For a country, or stock market, of your choice explore the evidence for or against the Capital Asset Pricing Model (CAPM). [USA]

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Final Year Project

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Abstract

This paper examines the Capital Asset Pricing Model (CAPM) for the US stock market. It consists of two complementary parts. In the first part, the CAPM theory is introduced and followed by main important empirical studies and debates of the validity of CAPM in US market. In the second part, after the discussion of some conceptual and technical issues, two different tests are conducted to test the US stock market in two periods, 2002 to 2006 and 1997 to 2001. Two opposite results are found for these two periods.

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Section 1 Introduction

CAPM is a model explaining the relationship between asset's return and its risk with market in equilibrium. The model originates from a portfolio selection theory of Markowitz (1952), known as Mean-Variance Analysis (MVA). It then is developed by Tobin (1958) who introduced the idea of linear efficient set and separation theorem. Finally, the relationship between asset return and risk were developed by Sharpe and Lintner in 1965. After the CAPM was introduced, researchers and scholars have made various empirical studies on it. For the US market, two waves of studies are especially well known: i) early empirical studies in 1960s and 1970s and ii) the second cycle studies after late 1970s. The model was supported by the first waves but rejected by the second. There are heavy debates regarding the results of these studies. Therefore, no clear conclusion was made.

The main purpose of this paper is to find empirical evidences in recent years in the US stock market for or against the CAPM. Specifically, two tests are conducted by using the US data in period 1997 to 2006. They are the two-stage test and the conditional-relationship test. This paper is organized as following. Section 2 will introduce the CAPM theory and same background behind. Section 3 reviews some main tests and results for the US market from the two waves of studies. Section 4 explains the testing techniques and discusses some conceptual and statistical issue in the tests. The choices of data and the results of two tests are presented in section 5. And finally conclusion is made in section 6.

Section 2 Theory and Derivations

2.1 Mean-Variance Analysis

The MVA is an important foundation of CAPM. It assumes that when investors are selecting portfolios, they only care about two aspects of assets: the expected return and variance of the return. This setting means that other aspects of securities, like skewness and kurtosis of the distribution, are not taken into account by investor. MVA also assumed that expected return (wealth) is good for investor and variance of return (as a kind of risk) is bad.[1] All in all, under these settings, investors will only select among assets (or portfolios) that provides maximum expected return for a given level of variance or provides minimum variance for a given expected return. Portfolios of this kind are said to be Mean-Variance (MV) efficient, otherwise are inefficient.

[1] There are also other assumptions (Jensen 1983): i) Investors select portfolio at time t-1 that produces random return R at t. ii) All assets are perfectly divisible and perfectly liquid. iii) There are no taxes. iv)

In a market where exists n-risky assets, given the expectations of return, variances and covariances of all assets, one can derived all the MV efficient portfolios. This set of portfolios constitutes what is called the efficient frontier. The frontier is derived from the following. Suppose there are n risky assets. The investor has expected return of assets j μ_j, covariance between two assets σ_{ij}. Then using the probability theory we can derive the expected return and variance of a portfolio as the following formulas:

$$\mu_p = \sum_{j=1}^{n} a_j \mu_j$$

$$\sigma_p^2 = \sum_{j=1}^{n}\sum_{i=1}^{n} a_j a_i \sigma_{ij}$$

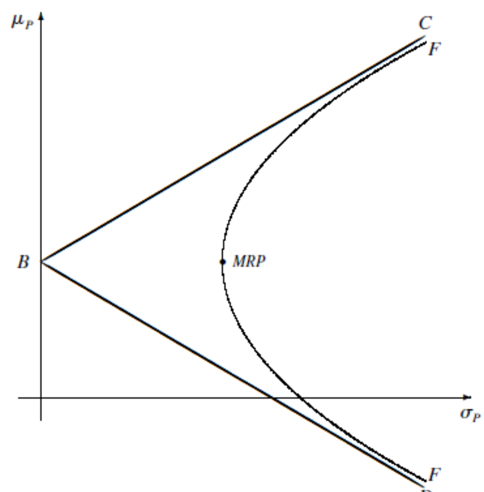
Where a_j is proportion of portfolio invested in asset j with $\sum_{j=1}^{n} a_j = 1$. Given a value of the expected portfolio, we can choose portfolio proportions, $a_1, a_2, ...a_n$ to

$$\text{Minimize } \sigma_p^2 = \sum_{j=1}^{n}\sum_{i=1}^{n} a_j a_i \sigma_{ij}$$

$$\text{Subject to } \mu_p = \sum_{j=1}^{n} a_j \mu_j \text{ and } \sum_{j=1}^{n} a_j = 1$$

By sorting this optimal problem, we will obtain the proportions and hence the minimal variance for that expected return. By doing this for different values of expected return, the efficient frontier will be traced out. The frontier will looks like the following.

Figure 2.1The Efficient Frontier



Source: R. E. Bailey (2005)

The point MRP represents the minimal risk portfolio. Only the FF curve on the upper part of MRP is the efficient frontier. Portfolios on this part are efficient. Those on the other part and those inside the FF curve are inefficient. Portfolios outside FF curve are infeasible.

2.2 The capital market line and the separation theorem

When there are n risky assets and a risk-free asset, the set of efficient portfolio is a straight line. This set of efficient portfolio can be derive from the following optimization problem: for a given value of μ_p, choose portfolio proportions, $a_1, a_2, \ldots a_n$, to

$$\text{Minimize } \sigma_p^2 = \sum_{j=1}^{n}\sum_{i=1}^{n} a_j a_i \sigma_{ij}$$

$$\text{Subject to } \mu_p = \sum_{j=1}^{n} a_j \mu_j \text{ and } \sum_{j=1}^{n} a_j = 1$$

Solve this optimal problem for each value of μ_p trace out a linear efficient set.

The derivation can be seen in a more intuitive way. Let us consider a combination of any risky portfolio and the risk-free asset. Suppose we have a risky portfolio g, the expected return of which is $E(R_g)$. Then the expected return and combined portfolio P is

$$E(R_p) = (1-a)R_f + a\,E(R_g)$$
$$\text{or } E(R_p) = R_f + a(E(R_g) - R_f) \quad (2.1)$$

The variance and standard variance of P are

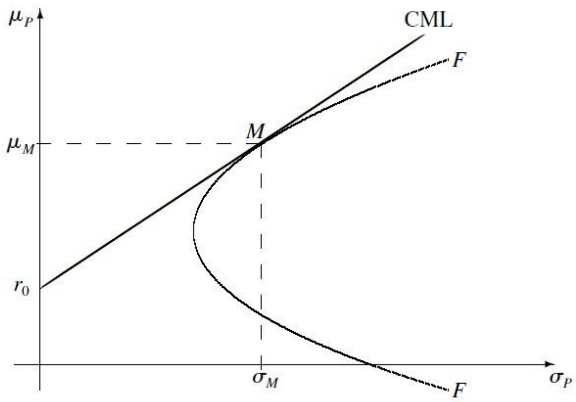
$$\sigma_p^2 = a^2\ \sigma_g^2$$
$$\text{and } \sigma_p = a \quad \sigma_g \ (2.2)$$

Define $\theta = \frac{(E(Rg) - Rf)}{\sigma_g}$ and subtract a in (2.2) into (2.1) we can obtain the expected return of portfolio P as:

$$E(R_p) = R_f + \theta\sigma_p$$

In a mean-variance diagram, this equation is a straight line goes through the risk free asset and portfolio g with slope equal to $\theta = \frac{(E(Rg) - Rf)}{\sigma_g}$. Different points in the line are portfolios with different proportions invested in the risk-free asset and the risky portfolio g. The slope is known as Sharpe ratio which measures how much excess return compensate for one unit risk (standard deviation). Obviously, the higher is the Sharpe ratio, the higher will be the excess return of a portfolio given the risk level. Hence the efficient portfolios must be portfolios that have the highest Sharpe ratio. Since θ is the slope, the efficient frontier is represented by the steepest line that tangent the efficient frontier of risky assets. It is called the capital market line (CML).

