The behaviour of players in games with a mixed strategy Nash equilibrium: Evidence from a stylised Poker experiment.

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Abstract

This paper analyses and compares the strategies chosen by experienced and inexperienced card players, participating in a simplified construction of the game Poker. The current research investigates whether mixed strategies can be used in real life situations, and whether they can yield a higher utility than any other strategy. Taking the research away from the sporting analysis, that has already received much scrutiny by other economists; we study the behaviour of players gambling at poker games. Although some research has already been undertaken; this study researches a specific construction of Poker and has allowed for empirical data to be analysed. The experiment carried out, compares two different groups of players (experienced and inexperienced) in a Stripped Down Poker game. This simplified construction generates interesting results in view of a deeper understanding of mixed strategy Nash Equilibria. Although many believe it is impossible to adopt a mixed strategy in real life situations, the findings from this experiment suggest that mixed strategies can be used to gain the highest utilities.

Contents

1. Introduction

John Nash (1951)stated that all games had an equilibrium that could be reached, and this is where the theory of Nash equilibrium was formed. When game theory first came about, economists believed that all equilibriums were Pure Nash equilibriums, however after more research they realised that some equilibriums were gained from using mixed strategies. Mixed strategies are odd concepts, and many people believe that they are impossible to implement, due to the fact it requires people to be indifferent between their possible actions. Using economic models it is easy to see how mixed strategies can be used to increase utilities, rather than looking into real life situations. Although some studies have taken place, that look into mixed strategies in real life situations; I have decided to further research into this topic. This paper will look into the effects of using mixed strategies when gambling and how it can affect the chances of winning.

1.1. Motivation

Game theory is a widely used economic concept that is used by everybody in day to day life. Interacting with other people (players), and trying to create positive utilities is the basic idea of a game, and generally people are playing games in real life almost every day. In most daily situations people are trying to gain a positive payoff from the actions they are taking and the people they are interacting with. Because of economic rationality it is believed that everyone aims to gain their highest utility possible from any situation. In real life situations this may be translated to being able to consume more goods or even being able to win a political argument. To try and gain the highest payoff in these situations, all players need to decide upon a strategy that will provide them with their best responses. Game theory and the study of player's trying to reach their highest utilities is known as strategic behaviour.

Contributions to strategic behaviour started to be produced in the 1920's as many realised the importance of game theory, and how it has always been used throughout history; dating as far

back as the third century BCE (Schmidt, 2002, pg 16). With more research and studying taking place within game theory, it allowed economists to analyse in more depth the behaviour of players and to start to gain a better understanding of how players can use strategic behaviour. One of the most influential economists studying game theory was J. F. Nash, and in 1951 he stated that every zero-sum game has an equilibrium. In his article "Non-cooperative games" (1951) he examines the use of mixed strategies and realises that in certain situations they are the only way to reach an equilibrium. Although Nash had published an article on equilibria, many economists refused to accept his theories, as they differed from the early works on game theory.

The main reason Nash's work was at first dismissed, was because it not only differed to previous game theory interpretations, but it also didn't make sense to many economists. To be able to use a mixed strategy, when deciding what action to take, it requires that the player is indifferent between each action. "Rational players never randomise between two pure strategies unless they are indifferent between them. If one strategy were better, the inferior strategy would never get played" (Binmore, 2007, pg 25) Theoretically mixed strategies could be seen to work in certain situations, however many economists believed that in real life, no one can be indifferent between certain actions. Therefore in this paper I plan to examine the behaviour of players using mixed strategies, and how it affects their utilities as a result.

Most studies that have researched the mixed strategy phenomenon have, in some way, involved sports players. Therefore I would like to branch out into a different field, in terms of how they can be used in casino games. I feel that gamblers are equally dedicated to gaining high payoffs as much as sport players. Throughout this paper I will discuss how sports players have been able to gain higher payoffs from implementing mixed strategies into their game play to allow them to win a game. Gamblers equally may play games with large risk taking opportunities, allowing them to win large amount of money. Therefore, I will assume that when I test players who are

gambling on a simple card game, they will be playing their best strategies in order to win the most money.

From analysing the behaviour of players gambling, I will show whether using a mixed strategy will increase the chances of gaining a higher utility. Once I have analysed the data and formed a conclusion, I will have enough evidence to show whether mixed strategies can be used in real life and if players can be indifferent about what actions they will play. I will also look to see whether using a mixed strategy will increase the chances of gaining a positive payoff.

The rest of this paper is going to predict the possible outcomes from Players using a mixed strategy when playing Stripped Down Poker. I will research into the use of mixed strategies and calculate the equilibrium probabilities for when both players would decide to bet, call, or bluff if they were to actually use a mixed strategy. In doing this I will then be able to construct a hypothesis that I will be able to use to test my real data against. What this study will show is if a mixed strategy is the best strategy to use when trying to increase your payoff. To help me test this I will be comparing experienced players (who are the top ten players of the University Poker Society) against inexperienced players (who have very little or no experience with card games). Once I have collected and analysed my data, I am hoping that it will leave the opportunity for further study to take place and test behaviour of players using mixed strategies even further.

The rest of this paper is going to unfold in the following way: An in-depth literature review examining previous contributions to strategic behaviour, a detailed plan of the experiment being carried out, further background knowledge on strategic behaviour and numerical reasoning showing the theoretical argument for using mixed strategies, an aim of the experiment followed by a set of hypotheses, the results with a critical analysis of the findings, finally leading onto a suggestion for further study and conclusion. Section 2 provides the literature review, Section 3 a plan, Section 4 economic reasoning, Section 5 the hypothesis,

Section 6 the results, Section 7 an analysis, Section 8 the conclusion followed by Sections 9 and 10 with the appendix and bibliography.

2. Literature Review

This literature review is going to be looking at how mixed strategy equilibria can be used in real life situations. I will first explain, and describe, what a mixed strategy is and how it differs to pure strategy equilibrium. I will then describe case studies that have been carried out to see whether mixed strategies can be applied to real life situations, and in particular when people gamble. Although gambling is generally popular, in some parts of the world it is frowned upon, as some argue that Casinos take advantage of the players participating in their games. As a result I want to see if experienced players can in fact increase their chances of winning, by using mixed strategies and avoid the unfair statistics of losing games and money when gambling.

