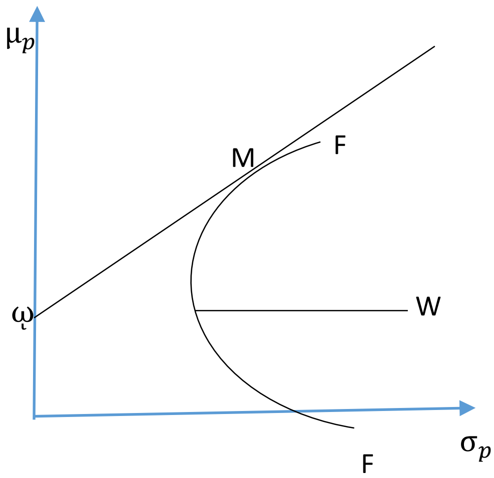


*Implication two: The characteristic line.*




The expected excess returns of asset j ($\mu_j - r_0$) are plotted against the expected excess returns of the market portfolio ($\mu_m - r_0$). The slope is beta and therefore differs depending on the asset. The CAPM predicts the intercept is 0.

*Modification: The Black CAPM*




Fischer Black (1972) created the BLACK CAPM in response to the unrealistic assumption of unlimited risk-free borrowing and lending. The term $r_0$ is replaced with $\omega$, which represents the expected rate of return on a zero-beta asset or portfolio, mathematically demonstrated below. The horizontal W line represents the feasible zero beta portfolios.

$$\beta_{jm} = \beta_j = \frac{Cov(j,m)}{Var(m)} = \frac{\sigma_{jm}}{\sigma^2{}_m} = \frac{0}{\sigma^2{}_m} = 0$$

$$\mu_j = \omega + (\mu_j - \omega)\beta_j$$

$$\mu_j = \omega + 0$$

Other modifications to the CAPM also exist such as the ICAPM or CCAPM. However, the majority of this paper will focus on the Sharp-Lintner and the Black CAPM. Although a brief explanation of the ICAPM will be given later in this paper.

*Some CAPM predictions*

1. Model predicts the expected rate of return and the beta coefficients for all assets and portfolios are located along the SML line.
2. No other explanatory variable exists in determining the expected returns of asset j.
3. The market portfolio is mean-variance efficient. Therefore any uncorrelated asset (zero beta) has less expected return than the market portfolio.
4. The Sharpe-Lintner CAPM also predicts that if an assets expected return is uncorrelated with the market portfolio return it is equal to the risk free rate;

   $\mu_j = r_0 + 0(\mu_m - r_0) = r_0$

   The intercept of the SML is the risk free rate or zero depending on the Y axis value. The beta risk premium is also given by the Market return minus the risk free interest rate $(\mu_m - r_0)$.

*CAPM tests*

Using time series data observations of rates of return for a set of dates can be used to measure the excess return on asset j and the market portfolio. Beta can then be estimated and the data fit is investigated. The Sharp-Lintner CAPM predicts the alpha coefficients will be zero for every asset. Blacks CAPM can be investigated by observing if the alpha coefficient's pattern. Including additional variables such as economic growth can also be used to test the CAPM with time-series data. CAPM predicts only beta should be significant.

Cross sectional data is generally used to test of the security market line. Average excess returns (uj-r0) and estimates of beta are relied upon. The expected excess return on the market portfolio are estimated and then compared to true values. The aim being to examine the slope (effectively betas significance). Another popular test involves the inclusion of additional variables, the CAPM predicts their coefficient should be zero.

*Critique of the model*

The assumptions of the CAPM are unrealistic, this makes real world application of the model difficult. We do not live in a world comprised of the assumed assumptions. Such failing assumptions being the following;

- Frictionless financial markets do not exist. For example if I wanted to execute a trade I would be required to pay a commission to a broker. Certain regulation will also be present which violates this assumption.
- The unlimited risk-free borrowing and lending assumption is unrealistic. However this assumption is dropped in the BLACK CAPM.
- Neutral taxes do not exist. Investors and investments do not face the same tax rates. For example the dividend upper rate vs. the ordinary rate.
- All investors do not optimize a mean-variance objective such as noise traders who invest irrationally. The homogenous belief assumption is therefore also violated.