Figure 2.2 The Capital Market Line



Source: R. E. Bailey (2005)

Portfolios lying on this line are all MV efficient. The point on the risk-free asset is the portfolio which consists only the risk-free asset. The tangent point is the portfolio contains only risky asset. The derivation of the efficient frontier hints a theorem known as the separation theorem. This theorem says: ‘‘the expected return and the variance of every portfolio in the efficient frontier can be obtained by combining any two other portfolios on the frontier.’’ Hence, the process of choosing portfolios can be broken into two stages: ‘‘first, the choice of a unique optimum combination of risky assets; and second, a separate choice concerning the allocation of funds between such a combination and a single riskless asset’’ (Sharpe 1964, pp.426)

2.3 Derivation of the CAPM

Based on the asset selection theory of Markowitz and the development of Tobin, Sharpe, in 1964, and Lintner, in 1965, independently derived the model. In here, only the derivation of Sharpe will be represented.

First, it is assumed that there is homogeneous belief: all investors have the same expectations of returns, variances and covariances of all assets, then all investors will face the same FF frontier and therefore choose the same unique optimum combination of risky assets. Hence the tangency portfolio is the market portfolio.

Then let us consider a combination of the market portfolio and any portfolio or individual asset i. The expected return of this combined portfolio, denoted as Z, is

$$E(R_z) = (1-a)E(R_m) + a\,E(R_i) \quad (2.3)$$

The standard deviation therefore is

$$\sigma_z = [(1-a)^2\sigma_m^2 + a^2\sigma_i^2 + 2a(1-a)\rho_{mi}\,\sigma_i\sigma_m]^{1/2} \quad (2.4)$$

Where $\rho_{mi} = \frac{\sigma_{mi}}{\sigma_i\sigma_m}$ is the correlation between the market portfolio and asset i (the standardized covariance σ_{mi}). The combination of two risky assets should be a curve that goes thought the market portfolio and asset i in mean variance graph as long as $-1<\rho_{mi}<1$

and $\rho_{mi} \neq 0$. This combination curve does not intersect the minimum variance frontier because when there is no risk-free assets, portfolios on the minimum variance frontier are efficient and no portfolios would lie on the left of it. Therefore the curve is tangent with the minimum variance frontier at the point of market portfolio. Hence the slopes of this curve and the frontier are equal at point M.

The slope of the minimum variance curve, as we know from the previous sub-section, is $\theta = \frac{(E(Rm)-Rf)}{\sigma_m}$. The slope of the curve for the combined portfolio Z is given by

$$\frac{\partial Rz}{\partial \sigma_z} = \frac{\frac{\partial Rz}{\partial a}}{\frac{\partial \sigma_z}{\partial a}}$$

Take the derivative of (2.3) with respect to a, we have

$$\frac{\partial Rz}{\partial a} = -E(R_m) + E(R_i)$$

Take derivative of (2.4) with respect to a and rearrange, we can get

$$\frac{\partial \sigma_z}{\partial a} = \frac{1}{\sigma_z}[-(1-a)\sigma_m^2 + a\sigma_i^2 + (1-2a)\rho_{mi}\,\sigma_i\sigma_m]$$

Because at point M, the portfolio Z contains only the market portfolio, a = 0 and $\sigma_z = \sigma_m$. And because $\rho_{mi}\,\sigma_i\sigma_m = \sigma_{mi}$, $\frac{\partial \sigma_z}{\partial a}$ reduces to

$$\frac{\partial \sigma_z}{\partial a} = \frac{[\sigma_{mi} - \sigma_m^2]}{\sigma_m}$$

Therefore

$$\frac{\partial Rz}{\partial \sigma_z} = \frac{[-E(Rm) + E(Ri)]\sigma_m}{\sigma_{mi} - \sigma_m^2}$$

Let the two slopes equal, we get

$$\frac{[-E(R_m) + E(R_i)]\sigma_m}{\sigma_{mi} - \sigma_m^2} = \frac{(E(R_m) - R_f)}{\sigma_m}$$

$$E(R_i) = \frac{\left(E(R_m) - R_f\right)(\sigma_{mi} - \sigma_m^2)}{\sigma_m^2} + E(R_m)$$

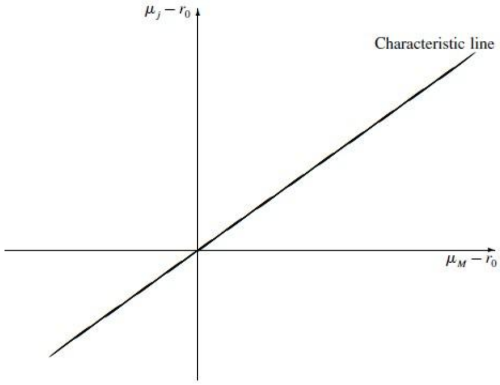
$$E(R_i) = (E(R_m) - Rf)\frac{\sigma_{mi}}{\sigma_m^2} - E(R_m) + R_f + E(R_m)$$

Hence

$$E(R_i) = R_f + \beta_i(E(R_m) - Rf) \text{ where } \beta_i = \frac{\sigma_{mi}}{\sigma_m^2}$$

This is relationship is the central prediction of CAPM. It can be looked as a function between expected return of asset i, and the expected market return, with slope equals to β_i. In this case it can be presented as the so called characteristic line:

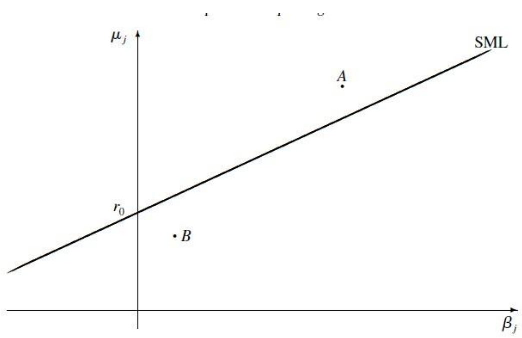
Figure 2.3 The Characteristic Line



Source: R. E. Bailey (2005)

More commonly, the relation is interpreted as a relation between $E(R_i)$ and β_i. This relationship can be graphically presented as the security market line (SML) below.

Figure 2.4 The Security Market Line



Source: R. E. Bailey (2005)

Beta is a measure of risk and, since$\beta_i = \frac{\sigma_{mi}}{\sigma_m^2}$, the CAPM states that the risk of an asset is its covariance with the market portfolio. Because it can be showed that in a portfolio contains N assets, the variance of a individual asset would not contribute to the variance of that portfolio as it can be diversify to zero and the only thing that contributes to the variance of the portfolio is its covariance with other assets.

The model gives the following testable implications according to Fama and MacBeth (1974)

1) The relationship between the expected return on a security and its risk, beta, is linear.
2) Beta is a complete measure of the risk of a security, hence other variable will not have explanatory power and intercept should equal to risk-free return.
3) Higher risk should be associated with higher expected return. That is $E(R_m) - R_z > 0$

Soon after the born of CAPM, a modified version of CAPM was derived by Black. In black's CAPM, the assumption that investors can borrow or lend any amounts is released (the risk-free asset dose no exist). Black derived a similar prediction. It says that in equilibrium the expected return on any asset i is

$$E(R_i) = R_z + \beta_i(\mathrm{E}(R_m) - R_z),$$

where R_z denote the return on the assets that has a beta equal to zero. A zero beta means that its return is uncorrelated with the market portfolio. The testable implications of this model are similar to the pervious three except that the intercept is not required to be risk-free rate.

Section 3 Review of Empirical Tests

3.1 Early Empirical Tests on US Markets.

Early Empirical Tests are in favour of the model to some degree. In order to implement tests, expected variables in the CAPM equation are replaced by their realized counterpart. The model used to test is

$$R_i = R_f + \beta_i\left(R_m - R_f\right) + \varepsilon_i$$

Where R_i and R_m are the realized return of assets i and market portfolio respectively. ε_i is a disturbance term with expected value equals to zero. The justification of this transfer will be discussed in section 4.1.1.

The earliest empirical study of CAPM is the study of Linter in 1965. Linter proposed a widely used, two-stage procedure for testing the CAPM. In the first stage, time series regressions are run: monthly returns of each asset are regressed on the monthly excess market returns. By doing so, he obtains betas for each asset. In the second stage, the average returns of each asset are calculated and the following cross-sectional regression is run:

$$\overline{R}_i = a_1 + a_2\beta_i + a_3 S_{ei}^2 + \varepsilon_i$$

Where S_{ei}^2 is the variance of residuals in the regression for asset i in the first stage. It is included to test if the beta is the only explanatory variable (it should has no effect on return). He found a positive and significant coefficient of beta, which is consistent with the prediction. However, this coefficient is smaller than the excess market return. The intercept which should equal to risk-free rate, is also much lower. Moreover a_3 is significant.