A game is an abstract view of a situation where players use strategic behaviour in decision making. Nash equilibriums can be formed when a player chooses a strategy that maximises their payoff given the strategy of the other player. Aumann and Brandenburger have demonstrated that "in a two person strategic game, if the structure of the game and player's beliefs and rationality are mutual knowledge, the beliefs profile constitutes a Nash equilibrium" (Lo, 1991, pg622). Sometimes players use a pure strategy, where they will have a specific action to take at every decision but if this is not possible, players can use a mixed strategy where they randomise the actions that they take during every decision. However for a mixed strategy equilibrium to hold "an agent must be indifferent between each of the pure strategies that are played with positive probability in the mixed strategy, as well as any combination of those strategies" (Chiappori, Levitt, Groseclose, 2002, pg1138). John Nash explains the difference in measuring mixed strategies between the normal form (Theory of Games and Economic Behaviour) and behaviour parameters. "In the normal form representation two mixed strategies

of a player may be equivalent in the sense that each make the individual choose each available course of action in each particular situation requiring action on his part with some frequency." (Nash, 1950, pg293) When looking at behaviour parameters probabilities are given for each possible action taken rather than their frequencies. Because in my study I will be examining the behaviour of the players I will be using probabilities and not frequencies.

The Nash Demand Game as described by Malueg shows that "proper mixed strategy equilibria are significant in that they offer a possible explanation for disagreement while relying on demands that are more plausible as they offer potential benefits to both players" (Malueg, 2009, pg248) In this paper Malueg describes that in order to have a mixed strategy equilibrium the players' sets of possible strategies must be balanced. This means that the demands from both players will be equal to one and when applied to the Nash demand game both players will expect a positive payoff. The Nash demand game is based upon two players dividing an amount of money between them. Simultaneously they say what share of the money they demand; if their demands are equal to each other then they are satisfied and they receive the money they have demanded; but if the demands do not equal each other, then the players will receive nothing. Disagreement is a fundamental contribution to how this game is played because if the players disagree then they will receive zero payoffs. For a mixed strategy to work the players must be indifferent between every single possible action they could play; therefore the demands from each player would not always equal. In the case of this game it means that they players won't always receive a payoff, however in real life situations if players disagree in the game it can allow one or more players to receive a positive payoff.

Roth (1985) argued that players would disagree and have different demands due to a lack of coordination and would therefore need to adopt a mixed strategy. Roth in a binary lottery game study, he found that two focal points would arise and that the player's expectations would be a contributing factor to any disagreements. The aim of the experiment was to test the classical theorist's hypotheses, which stated that "the outcome of a game can be predicted from the set of

feasible utility payoffs and strategic possibilities, implying that, in this experiment, the different experimental conditions should have no continuing effect." (Roth, 1985, pg264) In fact the results of the experiments showed that the player's individual utility predictions were the uncontrolled factor and were the reason for the difference in their demands. This resulted in the players receiving a zero payoff. If there had been coordination between the players and they had been able to use mixed strategies, then they might have been able to receive positive payoffs. This is what can happen in the case of real life situations.

A real life study conducted by Chiappori, Levitt and Groseclose (2002), examines the behaviour of footballers shooting penalty kicks in a football game. This I feel is a perfect example of how mixed strategies can be used in everyday activities as many believe that it is impossible for people to be truly indifferent between two different strategies. This study shows that this isn't strictly true. This paper was one of the first attempts to test mixed strategies outside of a controlled environment. "Although there are clear advantages provided by a well – conducted laboratory experiment, testing game theory in the real world may provide a unique insight." (Chiappori, Levitt and Groseclose, 2002, pg1150) Chiappori, Levitt and Groseclose studied games between penalty shooters (maximising the probability of scoring) and goalies (minimising the probability of a goal being scored). Generally professional players will have a vast amount of data available to them that relates to the history of each players game strategies i.e. a goalie may seem to prefer to jump to the left when trying to save a goal. However both players usually will be indifferent to their possible strategies of: shooting to the left, right or centre and for the goalie to jump to the left, right or centre. In this zero sum game the players act simultaneously which means that the actions taken by one another cannot impact on the other's decision because for example if the goalie waits to see which side the ball is being shot at; it will be too late to jump and save the ball. The results of this study show that no penalty shooters shot in the same direction more than four times which demonstrates that a mixed strategy was being used when deciding where to shoot. Also there was only one goalie throughout the whole study that jumped in the same direction for every single shot and ended

up conceding more goals than any of the other goalies studied. These results then show that using a mixed strategy in real life can in fact increase your chances of receiving a positive payoff and in this case it was through shooting or saving goals.

Another similar study showing that mixed strategies can easily be related to real life situations was created by Walker and Wooders on the strategies played by tennis players. Tennis players must mix their serves to the receiver's forehand and backhand sides; if the receiver knew where the serve was coming, his returns would be far more effective (Walker and Wooders, 2001, pg 1521)Here this study shows that most experienced tennis players will have an equal probability to which of their opponents hands they will serve to. By implementing a mixed strategy it means that the serves are randomised to the opponent resulting in them not being able to guess where the ball is going. By using a mixed strategy it increases the probability of the receiver not being able to return a good ball. Therefore studying tennis players provides a reasonable demonstration of mixed strategies being used in real life and that it can increase the chance of winning.