The sole intention of the model is to provide a guide for investment via an assets required risk premium. These unrealistic assumptions could result in the failure of this. Beta may therefore not be significantly different from zero. This claim is investigated in the ensuing section.

Roll (1977) argues previous observed methods of testing the CAPM are incorrect and there is practically no future possibility to test the model unless the true market portfolio is known, which will include all investable assets such as real estate. It's a market portfolio identification problem; using a Proxy such as S&P 500 creates two major problems;

1. S&P 500 might be mean-variance efficient while the market portfolio is not.
2. S&P 500 may turn out to be inefficient, but this implies nothing about the true market portfolios efficiency.

3. Empirical results

*Early support*

Outlined in a paper by Jagannathan and McGrattan (1995) a study by Black, Jensen, and Scholes (1972) made use of the security market line for their test of the model. The analysis involved regressing historical excess returns onto a proxy for the market risk premium. The two proxies being the NYSE (market portfolio) and the 30-day T-bill rate (risk free rate)

$(\mu_j - ro) = \beta_j(\mu_m - r_0)$


Source: Jagannathan and McGrattan (1995)

This leaves beta as the only unknown and therefore possible to calculate, the results are shown in figure one. The analysis concludes the data is consistent with the model, however the intercept is not zero. This provides support for the less restrictive BLACK CAPM and supports beta's explanatory power.

A paper by Fama and Macbeth (1973) also supports this theory. The testing involves regressing the rate of return of a portfolio onto three variables; the beta estimate, the squared beta estimate, and the squared residual variance. The use of portfolios instead of assets gains the benefit of diversification, and therefore reducing the beta standard error which later becomes standard practise. The analysis concludes that only the beta coefficient was shown to be statistically significant.

*Beta opposition.*

Banz (1981) provided the first real objection to the explanatory power of beta. The study made use of the following formulae in which the additional term ($¥_p$) represents the size effect (value of the market equity);

$$r_p = \Upsilon_0 + \Upsilon_1 b_p + \Upsilon_2 ¥_p + \varepsilon_p$$

| | |
|---|---|
| $r_p$ = Expected excess return on portfolio p<br>$\Upsilon_0$ = Expected return on an asset with zero beta (no risk)<br>$\Upsilon_1$= Risk premium for bearing one unit of beta risk<br>$b_p$ = Estimate of beta for portfolio p | $\Upsilon_2$ = Risk premium for size risk<br>$¥_p$ = Relative size of portfolio p<br>$\varepsilon_p$ = Random disturbance term |

The CAPM predicts the $\Upsilon_2$ value should not be statistically different from zero, i.e. no other explanatory variable exists. However, the study demonstrated a statistically significant relationship existed. Most importantly regarding beta, the size effect was shown to explain more cross-sectional variation than beta. Banz (1981) conducted a second test which verified his results and thus concluded CAPM as miss-specified. It is therefore argued beta is not as useful a guide as previously thought.

Following the result Fama and French (1992) advance this argument with a similar study comprising of more recent data. Size was shown to be statistically significant (the $\Upsilon_2$ T-statistic being -3.41). The beta coefficient's ($\Upsilon_1$) T-statistic was -1.21, indicating the beta coefficient was not significantly different from zero.

Fama and French (1992) further investigate with the inclusion of a third variable (BE/ME); the ratio of the book value of a firm's common equity to its market size. Average returns are regressed onto β, ME, and BE/ME. However, in one regression β is excluded.