Douglas (1969) and Miller and Scholes (1972) both replicate and extend the study of Lintner by employing a larger sample. And both found very similar results. Miller and Scholes also analyze various potential statistical problems and come to conclusion that these tests are subjected to several statistical problems like omitted variable bias, heteroscedasticity, autocorrelation, and measurement error in variable. Thus little can be said about these test results. These statistical problems will be discussed in more detail in section 4.3

Fama and MacBeth (1972) (FM) developed a much more sophistic method for testing. In order to tackle the beta measurement error problem, they conducted the test by using 20 portfolios constructed from securities on NYSE. The portfolios are constructed in a way that minimizes the measurement error of the portfolio beta. Specifically, they divided 15-years period in to three 5-years sub-periods. In the first sub-period, they estimated beta of each stocks by the first stage procedure and then grouped stocks into 20 portfolios according to the ranking of betas. Next, they re-estimated the beta of each portfolio by using the data in the second 5-years sub-period. This ensures that the estimated betas of each portfolio are unbiased.[2] Finally, they regress returns of 20 portfolios in the third sub-period on their betas. (Brealey and Mayers 1988) The exact regression they used is following:

$$R_{it} = \gamma_{0t} + \gamma_{1t}\beta_i + \gamma_{2t}\beta_i^2 + \gamma_{3t}\overline{\sigma}_{ei} + \varepsilon_{it}$$

Where $\overline{\sigma}_{ei}$ is the average residual standard deviation for each portfolio i that obtained from the time series regression used to estimate beta. It is included to test whether risks other than beta plays no role in determining the asset return. β_i^2 is the squared beta of each portfolio i. It is used to test linearity. This cross-sectional regression is conducted separately for each month of the third sub-period thus they obtain those coefficients for each month. They then calculate for each coefficient the average value of all months and employ the t-test to examine whether they are significant.

If the model holds, those coefficients should be such that 1) $E(\gamma_0)=R_f$, 2) $E(\gamma_1)=E(R_m)-E(R_f)>0$, 2) $E(\gamma_2)=0$, and 3) $E(\gamma_3)=0$. Their result failed to show that $E(\gamma_0)$ equal to R_f but this does not contradict the Black CAPM. In addition, $E(\gamma_1)$ is positive and significant while $E(\gamma_2)$ and $E(\gamma_3)$ are not significantly different from zero. Thus, this study supports the CAPM by showing that there is a positive relation between returns and betas while β_i^2 and residual variance do not explain the variation in return.

Black, Jensen, and Scoles (1972)(BJ&S) employed a similar method. They tested the CAPM as well as the zero beta model of Black by using time-series and cross-section method. In the cross-sectional test, because the period (from 1926-1966) they used is long, they conduct the test for several sub-period and the whole period. Whereas their results vary across the various sub-periods, for the whole period, they found a strongly positive relationship between average returns and beta. Moreover, beta explains most of the variation in return as the R-square 0.98. However the slope and the intercept are

[2] E.g. Because low betas estimated in the first five-years sub-period are likely to be realized betas that is lower than the true value in that period, the beta of portfolio calculated by using data in that period will be lower than the true beta of that portfolio.

significantly different from what is predicted. In the time series test, where the excess returns of each portfolio are regressed against the excess market return, they found that the intercepts of the 10 regressions (one for each portfolio) mostly insignificantly different from zero. This result is consistent with prediction. However, the change in the intercept is not random: it is negative for the 5 high risk portfolios and positive for the others. Hence the strict CAPM is rejected. However the Black CAPM, in which the intercept is not a concern, is not. BJ&S showed that the positive relation in the zero-beta model is hold hence suggests that the Black model fits the empirical results better.

3.2 The Second Cycle of Tests

The second cycle of studies provides some strong evidences against the CAPM. First it has been found that the returns on stocks of firms with relatively small market value are above the returns predicted by CAPM, and the opposite holds for large firms. Banz (1981) is one of first studies that found this well-known small firm effect. Banz regresses returns of 25 portfolios on their betas and their relative market values for 6 sub-periods and a whole period from 1926-1975. He finds that 4 out of 6 sub-periods and the whole period give a negative and significant coefficient of the market size. Thus the beta dose not capture the whole risk.

In the same year, Reinganum(1981) published a paper regarding the same issue. He analyzes various anomalies that contradict CAPM. He finds that when portfolios are grouped by either the price/earning(P/E) ratio or by market value, portfolios with low P/E or market value tend to have returns that are higher than the prediction. He further finds that when P/E and market value are run together with beta, the effect of P/E become negligible. Hence he concludes that firm’s size, which associated with P/E and market value, is likely the missing factor, a conclusion that is consistent with Banz.

Another prominent study is the study of Fama and French (1992) (FF). Fama and French regress returns on various combinations of explanatory variables, including beta, size, earings to price ratio(E/P), book to market ratio(B/M), debt to equity ratio. Analyzing firstly the beta and market size, they find that the coefficient of beta is negative and insignificant while the market size is significant and negative, consistent with the size effect found in other study. Moreover, when beta is left alone as the only variable, it does even worse: its t-statistic is even smaller indicating that beta dose no play a role in explaining returns at all. This contradicts the early study of FM. The explanation suggested by the authors is that the different time period makes the result different. One of the sample periods may be not suitable to represent the population. Beside, in contrast to the negligible and insignificant role of beta, Fama and French show that when the size, B/M and E/P are regressed alone, they all play significant role in explaining returns.

When these three variables are regressed together, the size and B/M seem to absorb the effect of E/P. Therefore they conclude that size and book to market are crucial variables in explaining returns, on the other hand, beta has little explanatory power.

3.3 Responses

After those negative evidences were published, doubts were casted regarding different aspects of these studies. Amihud et al (1992) suggest that previous studies like FF are subjected to "survivorship bias'' and this bias seriously affected the results. Survivorship bias occurs because stocks of firms that are merged or firms that go bankruptcy during the testing period are naturally not included in the studies. Therefore, among the high risk stocks, only those survived are recorded. This will make the average returns of stocks in that risk level higher than the return they actually are. Moreover, Amihud et al also argue that the test did not take into account statistical problems like heteroskedasticity and autocorrelation. Their test results may not valid. Thus they develop a method that take the bias into account and employ a GLS methodology to tackle the statistical problems. They re-test the model using the same time period as FF and find that the positive relationship exist and is significant.

Pettengill et al.(1995) argue that many tests that use realized return are not directly testing the CAPM because the implied positive relationship is about the ex-ante terms while these tests are testing positive relation upon realized variables. A negative relation between realized assets returns and betas (that is a negative excess market return)can not disprove the model, because the expected excess market return in the model implies that realized market return must, with some probability, be below the risk-free rate.(If investors were certain that market return would always be greater than the risk-free rate, no one would hold risk-free securities.) Pettengill et al.(1995) therefore suggest that while there is a positive relationship between realized returns and betas in up-market time (when realized excess market return is positive), an inverse relationship should exist in down-market time (when realized excess market return is negative).

To test the CAPM, Pettengill et al. first test the systematic conditional relationship between betas and realized returns (positive coefficient in good time and negative coefficient in bad). Secondly they test whether there is a positive long run trade off relationship. A positive long run trade off would indicates that high beta assets, on average, earn higher returns than low beta assets. If this is so, the CAPM, in some degree, is supported. Consistent with their expectation, firstly their results show a significant positive beta-return relation when market return excess risk-free rate and a significant negative relation when market return below. Secondly, they found that the excess market returns on average are significant positive for the total period and for most of the sub-periods. Moreover, two coefficients of betas (each one catches the positive relation between beta and asset returns in up market and negative relation in down market)are symmetric. These mean 1) on average there is an up market 2) the amounts of gain for per unit beta risk in good time and the amounts of loss in bad time are the same. Hence it suggests that high beta assets, on average, earn higher returns than low beta assets.

Response to the poor results of FF, Black (1993) suspected that the size effect exist in FF's study is due to data mining. Namely, the result is sample specific. Because when FF extent the time period used by Benz to 1990, they found no size effect exist in period 1981-1990. Moreover, as discussed in Jagnannathan et al(1995), Kothari et al (1995) points out that the standard errors of the coefficients on beta in FF's test are too high therefore their estimation of beta is imprecise. And in fact, Amihud et al(1992) finds that when a more efficient estimating method is used, beta is positive and significant. One more explanation for the small firm effect is the investment horizon. Most of empirical studies employed monthly data, while many studies (Levy 2011 p225) showed that the average holding period of stocks is more than one year. The use of monthly, as showed theoretically and empirically in Levhari and Levy (1977), will lead to biased results.