Rubinstein (1991) in his interpretation of game theory believes that for randomisation to work in real life as part of a mixed strategy is an unreliable explanation for people's behaviour as no one can truly make decisions at random. "One of the reasons why game-theoretic ideas have not found more widespread application is that randomisation, which plays a major role in game theory, seems to have limited appeal in many practical situations" (Rubinstein, 1991, pg912 – 913) In many real life situations it is usually unlikely that someone will be able to randomise their decisions as most strategies will have different payoffs. Therefore the strategies will be pure strategies as the person will not be indifferent between them and will choose the strategy with the highest pay off. The example Rubinstein uses is a frequent problem employees face whilst at work and I think it explains his point well. An employer has the choice of monitoring two employees to check that they are not shirking. He has two strategies; to either monitor employee A 100% of the time as a representative for both employees or to monitor employer A

and B 50% each. Therefore the first strategy holds a probability of p=1 and the second strategy holds a probability of p=1/2; with the employer preferring the second strategy. Here we can see a real life example of how someone is not indifferent between two strategies showing that mixed strategies are sometimes impossible to implement. Another reason why Rubinstein feels that mixed strategies are hard to implement is because "if implementing a mixed strategy is costly for the player, then he will strictly prefer to use any of the pure strategies which appear in the support of the mixed strategy rather than to waste the resources associated with implementing the lottery" (IBID). Although Rubinstein provides a strong argument against mixed strategies being used in real life games, I believe that there are many situations where mixed strategies are the only way to describe player's behaviour. The main topic that I feel shows mixed strategy equilibriums in its truest form is with gambling, in particular with poker games.

From reading articles looking at the behaviour of players in a mixed strategy equilibrium I have found some particularly interesting articles based on games of poker and how a mixed strategy can be applied to these games. I think this is a good way of determining the best outcomes because when playing poker the players are betting with money. This means that every decision they make is as important as the last one. The most common form of mixed strategy used in poker is when players decide to bluff and this has created a large interest in studies based on when and how players bluff. "The interest arises because it is quite clear to those who have played much poker that some sort of mixed strategy with regard to bluffing must be used" (Friedman, 1971, pg 764)

In the most extensive form of Texas Hold'em poker there are probabilities for every single hand played; the chance of winning is based on whether you decide to bet, bluff or check. However, this range of probabilities is almost impossible to remember when playing a real life game, and there isn't enough time to calculate the answers before making a decision. Therefore many economists have formed simpler versions of the game, or created other games involving betting

and bluffing, to allow them to analyse the behaviour and strategies that the players make. Mike Caro (2011) explains that fixed strategies will never be able to truly explain the behaviour of players in a game of Texas Hold'em because there are too many things to consider for every game. He also describes that game theory only provides a solution when the opponent plays perfectly. Thereby if an opponent starts to bluff throughout the game you will have to adjust your strategy. "If you stick to your predetermined game plan, you win more when your opponent bluffs too much. However, you won't win as much as you would if you simply said, "To hell with this game-theory stuff. I'm going to call all the time"" (Caro, 2011, Accessed online) Although Caro believes that it is impossible to apply game theory to real life poker, he does believe that the use of mixed strategy will help in chances of winning. This is because if you always fold when you have a weak hand and you bet when you have strong hand it won't take very long for your opponent to work out your strategy. As a result you need to introduce bluffing using a mixed strategy so that your opponent won't be able to work out whether you have a weak or strong hand.

One article that looks into bluffing during poker is based on a simpler game of stripped down poker where it shows a model of how to form a mixed strategy equilibrium. Here Reiley *et al* (2008) discusses how to demonstrate certain game theory traits when playing a very basic version of the game. These traits include signalling, bluffing and mixed strategies. The model differs to a real game of poker as it is played between one student and the researcher. The game requires only four kings and four queens in the deck and for only the researcher to be dealt a card during the game. The student and researcher can bet, fold or call during the game and then after they have both played the researcher wins if the card is a King but the student wins if the card is a Queen. Where the mixed strategy comes into this game is whether the researcher decides to bluff when they receive their card. If the strategy for the researcher was to always fold with a queen and bet with a king; then the students will very quickly learn the researcher's strategy and so neither players will be able to win money. However if the researcher uses a mixed strategy to determine whether he should bluff or not, it allows him to

earn money though out the process. However for the researcher to be "willing to mix over two strategies, they must be indifferent between them; otherwise, they would play their preferred pure strategy" (Reiley, Urbancic, Walker, 2008, pg329) So because the researcher won't earn any money from playing a pure strategy they use an external randomisation device i.e. bluff if the second hand on a clock is before 20 seconds and fold once it has passed 20 seconds that will allow for a mixed strategy equilibrium. In this case the payoff for the researcher will be 1/3 and for the student -1/3.This model shows that because of the asymmetric information the game is unfair as the researcher is secretly playing a mixed strategy.

3. Plan

This model using stripped-down poker can be easily related to the real game of Texas Hold'em, where players use mixed strategies, to decide their bluffing and signalling moves. "Real world poker is much richer and complicated, with multiple rounds of betting and multiple rounds of cards being dealt" (Reiley, Urbancic, Walker, 2008, pg333) Because it would be far too complex to collect data on a real game of poker, I want to collect my own data based on players using mixed strategies; when they decide to bet/fold that follows a similar model to the stripped-down poker model. Therefore, I am going to carry out an experiment following the same procedure as Stripped Down Poker. However, to make the results more reliable and verifiable I plan to test two different groups of people (experienced and inexperienced). I have chosen to go down this route, as I am testing to find out if there is an effect on the utility gained, based on whether a player is using a mixed strategy or not. I'm hoping that the comparison of behaviour between the two groups, will allow me to test which strategy is the best to use for gaining a higher payoff. From comparing the two groups of players I plan to prove whether using a mixed strategy really is the best strategy to follow, in order to enable a higher payoff. I will also find out whether it is possible to use mixed strategies in real life situations, as this has been a concern among many economists.

In my study the game of Stripped Down Poker is played with ten experienced card players, as they will have more experience and knowledge about what the best strategy is. I will then be able to test whether they are using a mixed strategy and whether it can increase the chances of winning. I have asked the top ten players from Essex Universities' Poker Society, who have been part of the team for at least two years, to take part in my study. This is because the players will have learnt how to use bluffing techniques to try and signal to the other players that they have a good hand, even if they may not. If the player who receives the card in the long run ends up winning the majority of hands, then I can assume that using a mixed strategy is the best strategy. This is because there is 0.5 chance of being dealt a King or Queen so if Player 1 is able to win more than 0.5 it will test whether mixed strategies do in fact increase the chance of winning. I will collect my data at the weekly poker society game night so that all the players are in the gambling frame of mind. To enhance my results and test my theory I will conduct the same experiment with inexperienced players, who have little or no experience with card games. If I can find players who have no experience with any form of card games, where bluffing is used as a technique to put off other players, then my results will hopefully show that in the long run there will be an equal chance of winning between two players. This will then further enhance my theory that using mixed strategies can increase the chance of receiving a positive payoff.