In all three results the beta coefficient was not statistically different from zero, the t-test values being -0.62, 0.25, and -1.17 respective to each data set. The Book-to-market ratio was also shown to be more significant than the size effect. The result regarding beta are described by the authors as follows;

"We find that the simple relation between β and average return disappears during the more recent 1963-1990 period"

Fama and French (1992, p449)

*Death.*

A paper by Jagannathan and McGrattan (1995) describes Jagannathan and Wang (1993) response to the results. The authors believe this lack of support for the CAPM may be due to the inappropriateness of some of the models assumptions. For example, a true market portfolio must include a measure of all agents in the economy. Non-tangible assets such as human capital are not captured by a stock market index which is the usual proxy.

Following this, Jagannathan and Wang (1993) include human capital in their measurement of the market portfolio via the growth of labour income as a proxy. The result being a jump from 1.4% of cross-sectional variance explained to 28% and thus human capital makes a significant difference. A time varying beta is introduced because of the systematic varying of assets betas, this results in a further jump to a 57% explanation of the cross-sectional variation.  This result implies more than one beta could be required and therefore miss-specification is present.

*ICAPM dual beta model.*

The Intertemporal CAPM model was created in 1973 by Robert Merton. The Model is not static and attempts to better model investors behaviour by taking future expectations into consideration, such as hedging against uncertainties. These uncertainties are not captured within the original model and thus explains Banz (1981) results.

Campbell and Vuolteenacho (2003) claim in order to capture the systematic risk two betas must be used rather than one. A "good" beta which reflects news of the market discount rate, and a "bad" beta which represents news about the markets future cash flow.

Their analysis uses data from 1963:7 to 2001:12 with a restricted (proxied by the Treasury bill rate) and unrestricted zero beta rate. The test concluded the two beta model had a significantly higher explanatory power relative to the static CAPM. The results are tabulated below;

| | Restricted zero beta test | Unrestricted zero beta test |
|---|---|---|
| Two-beta ICAPM | **$R^2$ = 49.26%** | **$R^2$ = 47.41%** |
| CAPM | $R^2$ = 3.1% | $R^2$ = - 61.47% |

The relative success of the Static CAPM pre-1963 relative to post-1963 is argued by Campbell and Vuolteenaho (2003) to be due to the "bad" beta being a constant fraction of the CAPM beta across the assets. This is not the case post-1963 and hence the low explanatory power of beta.

## 4. Conclusion

Early empirical analysis showed support towards beta's explanatory power. A later paper by Banz (1981) indicated other variables may be statistically significant in predicting an assets risk premium. This view was further amplified by Fama and French (1992) who concluded that beta was not significantly different from zero and therefore arguably dead.

One year later Jagannathan and Wang (1993) argue the inclusion of human capital is required for an accurate market portfolio, which results in a significant increase in the explanatory power of beta. However, the introduction of a time varying beta also proves significant. This implies the CAPM is miss-specified.

A two beta ICAPM developed by Campbell and Vuolteenacho (2003) is able to demonstrate its success and betas low explanatory power. However, the market portfolio proxy used is arguably still significantly different from the true market portfolio.

However, Fama and French (1996) claim the bad market portfolio proxy argument does not justify its current use in the real world. These bad market proxies are generally the proxies used in the applications of the model. Therefore, empirical analysis showing evidence against the model and therefore beta should not be ignored. If β is shown to have little explanatory power it should not be relied upon in the real world.

Levy (2012) supports the view the CAPM cannot be rejected on an empirical basis. The empirical tests do not account for the difference between ex-post and ex-ante values, observed returns are only a proxy for the past expected returns for both the asset and the market portfolio. Any deviation between these values can result in a change of the beta and therefore bias the result and support rejection of the model and therefore beta.

On a theoretical standpoint beta has not been proven dead. Also the lack of an adequate market portfolio proxy should not result in the disregarding of the model. However, the current real-world use of beta should be used with caution. Perhaps a model which is more easily transferable into the real world could provide a better guide to investors, such as the one suggested by Campbell and Vuolteenacho (2003).

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