Section 4 Issues and Techniques for testing the CAPM

4.1 Conceptual Issues in the Tests

4.1.1 Ex ante & ex post

The CAPM predictions are stated in terms of ex ante expected returns and beta:

$$E(R_i) - R_f = \beta_i [E(R_m) - R_f] \quad (4.1)$$

Where $E(R_i)$, $E(R_m)$ are expected returns of asset i and market portfolio. They are naturally not observable. In order to implement tests, the fair game hypotheses should be made to transform the model from ex ante to ex post.

In view of time series regression, it is assumed that the observed return in time t is equal to the expected return plus the stochastic disturbance:

$$R_{it}=E(R_{it})+ e_t$$
$$R_{mt}=E(R_{mt})+ w_t$$

Where ε_t and w_t are i.i.d. Substitute (2)(3) into (1), we have the ex post model:

$$R_{it}=R_f+\beta_{im}(R_{mt}-R_f) + \mu_t$$

where $\mu_t = \beta_{im} w_t+ \varepsilon_{t,}$. The fair game hypotheses in ex post CAPM are:

$$E(\mu_t)=0$$
$$Cov(\mu_t ,R_{mt})=0$$

In view of cross-sectional regression, the real return of each asset is also assumed to be drawn from the expected return with stochastic disturbance ε_j.Then the ex post CAPM is

$$R_j = \gamma_0 + \gamma_1\beta_j + \varepsilon_j$$

With the interpretations $\gamma_0 = R_f$,$\gamma_1 = [E\,(R_m) - R_f\,]$ and $E(\varepsilon_t|\,\beta_j)=0$ Hence the use of realized returns are justified.

4.1.2 Roll's critique

In the model, the market portfolio is the portfolio consists of all risky assets available , including stocks, bonds, real estate, and human capital etc. Roll (1977) in a well-known paper underlined that testing the CAPM is testing whether the market portfolio is Mean-Variance efficient. Empirical studies which use proxy actually only test the efficiency of the proxy rather than test the CAPM. Since the real market portfolio is never possible to be observed, strictly speaking, testing CAPM is never possible.

Fortunately, as presented by Campbell et al. (1997), some studies show that as long as the correlation between market proxy return and true market return exceeds about 0.70, then the rejection of the CAPM with a proxy implies also the rejection of CAPM with the true market portfolio. And a good proxy does not necessary have to include a wide range of assets. Studies like Stambaugh(1982) showed that inferences are similar whether one uses a proxy including only stocks or including also bond ,real estate and human capital. Thus Roll's concern is not an empirical problem.

4.2 The Two-stage Test

This paper will use the two-stage procedure as many of the early tests did. The procedure is following. Suppose I have N assets and each asset has T returns for T time period. I also have T market returns. In the first pass, following time series regression is run for each asset:

$$R_{it}=R_f+\beta_{im}(R_{mt}-R_f) + \mu_t$$

R_{it} for asset i is regressed against the excess market return,$(R_{mt}-R_f)$(only R_{mt} when testing the Black CAPM). By doing this for each asset, I obtain N coefficients, which are betas, for each asset.

In the second pass, the average returns of assets are run against their betas:

$$R_j = \gamma_0 + \gamma_1\beta_j + \varepsilon_j (4.1)$$

By doing so, we obtain the estimated γ_0 and γ_1. Thereafter the standard t- test will be used to test the CAPM prediction that γ_0 will equal to risk-free rate and γ_1 will be positive and equal to excess market return.

For this testing procedure to be valid, we should assume that all the standard econometric-assumptions are held. In particular, in the first regression, I assume that μ_t is uncorrelated with R_m so that the estimated beta can be unbiased. However, it should be admitted that this assumption is hardly held because the market return is measured with error. The same assumption should be assumed for the second regression (the residual should be uncorrelated with beta). Moreover betas obtained from the first pass should be the true beta so that estimator of γ_0 and γ_1 can be unbiased. Furthermore the residual in the second regression should have constant variance and uncorrelated across assets so that the estimated coefficients will have asymptotically normal distribution and the standard t-test therefore can be used. The last two assumptions are also not likely to hold. Fortunately, some techniques can be used to correct the problem caused by the failure of these two assumptions. These techniques will be discussed in the next section.

One more assumption of the CAPM is explicitly made in this test. It assumes that the true betas of each asset and relationship between returns and betas in population are constant during the time period. This assumption is not realistic especially for a long period of time because the relationship (risk premium) and the betas (risk of assets) are likely to change as the economic environment changes. However, for a short period of time, as will be discussed in 5.1.2, making this assumption is not too controversial.

4.3 Econometric issues in the two-pass Test

4.3.1 Heteroscedasticity and Autocorrelation

As mentioned above, one econometric issue of the test is that the disturbance terms in (4.1) are likely to be heteroscedastic and correlated. This causes two problems. Firstly, the OLS estimator would be inefficient. It would have higher variance compare to other estimators. This means that when we are testing the estimated coefficient, there is a wider range in which we can not to reject the null hypothesis. The possibility that we fail to reject null hypothesis when the alternative hypothesis are true is high. Second, the variances of coefficients would be biased if the standard OLS formula is used to calculate the variances. Hence the t-statistic would be biased and the T-test is invalid. (Wooldridge (2009))

One solution to the heteroscedasticity problem is to use robust standard error. A general solution to both heteroscedasticity and autocorrelation problem is to use the General Least Square estimation. This method is taken by Sauer & Murphy (1991) and Mankin & Shaparo (1986). The GLS procedure basically is to modify variables such that they can form a regression without heteroscedasticity and autocorrelation. These modified variables are then regressed by using the usual OLS procedure. Since the conditions of

Gauss-Markov theorem have been induced to hold by the transformation, the estimators from GLS are BLUE.

To see this, rewrite (4.1.1.5) in matrix form as

$$\mathbf{R}=\mathbf{X}\mathbf{b}+\mathbf{u} \quad (4.2)$$

Where $\boldsymbol{R} = \begin{bmatrix} R1 \\ \vdots \\ Rn \end{bmatrix}$, $\boldsymbol{b} = \begin{bmatrix} \gamma 0 \\ \gamma 1 \end{bmatrix}$, $\mathbf{X} = \begin{bmatrix} 1 & \beta 1 \\ \vdots & \vdots \\ 1 & \beta n \end{bmatrix}$, $\mathbf{u}=\begin{bmatrix} \varepsilon 1 \\ \vdots \\ \varepsilon n \end{bmatrix}$

Assuming that Var($\mathbf{u}$)=E($\mathbf{u}\,\mathbf{u}'|\mathbf{X}$)=$\sigma^2\boldsymbol{\Omega}$, where $\boldsymbol{\Omega}$ is a $n\times n$ symmetric and is positive definite with diagonal values that are not constant (heteroscedasticity) and the off-diagonal values are not zero (autocorrelation). Suppose that the variance covariance matrix of the disturbance is known. The inverse of Ω can be decomposed as $\boldsymbol{\Omega}^{-1}=\mathbf{K}\mathbf{K}'$. Multiply both sides of (4.2) by $\mathbf{K}$ yields:

$$\mathbf{R}^*=\mathbf{X}^*\mathbf{b}+\mathbf{u}^*$$

Where $\mathbf{R}^*= \mathbf{KR}$, $\mathbf{X}^*= \mathbf{KX}$, $\mathbf{u}^*= \mathbf{Ku}$.
Note that E ($\mathbf{Ku}\,|\mathbf{X}$)=$\mathbf{K}$E($\mathbf{u}\,|\mathbf{X}$)=$\mathbf{0}$, the variance-covariance matrix of residual is

$$\text{Var}(\mathbf{Ku})=\text{E}(\mathbf{Kuu'K})=\mathbf{K}\text{E}(\mathbf{uu'})\mathbf{K}= \sigma^2\mathbf{K\Omega K}= \sigma^2\mathbf{I_n}$$

$\mathbf{u}^*$ is then homoscedastic and have no autocorrelation.
$\mathbf{b}$ is obtain by the OLS:

$$\begin{aligned}\mathbf{b}&=(\mathbf{X}^{*\prime}\,\mathbf{X}^*)^{-1}\,\mathbf{X}'\,\mathbf{R}^* \\ &=(\mathbf{X'K'KX})^{-1}\,\mathbf{X'K'KR}\end{aligned}$$

It can be showed that the variance of this estimator is smaller than the original OLS estimator therefore the GLS estimator is more efficient.