3.1. Choice of game to be played

As explained above, I have decided to test the theory of mixed strategy equilibriums using the Stripped Down Poker game. It is a very simple card game that has not yet been properly tested, or been analysed with real data. Therefore I feel that my research and analysis will be allow me to come to a formal conclusion, testing a hypothesis. Due to the time constraints of writing this paper, I have chosen to play an extremely simple version of Poker. This is because I will need enough time to collect sufficient data, that will allow me to statistically analyse it and form a conclusion. Stripped Down poker is an extremely basic version of the real game Texas Hold'em, and although this game is slightly different from the more complciated version, it should still be

verifiable to test the theory of using mixed strategies. Due to the fact it is simplicic, I feel it will allow Players to pick up the game very quickly, thereby I will only need a small sample of experienced and inexperienced players. If the game was more complicated, I would have to let each player play the game for a longer time to be able to gain a better understanding of the game and rules, and so inplement their preferred strategy. Therefore, as I have such a short amount of time to present this study, I will use the Stripped Down Poker to test my theory. Not ony will my experiment show me the effects of using mixed strategies, but it will also allow me to gain a better understanding of strategic behaviour and how experience can effect this behaviour.

Stripped Down Poker involves two players having two actions. In this game both players place a poker chip each into the pot and then Player 1 will be dealt a card. It will either be a King or a Queen. They then have the choice to bet (place a poker chip into the pot)or fold. If they fold then then Player 2 wins the chips. If they bet then Player 2 has the choice to either bet or fold. If they fold then Player 1 wins the pot. If they bet then they place another chip into the pot. If both Player 1 and Player 2 have chosen to bet then the face on the card determines who wins the pot of poker chips, If the card is a King then Player 1 wins, but if the card is a Queen the Player 2 wins. This game is extremely simple so I will therefore watch the game be played for ten rounds, per couple. This will allow me to collect enough data to be able to make an analysis from, as there will be ten rounds for ten different couples of Player 1 and Player 2. This will give me one hundred rounds of data for both the experienced and inexpereinced players. When assigning who will be Player 1 or Player 2 I will randomly choose which players will be playing who, and they will remain either Player 1 or Player 2 for the complete ten rounds. Once the ten rounds are over, I will randomly assign the next couple to play.

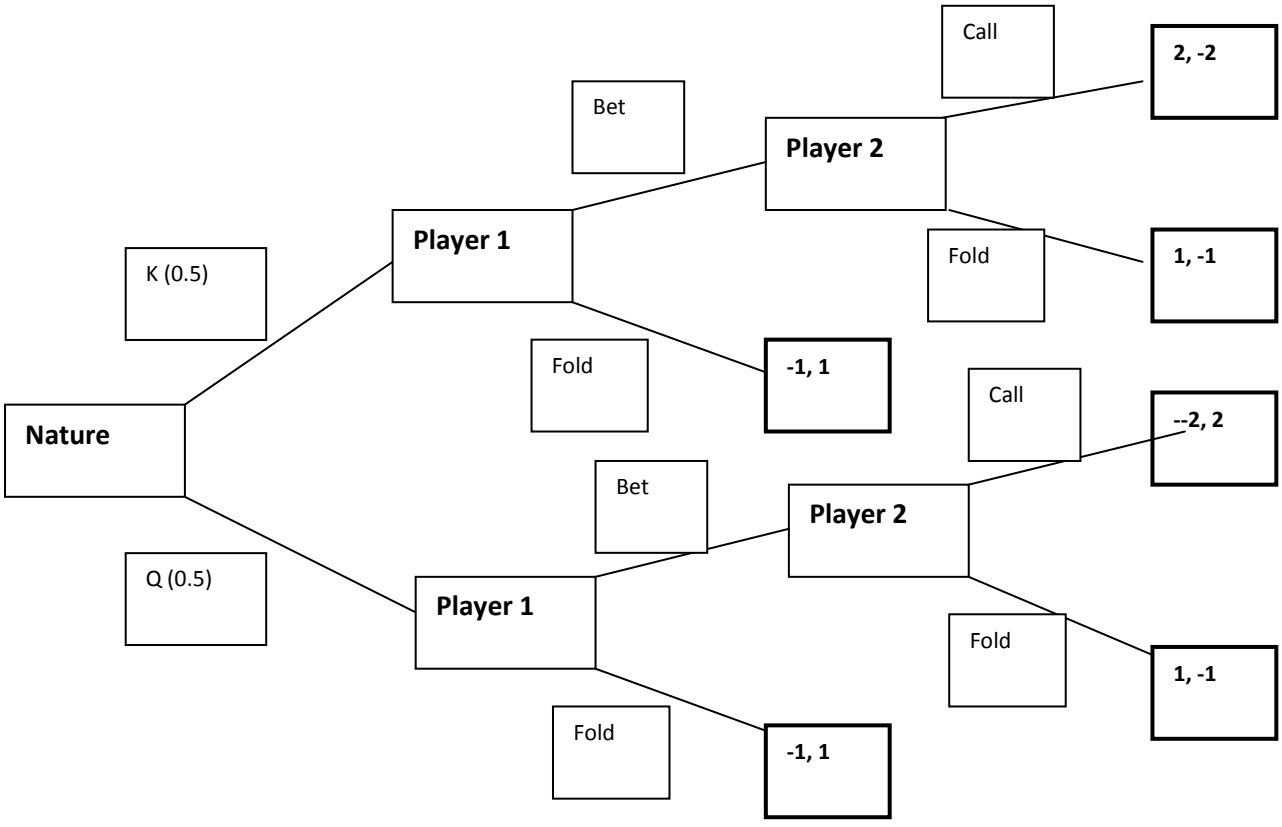
Within the literature review I have explained the rules of the game, and what the possible action sets are for both Player 1 and Player 2. Nature will determine what card is dealt to Player 1, but

for the rest of the game, it is up to Player 1 and Player 2 how the rest of the game will be played. A diagram below shows in the extensive form how the game will be played.