Unfortunately this GLS method is not easy to implement as Ω is unknown and must be estimated. To estimate the variance-covariance matrix, it is often assumed that the matrix takes a certain special form. If this form is not right, heteroscedasticity may still exist. Moreover with an estimated covariance matrix, the GLS can be largely inefficient in small sample. Hence this paper will just used robust and assumed that there is no autocorrelation. (Huang & Litzenberger (1988))

4.3.2 Measurement error in beta

In the cross-sectional regression, the explanatory variable, beta, is not observable. It is usually estimated either according to the formula$\beta j = \frac{cov(\text{Rj,Rm})}{Var(Rm)}$, or by the first stage regression of the two-stage test. Although the estimated beta could be unbiased but it inevitably contain measurement error which will cause the OLS estimators downward biased and inconsistent. To see this, suppose that the estimated beta is unbiased such that $\hat{\beta}_j=\beta_j+\omega_j$ where ω_j is i.i.d. The P lim of the OLS cross-sectional estimate is:

$$\text{P lim } \hat{\gamma}_1 = \frac{Cov(R_i, \hat{\beta}_i)}{Var(\hat{\beta}_i)}$$
$$= \frac{Cov(\gamma_0 + \beta_i\gamma_1 + \varepsilon_i, \beta_i + \omega_i)}{Var(\beta_i + \omega_i)}$$
$$= \gamma_1 \frac{Var(\beta_i)}{Var(\beta_i) + Var(\omega_i)} < \gamma_1$$

There are mainly two possible solutions to this problem. One is to use grouped portfolios rather than individual assets. Measurement errors in portfolios can be substantially less than that of individual assets. To see this, suppose that there L assets and the estimated beta of each asset is $\beta_j = \beta_j + \omega_j$, where β_j is the true beta of asset j, ω_j is the measurement error. If these assets are grouped equally into a portfolio g, it can be showed that the estimated beta of portfolio g is

$$\hat{\beta}_g = \frac{1}{L}\sum_{j=1}^{L}\hat{\beta}_j = \frac{1}{L}\sum_{j=1}^{L}(\beta_j + \omega_j) = \beta_g + \frac{1}{L}\sum_{j=1}^{L}\omega_j$$

If the measurement errors of different assets are not perfectly positively correlated, they will cancel each others. And if the measurement errors of the individual assets are uncorrelated, then the variance of the measurement error for the portfolio beta is

$$Var(\hat{\beta}_g) = Var(\frac{1}{L}\sum_{j=1}^{L}\omega_j) = \frac{1}{L^2}\sum_{j=1}^{L}Var(\omega_j) = \frac{1}{L}\sigma_{\omega}^2$$

We can see that as L $\rightarrow\infty$ the variance of $\hat{\beta}_g$ will go to zero hence $\hat{\beta}_g$ will converge to the true beta. (Fama & MacBeth(1973))

Another solution to error problem is to use instrumental variables (IV). This approach require to find an instrument z_i for β_i such that it is

- uncorrelated with measurement error: $Cov(z_i, \omega_i)=0$
- uncorrelated with regression error: $Cov(z_i, \varepsilon_i)=0$
- correlated with true beta: $Cov(z_i, \beta_i)\neq 0$

Suppose again that $\hat{\beta}_i=\beta_i+\omega_i$, the IV estimator is given by:

$$\hat{\gamma}_{1(IV)} = \frac{Cov(z_i, R_i)}{Cov(z_i, \hat{\beta}_i)}$$

Under those three conditions above, this estimator is unbiased as

$$\hat{\gamma}_{1(IV)} = \frac{Cov(z_i, \gamma_0 + \gamma_1\beta_\mathrm{i} + \varepsilon_i)}{Cov(z_i, \hat{\beta}_i)}$$

$$= \frac{Cov(z_i, \gamma_0 + \gamma_1\hat{\beta}_i - \gamma_1\omega_\mathrm{i} + \varepsilon_i)}{Cov(z_i, \hat{\beta}_i)}$$

$$= \gamma_1 \frac{Cov(z_i, \hat{\beta}_i)}{Cov(z_i, \hat{\beta}_i)} = \gamma_1$$

Hence, when the sample become very large, sample covariance will converge to population covariance and our estimates will converge to the parameters. (Huang & Litzenberger (1988))

Section 5 Empirical Study

5.1 Data Selection

5.1.1 The choice of market proxy

One issue about the choice of market proxy is whether value-weight or equal-weight index should be used. Recall that the expected market return is $E(R_m) = \sum_{i=1}^{n} w_i E(R_i)$, where w_i is the weight of asset i chosen by investors. The market portfolio is a value-weighted portfolio in theory. Hence value-weight is widely used by academic studies as well as commercial information providers.

Dividends are another issue. Naturally, dividends should be included since they are part of the reward to systematic risk. However, it is hard to take dividends into account in practice since most indices only account for the capital appreciation. Fortunately, the study of Bartholdy & Peare (2005) suggests that indice with or without dividend is highly related. The using index without dividend should not be critical.

In literature, the Standard & Poors 500 (S&P 500) index and the market index of the Center for Research on Security Prices (CRSP) are frequently used as a proxy. Because CRSP index is not available, I use the S&P Super composite 1500 for instead. It is a value-weighted index and, as stated in DataStream, it is designed to replicate the performance of the U.S. equity market such that covering approximately 90% of the U.S. market. It should be appropriate to use. Specifically, the total return of this index is used. The total return index is a kind of theoretical index which assumes dividends are re-invested to purchase additional units of the stock. To get the returns, I first calculate the difference of the index between time t-1 and t, and then I divided it by the index in time t-1.

5.1.2 Time Period and Data Frequency

For estimation, the more observations the more efficient the results would be. To obtain more observations, one way is to extent the tested time period as long as possible in the first pass estimation so that more observations will be obtained. On the other hand one can increase the sampling frequency. However, the relationship of the regression is likely to change over a long period of time. Estimation would be biased if we ignore this. Moreover frequent data, like daily returns, is too noisy and would reduce the efficiency of the estimates. A good collection of data should balance these two problems. It is widely accepted that the reasonable choice is five-year monthly data. This is because beta tend to be relatively stable over five-year time span (Berndt 1991) and monthly data make sure that there are enough observations and the returns are not too noise. Hence, monthly returns ranging from January 2002 to December 2006 are used in this paper. This time period is chosen because it is highly possible that the risks of firms (beta) and risk premium changed substantially across the financial crisis of 2007.

5.1.3 The Risk-free Rate

The short term U.S. Treasury bill rates are often referred as a risk free rate. Strictly speaking, they are not entirely risk free because the uncertainty of inflation makes the real return uncertain. However, compared to others, it is still a better proxy for risk-free rate. Supports can be found from studies like Mukherji (2011). Mukherji(2011) investigates the market risks and inflation risks of Treasury securities with different maturities and finds that the short term Treasury bills(within 5year) is the only one that has no market risk and has the lowest inflation risk over different investment horizons.

Corresponding to the monthly asset return, one month Treasury bill rates should be used. However, one month rate does not exist. Hence the 3 month T-bill rate is used as a proxy. Specifically, the T-bill rates at the first day of each month from 2002 to 2006 are used. Because they are quoted at annual rates, I add each rate by 1, take the 12th root and then subtract 1 to get the monthly rates.

5.1.4 Assets

As argued in section 4, portfolios can reduce measurement error in beta and their returns are much less volatile, portfolios will be used in this paper. Due to limitation of time, I use portfolios that are already available. Because there are indexes of different economic sectors available, each index can be viewed as a weighted portfolio that contains a few stocks of firms in that economic sector. I choose 30 indexes that contains at least 10 equities and calculate their monthly return. As the same as the returns of market proxy, these returns are also calculated from total return indices. Hence they all include dividends and are monthly returns from January 2002 to December 2006.

It should be admitted that there are two potential problems of using these portfolios. First, using these portfolios may make the estimation inefficient. Because, unlike Fama & MacBeth who groups portfolio by the ranking of betas, the portfolios I used are grouped by economic sectors. Grouping by the ranking of beta guarantees that the betas of portfolios will spread in a wide range. In here it is not. The beta values of different portfolios can be similar. This will reduce the efficiency of the cross-sectional estimation. Secondly, the measurement errors may not be substantially reduced. Because stocks are grouped by sectors, it is possible that the beta measurement error of stocks in the same sector may be correlated with each other in some systematic way. If the errors are all positively correlated, then the portfolios will not give us much accurate betas either.