4. Economic Theory

4.1 Stripped Down Poker in Extensive Form

To see how the game is played you can see below the Stripped Down Poker game, represented in an extensive form with the possible payoffs for both players.



This tree shows every possible payoff available to each player no matter how the game unfolds. From this tree diagram you would think that the optimal strategy for Player 1 would be to bet with every single hand, regardless of whether the card is a King or a Queen. However, in doing this it is not actually the best strategy to play. If Player 1 decided to bet for every round, regardless of what card they had been dealt, Player 2 would learn that their best response is to call the hand every time. If both players used these strategies it would result in both players winning 0.5 amount of the time, leaving them with a utility of zero. Because there are two nodes and player 2 only recieves a positive payoff from one strategy; I cannot use backward induction to work out the nash equilibriums. What also prevents me from using the extensive form and backward induction, is that nature will either deal a King or Queen, and there is no way of knowing beforehand what the card is. Due to the fact that Stripped Down Poker is a zero-sum game, if both players chose to bet and call for every hand, it would leave each player with a utility of 0. Therefore, I will represent the game in a strategic form showing all of the possible strategies that each player can play. This will also help me to explain how it is possible for Player 1 to increase their chances of winning and gaining a utility that is higher than zero, and realising that a mixed strategy is the only way to do this.

4.2. Stripped Down Poker in Strategic Form

		Player 2	
		Call	**Fold**
Player 1	**Bet K, Bet Q**	0, 0	1, -1
	Bet K, Fold Q	0.5, -0.5	0, 0
	Fold K, Bet Q	-1.5, 1.5	0, 0
	Fold K, Fold Q	-1, 1	-1, 1

By using strategic form rather than extensive form, it represents the possible payoffs in a more suitable way as it allows me to rule out certain strategies. The table above highlights that if

Player 1 were to bet every time, regardless of what card they had been dealt, they would receive a payoff of zero everytime. This is because Player 2's best response is to call every time. Therefore I believe representing the payoffs in a strategic form is the most suitable way of working out the equilibrium strategies.The calculations for the payoffs for playing each strategy for both players are shown in the appendix.

When calculating the payoff, I had to take in consideration that nautre would deal either a King or a Queen with 0.5 probability. Therefore the payoffs for each strategy are mulitplied by 0.5 and result in the expected payoff for every strategy played, represented in the strategic form above.

What the matrix shows is the payoff from a one off game for all of the possible strategies available to play. However we can see that there is no dominant strategy in the long run because there is a 0.5 chance that the card could be either a King or a Queen. There is no strategy that allows either player to have a positive payoff regardless of whether the card is King or a Queen. Therefore we can rule out that there are any dominant strategies.

Player 1's actions show that BB (Bet King, Bet Queen) and BF (Bet King, Fold Queen) ain positive payoffs, unlike the following strategies who gain negative payoffs: FB and FF. This shows that regardless of what Player 2 does, Player 1 is always better off playing BB or BF and we can therefore rule out that there are any dominant strategies. This therefore leaves us with a 2x2 matrix from ruling out the actions: FB and FF

From looking at the matrix it is easy to see that Player 1 will never choose to play the following strategies: (Fold K, Bet Q) or (Fold K, Fold Q). This is because Player 1 would never be able to gain a positive payoff. In real life it wouldn't make sense to fold with a King because there is no chance that Player 1 would ever lose if they decided to bet. Therefore, folding with a King will gain either a negative payoff or a payoff of zero for Player 1. Because this game is played sequentially it allows Player 1 to play their strategy first, this should mean that they always bet

with a King. From eliminating the two strategies (Fold King, Bet Queen) and (Fold King, Fold Queen) we are left with a 2 x 2 matrix.

	Call	**Fold**
Bet K, Bet Q	0,0	1, -1
Bet K, Fold Q	0.5, -0.5	0, 0

From this table we can see that there is no Nash Equilibrium and I have proven this below:

If Player 1 plays (Bet K, Bet Q), Player 2's best response is to Call

If Player 1 plays (Bet K, Fold Q), Player 2's best response is to Fold

If Player 2 plays (Call), Player 1's best response is (Bet King, Fold Queen)

If Player 2 plays (Fold), Player 1's best response is (Bet King, Bet Queen)

These results show that there is no possible Nash equilibrium for this game of Stripped Down Poker.

4.3. Testing Mixed Strategies

From these best response's of both players we can see that there is not a Pure Nash Equilibrium. Therefore we need to introduce the theory of Mixed Strategy Nash Equilibriums to explain how it would be possible to gain higher payoffs in the game, and not be left with a payoff of zero. John Nash (1951) believed that there should always be an equilibrium in every game and that there is a strategy each player can use to reach the equilibrium. In this game of Stripped Down Poker we have already worked out that there are no pure or dominant strategies, therefore a mixed strategy must be the only strategy that allows an equilibrium to be formed and a positive utility to be gained.

To test whether mixed strategies increase the chance of a higher payoff, I firstly need to test whether the experienced players use a mixed strategy, and that the inexperienced players do not use a mixed strategy. If this is the case, then I will be able to compare my results to test whether it is better to use a mixed strategy or not. By forming a theory to see if experienced players use mixed strategies, I will need to work out a probability for when Player 1 will bluff or fold. I will use: p = the probability that with a Queen they will bluff and p-1 = probability that with a Queen they will fold. Player 2 has the choice of either calling or folding depending on what Player 1 decides to do and I will denote: q = probability that Player 2 will call and q-1 = probability that Player 2 will fold.

4.4. Best Responses

I have already explained that for a mixed strategy to be used, Player 1 and Player 2 must be indifferent between both strategies. If this was not the case then we would see a pure strategy being used; however we have already been able to rule out pure strategies in this game. Because we know that both players' best responses will be based on a mixed strategy, we can find out the equilibrium probabilities of bluffing, folding and calling when using a mixed strategy.

Computing the two mixed strategy equations we can find out the probability of bluffing calling:

Player 1 will bluff: $0p - 0.5(1-p) = -p + 0(1-p) \longrightarrow 1.5p = 0.5 \longrightarrow p = 1/3$

Player 2 will call: $0q + 1(1 - q) = 0.5q + 0(1 - q) \longrightarrow 1 = 1.5q \longrightarrow q = 2/3$

These two equations have shown that Player 1 will bluff with probability 1/3 and will fold with probability 2/3 when using a mixed strategy. Player 2 will call with probability 2/3 and will fold with probability 1/3 when using a mixed strategy also.

If my results show similar probabilities for both players, then I can assume they have used mixed strategies. If they show different probabilities to those shown above, then I can assume they have not been using a mixed strategy.

4.5. Asymmetric Information

With Player 1 bluffing 1/3 of the time with a Queen, it should mean that they can increase their possible payoffs; making the game "unfair" for Player 2. This game is an uncooperative game as each player can only receive a positive payoff when the opponent receives a negative payoff. The reason for this is because Stripped Down Poker is a zero- sum game, so the gain in one player's utility is equal to the loss in the other players utility. In this game if player 1 uses a mixed strategy, then they should be the player who gains a positive utility, leaving Player 2 with a negative utility. Player 2 never knows what card Player 1 has been dealt so this is an example of asymmetric information. It is because of asymmetric information that gives Player 1 the advantage and allows them to trick Player 2 into believing they have a King, even if they have a Queen. Asymmetric information is a very important concept within economics and as a result has been "applied to the study of conflict where agents are differently informed, and to other numerous issues" (Sandler, 2004, pg127). In this study of Stripped Down Poker, Player 2 is at a disadvantage because they have no information about what card has been dealt. The only information they have is the history of Player 1's actions. Therefore due to the nature of this game and asymmetric information being involved, it should allow Player 1 to take advantage and use a mixed strategy, enabling them to gain a higher payoff.

Asymmetric information is seen everywhere in real life situations, varying from political problems to environmental issues. When deciding which strategy would be best to reach the highest utility, sometimes using a mixed strategy will ensure a higher payoff. Therefore this study is going to test whether in real life situations (not just theoretical) a mixed strategy will increase the chance of receiving a higher utility.

4.6. Aim

The results I collect should give me enough data to test this theory and analyse the effects of using mixed strategies. From analysing the experienced players, I should find in the data that if they use mixed strategies Player 1will bluff 1/3 of the time and that Player 2 will call 2/3 of the time. If my results show similar probabilities to this theory then I can assume that mixed strategies are being used. Once I have worked out whether the experienced or inexperienced players are using mixed strategies or not, I will work out the ratio of Kings they received to the number of games they won to judge whether their strategy has increased their chance of winning more games. I use a ratio of Kings to wins because we already know that Player 1 should always bet with a King as they will always win, no matter what player 2 chooses to do. Therefore if they have won more rounds than the amount of Kings they've had dealt, it means they have been successful in bluffing.

I am going to analyse two different groups of people; ten experienced card players and ten inexperienced card players. They will all play ten rounds each so by the end of the experiment I will have one hundred rounds for both groups. I will be able to compare the results for both groups and see whether experience has any effect on choosing a strategy. I will be able to analyse the behaviour of both players and see if it makes a difference to the amount of times they win each round. If the experienced player 1 bluffs approximately 1/3 of the time and wins 33% more times than the amount of Kings they are dealt; then I can assume that mixed strategies help to gain a higher payoff. If however Player 1 bets approximately 1/3 of the time and does not win a significantly higher number of rounds compared to Kings they are dealt; then I will not be able to assume mixed strategies help to gain higher payoffs. From comparing against inexperienced players I will be able to see whether there is any other strategy that could be played, other than a mixed strategy that will increase the chances of winning.

The theory of using mixed strategies to increase the chances of winning and gain a higher payoff could be applied to other aspects of life. I have specifically chosen a simple game of poker using

gambling as I feel the players will certainly use their best strategies to try and win, as they are playing for money. However in real life, there may be other examples of day to day games that have equally important payoffs. An example of this could be with professional sports players and politicians as they will always want to win and will use a strategy that they know will increase their chances of winning.

5. Hypothesis

Experienced Players:

Player 1: will be able to gain a higher payoff from using a mixed strategy technique. Player 1 will bluff 1/3 of the time when they have a Queen and will bluff all of the time when they have a King.

Player 2: will also use a mixed strategy technique to call 2/3 of the time when Player 1 decides to bet.

As a result Player 1 will win 1/3 more times than the amount of Kings they receive.

Inexperienced Players:

Player 1 will not use any form of mixed strategy and will bet all of the time whether they have a King or Queen.

Player 2 will not use any form of mixed strategy and will call all of the time when Player 1 decides to bet.

As a result Player 1 and Player 2 will win half of the games each and neither will gain a higher payoff.

6. Results

To test my theory I am going to work out the probabilities from both the experienced and inexperienced players of when they Bet, Fold and Call. This will allow me to see whether any of the players were using mixed strategies or not based on what the mixed strategy equilibrium probabilities are.

Experienced Players:

Player 1	
Number of Kings Player 1 received	47
Number of Queens Player 1 received	53
Number of times Player 1 Bluffed with a Queen	21
Number of time Player 1 Folded with a Queen	32
Probability that Player 1 Bluffed with a Queen	*0.396*
Probability that Player 1 Folded with a Queen	0.604
Ratio of Kings to Wins	*1 : 1.279*

Player 2	
Number of times Player 2 Called	44
Number of times Player 2 Folded	24
Probability that Player 2 Called	*0.65*
Probability that Player 2 Folded	0.35

Inexperienced Players:

Player 1	
Number of Kings Player 1 received	51
Number of Queens Player 1 received	49
Number of times Player 1 Bluffed with a Queen	36
Number of time Player 1 Folded with a Queen	13
Probability that Player 1 Bluffed with a Queen	*0.735*
Probability that Player 1 Folded with a Queen	0.265
Ratio of Kings to Wins	*1 : 1.09*

Player 2	
Number of times Player 2 Called	75
Number of times Player 2 Folded	12
Probability that Player 2 Called	*0.862*
Probability that Player 2 Folded	0.138

Summary for Experienced Players

	Player 1	Player 2
Prediction	Bluff 33%, Truth tell 67%	Call 67%, Fold 33%
Results	Bluff 39.6%, Truth tell 60.45%	Call 65%, Fold 35%

Summary for Inexperienced Players

	Player 1	Player 2
Prediction	Bluff 50%, Truth tell 50%	Call 100%, Fold 0%
Results	Bluff 73.5%, Truth tell 26.5%	Call 86.2%, Fold 13.8%

7.Analysis

From the data I have collected, I have been able to make some interesting finds. From testing my results against my hypothesis' I have below described and concluded the on the effects on using a mixed strategy.