5.2 Tests and results

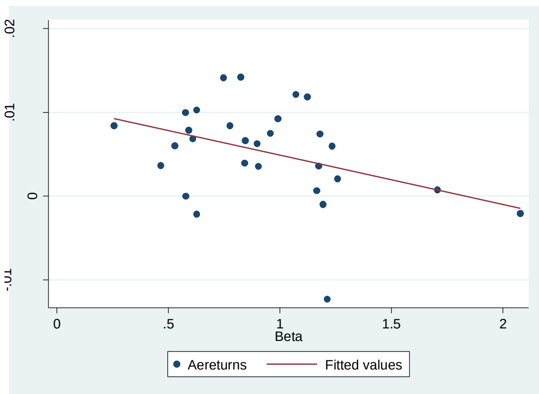
5.2.1 The two-pass test

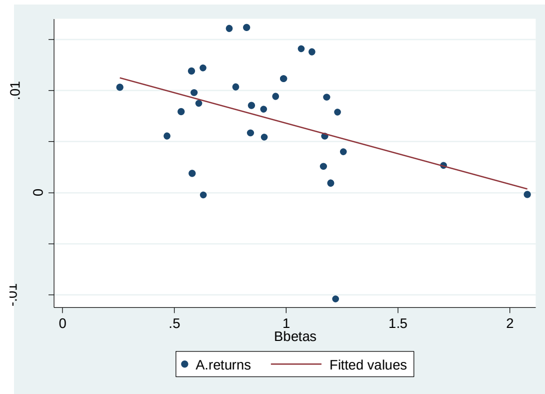
I first conducted the two pass test outlined in 4.2. For the problem of measurement error in beta, I use existing portfolios as mentioned above. For heteroskedasticity, I used robust t-statistic. Beside these two, I assume that all other assumptions hold. Hence monthly excess returns of 30 portfolios are run against monthly excess market returns (for the Black model, returns rather than excess returns are used).Thereby 30 betas for each portfolio are obtained. In the second pass, average excess returns of these portfolios from period 2002 to 2006 are computed and are regressed on their betas obtained in the first pass. We expect: 1) the coefficient of beta to be significantly positive and equal to the average of excess market return in the chosen period 2) the intercept to not be significantly different from zero (in S-L model). The result is following:

Table 5.1 Result of two-pass regression

Period 2002-2006	**γ0[standard error]**	**p-value (robust)**	**γ1[standard error]**	**p-value (robust)**	**R-square**
S-L Model	.0106 [.0026]	0.000	-.0053 [.0026]	0.038	0.1495
Black Model	.0125 [.0026]	0.000	-.0055 [.0025]	0.039	0.1573

Figure 5.1





(Lift: S-L; Right: Black)

We can see in the table that γ0 in the S-L Model is very significant. γ1 is negative and significant at 5% significant level in both models, indicating that the market premium for beta risk is negative. Figure 5.1 shows two negative relations. But we can see that the distribution is quite scattered. These results are not necessarily evidences against the model. They may due to the following potential problems:

1. The risk premium for the beta risk and the betas of firms may change substantially over the five-year period.
2. This result may be sample specific. This means that the sample period may be just a case that, for example, the market is not in equilibrium.
3. The measurement errors may be still severely affecting the estimation of γ1 because securities in the same sector may have beta-measurement-errors that are highly correlated with each other.

To tackle the first problem, I conduct a sub-period study. I divide the whole time period into five one-year periods. For each year, I calculated the average return of each portfolio and regressed these returns on their betas. Betas are still obtained by the five years time-series regression. (Betas can be obtained by running five yearly time-series regressions. This can avoid possibility that betas of firms change over time, but it will substantially reduce the number of observations available and hence cause inefficiency.)

Table 5.2 Result of two-pass regression for sub-periods

S-L Model	**γ0[standard error]**	**p-value (robust)**	**γ1[standard error]**	**p-value (robust)**	**R-square**
2002	.0005 [.0049]	0.924	-.0156 [.0047]	0.003	0.2760
2003	.0159 [.0041]	0.001	.0009 [.0035]	0.802	0.0023
2004	.0057 [.0034]	0.114	.0037 [.0031]	0.240	0.0490

2005	.0037 [.0039]	0.347	-.0050 [.0034]	0.157	0.0701
2006	.0053 [.0019]	0.007	.0010 [.0012]	0.407	0.0247

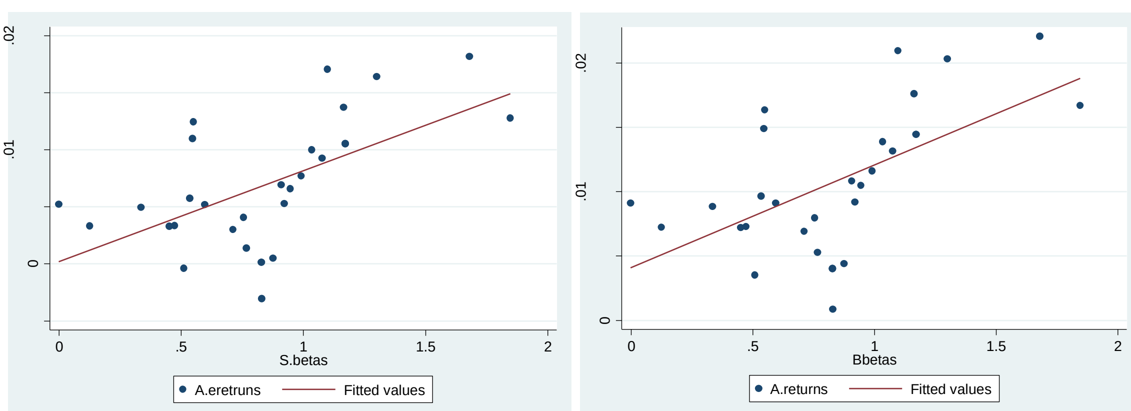
We can see that although there are positive risk premium γ1for 2003, 2004 and 2006, they are all insignificant at even 20% significant level. We failed to reject the null hypotheses that γ1 is zero. Moreover, all of their intercepts are significant. Except for period 2002, the R-squares are all very low, indicating that explanatory powers of these regressions are very poor. These tests again provide very negative results.

For the second potential problem, I collected data from 1997 to 2001 as a second sample and conduct the same two-pass test. An opposite result is found:

Table 5.3 Result of two-pass regression for periods 1997-2001

Period:1997-2001	**γ0[standard error]**	**p-value (robust)**	**γ1[standard error]**	**p-value (robust)**	**R-square**
S-L Model	.0002 [.0019]	0.915	.0080 [.0021]	0.001	0.3374
Black Model	.0041 [.0019]	0.043	.0080 [.0021]	0.001	0.3374

Figure 5.2



(Lift: S-L; Right: Black)

We can see γ1's in two models are both positive and have p-value equal to 0.001. There are significantly different from zero at 1% level. In S-L Model, γ0 has a very high p-value, indicating it is insignificant.

Since CAPM also have the predictions that:

- The coefficient of beta is the excess market return(in S-L Model)
- The relation between asset returns and betas are linear.
- Beta is a complete measure of the risk of a security. Other variable will not have explanatory power as other risks are idiosyncratic and can be diversified.

I also run the following regression.

$$R_j - R_f = \gamma_0 + \gamma_1\beta_j + \gamma_2 MV_j + \gamma_3 S_{ej} + \varepsilon_j$$

Two more variables, MV_j and S_{ej} are included in this regression. MV_j is the market value for portfolio j. It is the average of the market values of the underlying stocks in a portfolio. Following Fama and French (1992), this variable is included to test the size effect. S_{ej} is the variance of residual for portfolio j. It is estimated from the time-series regression of each portfolio in the first pass. If the CAPM holds, we expect γ_0 γ_2 γ_3 are all insignificantly different from zero; γ_1 is insignificantly different from average excess market return (or significantly positive at least). I obtained the following result:

Table 5.4 Result of two-pass regression with multivariable.