Firstly looking at the experienced players I can see that both the Player 1's and Player 2's have played the game similar to what I expected. From totalling the number of Kings and Queens dealt I could work out how many times overall a Player 1 bluffed with a Queen. It has shown that 21 out of 53 times they bluffed which as a probability equals 0.396. From my hypothesis and studying the mixed strategy Nash equilibriums I predicted that Player 1 would bluff with a probability of 0.33'. My results showed that in real life the probability is 0.396, so I can therefore say that the results are similar and prove my predictions for experienced Player 1's were correct. Experienced Player 1's do use a mixed strategy to try and increase their chance of winning.

Studying the behaviour of all the experienced Player 2's I worked out the probability of them calling on each round. The results showed that they called the hand 44 out of 68 hands equalling a 0.65 probability. I had predicted that Players 2's would call a probability of 0.66' based on what the mixed strategy Nash equilibriums had equalled. In the study it showed that Player 2's called a hand a probability of 0.65, so I can therefore say that the results are similar and prove my predictions for experienced Player 2's were correct. Experienced Player 2's also used a mixed strategy to try and increase their chances of winning.

Comparing my results against inexperienced players I have been able to test whether mixed strategies are able to increase payoffs. The results for inexperienced Player 1 have showed that altogether they bluffed 36 out of 49 times when they had a Queen. This means that the Player 1's bluffed with a probability of 0.735. This is considerably larger than what would be predicted

if a mixed strategy had been played. Using a mixed strategy the probability that Player 1 would bluff is 0.33'. It is clear to see that the inexperienced Player 1's were not using a mixed strategy as their probability is over double what should be expected of a mixed strategy equilibrium.

There has been a similar observation for the inexperienced Player 2's. Overall the Player 2's called 75 out of 87 hands resulting in a probability of calling equalling 0.862. The mixed strategy Nash equilibrium for this game is a probability 0.66'; therefore showing that the Player 2's called a considerable amount of times more than they would have if they had used a mixed strategy.

The data I have collected has proven that the experienced players have been using mixed strategies when deciding how to play the game. However the inexperienced have shown that they have not been using a mixed strategy as they have bluffed and called with probabilities dissimilar from the mixed strategy equilibriums. As a result I have been able to prove that using a mixed strategy will increase your chances of gaining a higher payoff. I have worked out the ratio of winning a game to the amount of Kings dealt for both the experienced and inexperienced Player 1's. A higher ratio will show that they have been able to bluff well and have won more games than the amount of Kings they have been dealt. This ratio is important because Player 1 has a probability of 1 of winning a hand if they have a King. Therefore is they have won more games than the amount of Kings they have; it means they have been successful in bluffing when they have a Queen.

Ratio of Kings dealt to games won	
Experienced Player 1	1 : 1.279
Inexperienced Player 1	1 : 109

This table shows that the experienced Player 1's were able to win almost a 1/3 more of the time than when they had Kings. This is down to them bluffing almost 1/3 of the time. The Player 1's

bluffed with a probability of 0.396 and won 1.279 games for every King they had. This shows that with using a mixed strategy to decide when to bluff or not they have been able to win more games as a result. This experiment has shown me that if you are willing to take the risk and bluff with a poor hand at the mixed strategy equilibrium probabilities; then you should be able to increase the chance of winning by that same probability. This is the best response Player 1 can use when they know the mixed strategy equilibrium probability for when Player 2 decides to call a hand. In this game the equilibrium probability for Player 2 to call a hand with a mixed strategy is 2/3 and this experiment has gained similar results to this. Both players have been playing their best responses and this has led to Player 1 being able to win more games.

For the inexperienced Player 1's they have not done quite as well. Their ratio of Kings to wins was only 1 : 1.109 showing that their strategy to bluff with a probability of 0.735 did not allow them to gain a considerably higher payoff. It is fair to say that from not using a mixed strategy the inexperienced Player 1's have not done as well as they could have if they had using a mixed strategy. In some cases of the game being played Player 1 would bet every time and Player 2 would call every time. This is an example of them playing their minimax strategies because in an infinite amount of games played; they would both receive a payoff of zero. Stripped Down Poker is a zero-sum game which means that if both players play their minimax strategies, they will not be able to gain a higher payoff than zero. If Player 1 always bets and Player 2 always calls it ends up that both players have a 0.5 chance of winning each game (depending on whether the card is a King or Queen). In order for Player 1 to increase their chances of winning, they need to bluff with a Queen some of the time to try and trick Player 2 into guessing they have a King. This will therefore make Player 2 fold and allow Player 1 to win more games than the amount of Kings they receive.

My experiment has shown some remarkable results. The first thing I found was that the experienced players seemed to be playing a mixed strategy when deciding when to bluff or call. This is because their results showed probabilities of the actions they took to be very similar to

those of the mixed strategy equilibriums I had worked out. As a result the experienced Player 1's were able to increase their chances of winning and as a result for every won 1/3 more games than what they had Kings for. In contrast, the inexperienced players did not seem to be playing a mixed strategy because their probabilities of bluffing or calling were very different to those predicted if a mixed strategy was being used.

Although we have seen that players using mixed strategies can increase their chances of winning; I feel that there needs to be more explanation with how bluffing has helped this to happen. Bluffing is an example of "Lying for Strategic Behaviour" and involves players being misinterpreted by another player's actions with no costs being made to the "lying" player. Lying for strategic behaviour should be used in a situation "in which the parties have predominantly conflicting interests, so that successful deception benefits the deceiver only at the expense of the deceived" (Crawford, 2003, pg134)

7.1Anomalies

My experiment has been able to test my hypotheses and has proved them to be correct. However this experiment may need additional research to ensure that the results are true, fair and reliant. Although I watched and collected data from fifty rounds of experienced players and fifty rounds of inexperienced players; I could carry out the whole experiment again and compare results to make the analysis more verifiable. The more tests I carried out would help me verify and prove that my theory is correct, as long as the results show the same patterns. If however the results differ, then I may need to do more research to test whether my hypotheses' are correct.