	γ_0[standard error]	**p-value (robust)**	**γ_1[standard error]**	**p-value (robust)**	**γ_2[standard error]**	**p-value (robust)**	**γ_3[standard error]**	**p-value (robust)**	**Adjusted R-square[1]**
1	.0002 [.0019]	0.915	.0079 [.0021]	0.001					0.3138
2	.0038 [.0014]	0.011			5.18e-09 [1.91e-09]	0.011			0.1802
3	.0049 [.0022]	0.035					.6166 [.6673]	0.363	-0.0051
4	-.0005 [.0019]	0.790	.0065 [.0021]	0.005	3.28e-09 [1.79e-09]	0.078			0.3671
5	.0002 [.0023]	0.923	.0079 [.0022]	0.001			-.0084 [.5886]	0.989	0.2883
6	-0.0013 [.0024]	0.568	.0059 [.0023]	0.019	3.68e-09 [1.92e-09]	0.066	.3720 [.5955]	0.538	0.3525

The first three rows are results of regressions on beta, MV, and residual variance respectively. It shows that beta and MV are both significant as the only explanatory variable at 5% significant level while residual variance is very insignificant. The effect of

MV on returns is positive which is inconsistent with general findings about the size effect. Moreover the effect of MV is not as significant as that of beta and the R-square of MV regression is also much lower than that of beta. These indicate that beta is more appropriate variable if we want to choose one as the explanatory variable.

When beta and other variables are run in one regression, we can see that beta is still significant at 5% level. MV however becomes insignificant. We can see, the p-value of MV increased to 0.078 in 4 and 0.066 in 6. Both are insignificant at 5% level. In term of residual variance, it remains insignificant in all cases. These results suggest that beta is the only explanatory variable. Moreover the intercept of these regressions also supports the CAPM predictions as it is highly insignificant when beta is included.

Furthermore, I tested whether the marginal effect of beta is equal to excess market return. The average excess market return is .0059092 during 1997-2001. T-test that using robust standard error is conducted to test H_0: γ_1= .0059092. The results show that p-values are 0.3372, 0.7570, 0.3662 and 0.9734 for regression 1, 4, 5 and 6 respectively. We can not reject the hypothesis in all cases.

Finally, I conducted a regression specification error test (RESET) to test the linear implication. I regressed the following model:

$$R_j - R_f = \gamma_0 + \gamma_1\beta_j + \gamma_2\bar{R}^2 + \gamma_3\bar{R}^3 + \varepsilon_j$$

Where $\bar{R}^2 and\ \bar{R}^3$ are the squared and cubed OLS-fitted values in (4.1). If (4.1) is correctly specified, the hypothesis that H_0: $\hat{\gamma}_2$=0, $\hat{\gamma}_3$=0 will be rejected. Hence I conducted a F test and found a F-statistic of 2.99 and a P-value of 0.0677. The null hypothesis is rejected at 5% level (although it failed to be rejected at 10%).

Regarding the third potential problem of my first test, I re-conduct the test by using the instrumental variable technique. Recalls from section 4.3.2, measurement error of different assets can be substantially reduced only if the measurement errors of assets in one portfolio are not all positively correlated. As the portfolios I used is sorted by economic sectors (each portfolio contains assets of firms that are only in that sector), it is possible that assets in a portfolio are correlated in some systematic way. If this is the case, those portfolios may still contain serious beta errors which would bias the estimates of coefficients. To tackle this problem, I use the instrumental variable technique suggested by Mankin & Shaparo (1986). Mankin & Shaparo divide the sample of T observations per portfolio into two sub-samples. One sub-samples contains the T/2 odd-month returns the other contains T/2 even-month returns. They then estimate the beta for each sub-samples and regress the average return on one beta using the other beta as an instrumental variable. Suppose the estimated odd and even betas are such that:

$$\hat{\beta}_{j,odd} = \beta_j + \omega_{odd}$$

$\hat{\beta}_{j,even}=\beta_j+\omega_{even}$.

Where the measurement error, ω, is assumed to be uncorrelated with the true beta. If we also believe ω's in different sub-samples are uncorrelated with each other (This assumption is approximately supported by the data in the study of Fama (1976)) and two estimated betas are uncorrelated with regression error then all the conditions listed in 4.3.2 are satisfied and IV technique will give us a unbiased estimators. Following by the technique of Mankin & Shaparo, I obtain the following results:

Table 5.5 Results of the IV regression

Period 2002-2006	**Instrumented [Instrument]**	**γ_{iv0} [standard error]**	**p-value (robust)**	**γ_{iv1} [standard error]**	**p-value (robust)**
S-L Model	**even [odd]**	-.0087 [.0051]	0.102	-.0083 [.0056]	0.150
S-L Model	**odd [even]**	-.0039 [.0072]	0.592	-.0122 [.0070]	0.104
Black Model	**even [odd]**	.0125 [.0032]	0.001	-.0059 [.0035]	0.109
Black Model	**odd [even]**	.0227 [.0070]	0.003	-.0150 [.0068]	0.073

The results show that in all cases the coefficients of betas are negative. And in the first three regressions, the p-values for γ_{iv1}become insignificantly different from zero at 10% level. And compare to the result in table 5.1, γ_{iv1}'s are even more negative. All in all these result fails to support the CAPM. This may be evidences against the model but it may due to the fact that this test is not efficient enough. Because two bates are estimated from sub-samples which contain only 30 monthly returns. Two estimated betas may be far away from their true values.

5.2.2 The conditional relationship test

In this sub-section, I test the conditional relationship proposed by Pettengill et al.(1995). According to Pettengill et al, the CAPM implies a conditional relation between realized assets returns and their betas: When the realized excess market return is positive there is a positive relation, when it is negative there is a negative relation. The regression used is:

$$R_{jt}= \gamma_0 + \gamma_1 * D * \beta_j + \gamma_2 * (1-D) * \beta_j + \varepsilon_j$$

Where D is a dummy variable that equal to 1 if excess market returns are positive(R_m-R_f>0), equal to 0 if excess market returns are negative(R_m-R_f<0). The β_j's are estimated by the first pass regression as in previous sub-section. R_{jt} is returns of portfolio j at period t. Hence, above relationship is estimated for each month by estimating either γ_1 or γ_2.

According to Pettengill et al, the CAPM will be supported if there are a systematic conditional relationship between betas and realized returns and a positive long run trade off relation. Therefore we expect that

1. $\gamma_1>0$ and $\gamma_2<0$
2. The excess market returns on average are significant positive for the total period
3. Two coefficients of betas are symmetric: $\gamma_1=\gamma_2$
   (2 and 3 are conditions for a positive long run trade off relation)

I conducted the regression for the whole period from 2002 to 2006 and for each year. The results are following:

Table 5.6 Result of conditional test(2002-2006)

	γ_1[standard error]	**p-value (robust)**	**γ_2[standard error]**	**p-value (robust)**	**R-square**
Total period	.0141 [.0028]	0.000	-.0382 [.0030]	0.000	0.2700
Sub-period 2002	.0222 [.0084]	0.009	-.0723 [.0082]	0.000	0.4454
Sub-period 2003	.0221 [.0065]	0.001	-.0421 [.0083]	0.000	0.2273
Sub-period 2004	.0055 [.0046]	0.232	-.0191 [.0050]	0.000	0.1240
Sub-period 2005	.0149 [.0050]	0.004	-.0227 [.0049]	0.000	0.2492
Sub-period 2006	.0092 [.0042]	0.031	-.0203 [.0049]	0.000	0.1645

The table shows that the relation between beta and realized return is positive and significant in up market, negative and significant in down market for the whole sample period at the significant level of 1%. These are consistent with the implication. For sub-period tests, all coefficients are also positive in up markets and negative in down markets. We can see that coefficients of period 2002,2003 and 2005 are also significant at 1% level, that of 2006 in up markets is significant at 5%. The only exception is the coefficient of 2004 in the down market condition. It is insignificant at even 10% level. All in all, these results meet our first expectation that $\gamma_1>0$ and $\gamma_2<0$.

To examine the average excess return, I collect all monthly excess market returns from January 2002 to December 2006 and estimate the mean excess market returns for the total period and for 5 sub-periods. By assuming that the distribution of excess market return is normal, I test whether they are different from zero by using t test.

Table 5.7 Result of testing average market excess return

	Mean Excess return	Standard Error	t-statistic	P-value
Total period	.0041	.0007	5.40	0.000
2002	-.0199	.0024	-8.04	0.000
2003	.0213	.0014	14.55	0.000
2004	.0076	.0010	6.99	0.000
2005	.00315	.0013	2.38	0.018
2006	.0082	.0008	10.07	0.000

Table 5.7 reports an average monthly excess return of 0.41%for the total period. It rejects the null hypothesis of zero excess return at 1% level. For sub-periods, average excess returns for 2003 to 2006 are positive and able to reject the null hypotheses at 5% level. The only negative result come from 2002, excess return of which is negative and significant at 1%. All in all, these results indicate that on average there is a positive reward for holding market risk. However the risk premium during sub-periods appears to be influenced greatly by the general economic conditions during the examined period.