7.2Further Study

The aim of my dissertation was to research and analyse the behaviour of people when using mixed strategies in day to day life, and to see whether it could increase their chances of receiving a higher payoff. Studies have already been conducted looking at people playing

football (Chiappori, Levitt and Groseclose, 2002) and tennis (Walker and Wooders, 2001) but I wanted to look at the results of using mixed strategies in gambling. I thought that gambling would show the best results of people using mixed strategies to win more and gain positive payoffs. This is because people will be betting with real money and money is a commodity that no rational player wants to lose. As a result I followed a similar version of the Stripped Down Poker game created by Paul Grimes, so that I could see whether bluffing with a mixed strategy could increase your chance of winning more money. Ideally I would have liked to study how players use mixed strategies when playing Texas Hold'em poker; however this game is extremely complicated to collect data from. There are so many different hands and probabilities of what each player could do during the game Texas Hold'em, consequently I decided to use a simpler version that would be more suitable.

Now that I have collected data and analysed the behaviour of both experienced and inexperienced players; it has created the opportunity to take this study further. If the game was made more complicated with more cards in the deck and slightly more complicated rules, it means that the study would be closer to studying the real Texas Hold'em game. This would then enable a more accurate conclusion about using mixed strategies in more complex games. In order to make Stripped Down Poker more complicated I would do the following: Add 4 Jacks into the pack of cards so now there would be 4 Kings, 4Queens and 4 Jacks. The game would be played just as normal, however if in the end Player 1 has bet and Player 2 has called then; if the card is a King, Player 1 wins, if the card is a Queen, Player 2 wins and if the card is a Jack, then both players split the pot. From introducing more cards and outcomes to the card game, it allows the study to become more true to life. It would still be useful to research the difference in results between experienced and non experienced players, as this will allow the study to become even more reliant and verifiable. If the game can continually be studied and researched further with the game being made more complicated. It will then show results that are more true to life and represent the way people react in real life games, better known as strategic behaviour. From comparing experienced to inexperienced players it will show whether using

mixed strategies will increase the chance of gaining a higher payoff against players using a different strategy. This is because I have already proved from my study that, experienced players use mixed strategies for bluffing close to the equilibrium probability and that inexperienced players do not use a mixed strategy.

8.Conclusion

The aim of this paper was to analyse the behaviour of players using mixed strategies and how using these strategies can reach Nash equilibriums. I also wanted to see that if people in real life situations, could truly be indifferent between a set of different actions. Although studies looking into sports have seen that some players do use mixed strategies, I wanted to see whether other aspects of day to day life could allow people to use mixed strategies giving them higher payoffs. I wanted to research further into how using a mixed strategy over any other strategy could affect the payoffs of players in certain situations and whether it is beneficial to use a mixed strategy. In particular I wanted to see if a mixed strategy could enable a player to receive a positive payoff from a zero-sum game, as here with one player gaining a positive payoff, the other player will receive a negative payoff. As a result I decided to carry out an experiment that studied the strategic behaviour of experienced and inexperienced people playing a game of Stripped Down poker, therefore allowing me to compare strategies and their effects on payoffs.

The results I collected showed that the experienced players used mixed strategies when deciding whether to bluff or not and that the inexperienced players did not use mixed strategies when they decided whether to bluff or not. Bearing this in mind it enabled me to test my hypothesis of whether using a mixed strategy will increase the chances of gaining a positive payoff from a zero-sum game. From my study I found that the experienced players who bluffed using mixed strategies, were able to increase the amount of times they won the game. The results showed that the experienced player 1's bluffed approximately 1/3 of the time which is

what I had worked out from both players' best responses. They also showed that as a result of their bluffing they won almost 1/3 of the time when they had a Queen. In comparison I found that the inexperienced player 1's did not use mixed strategies as they bluffed well over 2/3 of the time and as a result only won 1/10 of the time when they had a Queen. Therefore from this study I can say that using a mixed strategy was the best strategy Player 1 could play to enable a positive payoff. The results have compared using a mixed strategy, to not using a mixed strategy, and in this instance the mixed strategy proved to be the best option for Player 1.

Although my results have allowed me to test my hypotheses, and conclude that mixed strategies in this game were the best strategy; further study may be required to increase the reliability of the conclusion. Although the data I collected enabled me to conduct an interesting analysis of the game and how to increase payoffs through the use of a mixed strategy; some economists may say that this study alone is not verifiable enough to make such conclusions. As a result I have been able to suggest options for further study to test the theories in more depth and make the analysis more valid and reliable.

From this paper I feel that I have reached my aim to analyse the behaviour of players using mixed strategies and how it affects their utilities. I have not only proved that mixed strategies can be used in real life situations, but that they can also increase the chances of gaining higher payoff in certain games. I have also been able to prove that in some situations, a mixed strategy is the only way of reaching a positive payoff, particularly in zero-sum games.

9.Appendix

BBC: Player 1= 0.5(2) + 0.5(-2) = 0

Player 2= 0.5(-2) + 0.5(2) = 0

BFC: Player 1= 0.5(2) + 0.5(-1) = 0.5

Player 2= 0.5(-2) + 0.5(1) = -0.5

FBC: Player 1= 0.5(-1) + 0.5(-2) = -1.5

Player 2 = 0.5(1) + 0.5(2) = 1.5

FFC: Player 1= 0.5(-1) + 0.5(-1) = -1

Player 2= 0.5(1) + 0.5(1) = 1

BBF: Player 1= 0.5(1) + 0.5(1) =1

Player 2= 0.5(-1) + 0.5(-1) = -1

BFF: Player 1= 0.5(1) + 0.5(-1) = 0

Player 2= 0.5(-1) + 0.5(1) = 0

FBF: Player 1= 0.5(-1) + 0.5(1) = 0

Player 2= 0.5(1) + 0.5(-1) = 0

FFF: Player 1= 0.5(-1) + 0.5(-1) = -1

Player 2= 0.5(1) + 0.5(1) = 1

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