Another condition required for a positive trade off relation in long run is a consistent relation between risk and return. That is the coefficients of beta risk in up market and down market should be symmetric. I conduct a two-population t-test to test the following hypothesis:

H_0: $\hat{\gamma}_1 + \hat{\gamma}_2 = 0$

H_1: $\hat{\gamma}_1 + \hat{\gamma}_2 \neq 0$

The result shows a F-statistic of 19.64 and a p-value of 0.000. The null hypothesis is rejected. the coefficients are not symmetric. We can see from table 6 that $\hat{\gamma}_1$ is .0141 for the whole period, while $\hat{\gamma}_2$ is -.0382. The absolute value of $\hat{\gamma}_2$is almost three time greater than $\hat{\gamma}_1$. This indicates that the loss of a asset in the down-market time is greater than its gain in the up-market time when. The positive trade off relation between risk and return is therefore not supported.

Finally, I conducted the same test for the sample in period of 1997 to 2001. I only test the period as a whole. I found the following:

Table 5.8 Result of conditional test (1997-2001)

	γ_1[standard error]	p-value (robust)	γ_2[standard error]	p-value (robust)	R-square
Total period	.0479 [.0038]	0.000	-.0376 [.0039]	0.000	0.3020

	Mean Excess return	Standard Error	t-statistic	P-value
Total period	.00590	.0012	4.91	0.000

For testing the H_0: $\hat{\gamma}_1 + \hat{\gamma}_2 = 0$, I found a F- statistic of 2.03 and a p-value of 0.1540. Hence all three conditions listed in the beginning of this sub-section are met.

To sum up, in this conditional test I found evidences support the implication of Pettengill et al. in both two periods: there is a positive relation between beta risk and asset return in up markets and a negative relation in down market. However, for whether this systematic relationship translates into a positive reward for holding risk, the results did not give a positive answer for the first period. I found that 1) on average the excess market return is positive which indicating that on average the market is an up market. 2) The gain for every unit beta risk ($\hat{\gamma}_1$) in a up market is not as great as the loss for every unit beta ($\hat{\gamma}_2$) in a down market. This result therefore failed to provide support for a long run positive trade off predicted by CAPM. However, for the period from 1997 to 2002, a positive trade-off relationship is found.

Section 6 Conclusion

The main objective of this paper is to test the validity of the CAPM in the US stock market. Two main tests are conducted: the two-stage test and the conditional test. Two samples from two time periods are used: 2002-2006 sample and 1997-2001sample. I found that in the two-stage test, the CAPM is rejected in the 2002-2006 sample. The positive relation is not observed in a simple cross-sectional regression in the whole period test as well as several sub-period tests. I used the IV technique to try to correct the possible measurement error in beta and found that the result did not improve. In the conditional test, the result from the 2002-2006 sample is also not desirable. A positive trade-off relation between risk and return is not supported. On the other hand, the 1997-2002 sample gave positive results in almost all cases. It first supported the implications of positive relation and of an intercept which equal to 0 in the simple two-stage test. It further suggested that other variables like market value and residual variance does not have explanatory power in the multi-variable test, the linear relation is correctly specified in the RESET. Moreover, in the conditional test, data from this period also support the implication of positive trade-off in long run. All in all, regarding these two different results, I conclude that the CAPM in the US stock market holds from period to period. In fact in the literature, some studies which use the same methodology and test different time periods do come to opposite results[3]. The poor result of 2002-2006 sample may because of market anomalies and the CAPM may not be wrong as the results of 1997-2001 strongly support the model. However, it should be admitted that all the tests above takes many statistical assumptions that are not discussed for grant. Therefore no clear-cut conclusion can be made.

[3] One example is the small firm effect found by Bazns (1981). Some studies, like Fama& French(1992), used the period after Bazns (1981) and found no such effect.

Bibliography

Amihud,Y. Christensen, B. J. &Mendelson, H. Further Evidence on the Risk-Return Relationship, STANFORD UNIVERSITY, RESEARCH PAPER NO. 1248

Black, F. (1993) 'Beta and Return' The journal of portfolio management 20 no.1, fall 1993

Banz , R. W. (1981).The Relationship Between Return and Market Value of Common Stocks. Journal of Financial Economics, 9: 3-18

Black,F. Jensen, M. C. and Scholes, M.(1972.) The Capital Asset Pricing Model: Some Empirical Tests. inStudies in the Theory of Capital Markets, Michael C. Jensen, ed., Praeger, New York, 79-121

Baily, R. E. (2005) The Economics of Financial Markets, Chapter 5, 6, 8, 9 Cambridge Press

Berndt, E. R. (1991) The Practice of econometrics: classic and contemporary, Addison-Wesley Publishing Company

Bartholdy , J. and Peare , P. (2001). The Relative Efficiency of Beta Estimates Available at SSRN: http://ssrn.com/abstract=263745 or http://dx.doi.org/10.2139/ssrn.263745

Bartholdy , J. and Peare , P. (2005)Estimation of expected return: CAPM vs. Fama and French. International Review of Financial Analysis, 14: pages 407-427.

Campbell, J.Y. Lo, A.W. and MacKinlay, A.C.(1997) The Econometrics of Financial Markets. Princeton University Press, Princeton, NJ.

Cochrane, J. H.(2005) Asset Pricing, Princeton University Press; Revised edition (3 Jan 2005) Chapter 9, 10,

Fama , E. F. and MacBeth (1974) 'Return, and Equilibrium: Empirical Tests', Journal of Political Economy, 71

Fama, E. F. and French, K. R. (1992). The Cross-Section of Expected Stock Returns. Journal of Finance, 4 (2): 427-465,

Fama, E. F. and French, K. R. (2004)," The Capital Asset Pricing Model: Theory and Evidence", Journal of Economic Perspectives, Vol.18, No. 3, pp. 25–46

Huang, Chi-fu , and Litzenberger, Robert H. (1988), Foundations for financial economics, Chapter 10, Prentice Hall, 1988, Jagannathan, R. and Wang, Z. (1993) ‘The CAPM is alive and well’ Available at SSRN: http://ssrn.com/abstract=5444

Jagannathan, R. and McGratten, E.R. (1995) ‘The CAPM Debate’, Quarterly Review of the Federal Reserve Bank of Minneapolis Fall, pp. 2-17.

Jahankhani, Ali; Bey, Roger P. (1978) , capital asset pricing model : some econometric tests, http://archive.org/details/capitalassetpric498jaha

Levy , H. The Capital Asset Pricing Model in the 21st Century, Cambridge University Press, 2012, ch. 7

Reinganum, M. R. (1981) A new empirical perspective on the CAPM. The journal of financial and quantitative analysis vol. 16 no.14

Lim, Kian Guan (2011) Financial valuation and econometrics, Singapore ; Hackensack, NJ : World Scientific Pub., c2011.

Mukherji, Sandip, (2011) The Capital Asset Pricing Model’s Risk-Free Rate . The International Journal of Business and Finance Research, Vol. 5, No. 2, pp. 75-83, 2011. Available at SSRN: http://ssrn.com/abstract=1876117

Magnus, D. & Sander, J. (2011) ‘The Capital Asset Pricing Model:Theory, Econometrics, and Evidence’ http://pure.au.dk/portal-asb-student/files/36147960/Bachelor_thesis.pdf

Mankiw, G. and Shapiro, M. (1986), Risk and return: Consumption beta versus market beta, Review of Economics and Statistics 47, 452-459.

Pettengill, Glenn N. Sundaram, Sridhar and Mathur, (1995) Ike The Conditional Relation between Beta and Returns. The Journal of Financial and Quantitative Analysis , Vol. 30, No. 1, pp. 101-116 Article Stable URL: http://www.jstor.org/stable/2331255

Roll , R. (1977)‘A critique of the asset pricing theory's tests Part I’ ,Journal of Financial Economics, vol. 4, issue 2, pages 129-176.

Sauer, A. and Murphy, A. (1991) An empirical comparison of alternative models of capital asset pricing in Germany, Journal of Banking and Finance 16 (1992) 183-196. North-Holland

Shanken, J. (1992) On the Estimation of Beta-Pricing Models, The Review of Financial Studies 1992 Volume 5, number 1, pp. 1-33

Stambaugh, R. F. (1982), On the Exclusion of Assets from Tests of the Two-Parameter Model: A Sensitivity Analysis, *Journal of Financial Economics*, 237-268

Wooldridge, J. M. (2009) Introductory econometrics : a modern approach, 4th ed, Mason, OH : South-Western Pub, 2009